HIGHER ARITHMETIC

BY

JOHN C. STONE, A.M.

HEAD OF THE DEPARTMENT OF MATHEMATICS, STATE NORMAL SCHOOL MONTCLAIR, NEW JERSEY, CO-AUTHOR OF THE SOUTHWORTH-STONE ARITHMETICS, AND THE STONE-MILLIS ARITHMETICS ALGEBRAS, AND GEOMETRIES

AND

JAMES F. MILLIS, A.M.

HEAD OF THE DEPARTMENT OF MATHEMATICS, FRANCIS W. PARKER SCHOOL CHICAGO, CO-AUTHOR OF THE STONE-MILLIS ARITHMETICS ALGEBRAS, AND GEOMETRIES

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PREFACE

This book has been prepared for use in high schools and normal schools. The aim of the authors has been to furnish material that will develop skill in computation and the power to see and interpret the quantitative relations that arise in modern every-day life, and that will give a knowledge of modern business and industrial practices. Since the problems of any vocation are based upon the fundamental principles of arithmetic but vary somewhat in the different vocations, the authors have sought to lay a solid foundation for the arithmetic work involved in all vocations, rather than to prepare for any particular one. They hold that by obtaining a thorough grounding in the fundamental principles of arithmetic, by mastery of rapid and accurate computation, and by learning to interpret quantitative relations, one is much better fitted to make satisfactory progress in any kind of arithmetical work than if he had been made a mere automaton in some particular type of such work.

Efficiency in computation requires: (1) a quick and accurate mental response to the fundamental facts and processes; and (2) the ability to see number relations that will save figures and even whole processes. Both of these phases have been provided for in the text. The first three chapters give abundant drill in operations with whole numbers, fractions, and decimals. Chapter IV on "Short Methods of Computation" should cultivate the ability needed in the second requirement for efficiency in computation.

Most of the problems of the text are made from real conditions, are true to present-day customs, and thus contribute to social insight into business and industrial practices. Problems of this

type develop power to interpret references to business and industrial matters which are constantly met in general reading and in social and business intercourse.

A number of problems of another type, involving concrete situations familiar to the student, but dealing with indirect and hypothetical cases, are given in order to develop greater power to express and interpret the quantitative relations which exist among the magnitudes involved.

The authors gratefully acknowledge their indebtedness to a large number of business men who have so kindly given their time to a discussion of the practices in their particular vocations as well as to many teachers of secondary schools and normal schools who have made many helpful suggestions regarding the suitableness of the work for their students. Among those to whom they wish to acknowledge their especial indebtedness are: Mr. A. M. Baumgart, head accountant of L. Bamberger & Co., Newark, N. J.; Mr. F. L. Armstrong, Superintendent of Insurance Rating, State of New Jersey; Mr. H. N. Sheppard, actuary, Home Life Insurance Co., New York; Mr. Alfred T. Gibbs, cashier, First National Bank, Montclair, N. J.; Mr. C. R. Leach, of Millett, Roe & Hazen, New York Stock Exchange; and to Mr. Montgomery Rollins for kindly allowing the use of a page from his "Table of Bond Values" (page 259).

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JOHN C. STONE,
JAMES F. MILLIS.

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HIGHER ARITHMETIC

CHAPTER I

WHOLE NUMBERS

READING AND WRITING NUMBERS

1. The origin of number. — The concept of number must have originated with the ability to distinguish one thing from another. The idea of number, then, must have existed long before a symbolism by which to express it.

Number answers the question "How many", found by counting, and the question "How much", found by measuring.

2. Symbols by which number is expressed. — The early characters or symbols used to express number ideas must have been as varied as the tribes or nations using them. But the first system of characters used by any tribe or nation must have been some kind of *one-to-one-correspondence* system. That is, each individual thing in a group must have been represented by some sign or symbol. Thus, in the Roman system, three things are represented by three distinct signs (III), one sign for each thing represented.

3. One-to-one-correspondence systems. — In any of the one-to-one-correspondence systems, characters are used to represent one, ten, and powers of ten. In the Roman system, characters are also used to represent five, fifty, and five hundred. From these symbols other numbers are represented

by repeating the symbols whose values are added. This is called the *additive* principle. Some nations also used the ***subtractive*** and *multiplicative* principles.

In general, it may be said that the Roman system is representative of all of the older systems.

4. The Roman notation. — In the Roman notation,

I represents one. L represents fifty.
V represents five. C represents one hundred.
X represents ten. D represents five hundred.
M represents one thousand.

The Roman notation is of but little practical value to us. Its use is limited to recording dates on public buildings, to numbering the chapters of a book, to the characters on a clock face, etc. In this system, numbers are represented by combining the seven symbols or letters according to the following principles:

1. *The value of a letter when written after one of equal or greater value is added to the value of that letter.*

Thus, $XXV = 10 + 10 + 5 = 25$; $CCLX = 100 + 100 + 50 + 10 = 260$.

2. *The value of a letter when written before one of greater value is subtracted from the value of that letter.*

Thus, $IV = 5 - 1 = 4$; $XL = 50 - 10 = 40$; $CD = 500 - 100 = 400$.

NOTE. — When a letter in the Roman notation stands between two letters representing greater values, its value is always subtracted from the value represented by the letter at its right instead of being added to that at its left. Thus, $XIV = 10 + (5 - 1)$ and not $10 + 1 + 5$.

3. *A horizontal bar over a letter multiplies its value by one thousand.*

Thus, $\overline{X} = 1000 \times 10 = 10{,}000$; $\overline{D} = 1000 \times 500 = 500{,}000$.

5. The defects of the early systems. — It is easily seen that any kind of one-to-one-correspondence system serves merely as a means of representing or recording numbers, and that such a system is not fitted for the purpose of computation. With such a system, all computation must be some form of counting. The counting is done on some form of counting frame or *abacus*. The *swanpan*, used by the Chinese to-day, is an example.

6. The Hindu notation. — We use at present a *place-value* notation, which originated with the Hindus. This system is made possible by having a single character to represent each of the numbers below ten, the base of the system, and by having a character, *zero*, to show the absence of value. The unique feature of the system is that each character, or **digit**, used represents two values, one depending upon its *form* and the other upon its *position*.

Since the value of a digit in any order (*i.e. place* in the number) is ten times the value it would have in the next order (or place) to the right, the system is called a **decimal system**, from the Latin word *decem*, meaning *ten*.

7. The advantages of the Hindu system. — The *place-value* feature of the Hindu notation makes possible the methods of computation that we use, by allowing a number to be broken into parts and dealt with, part at a time. Thus, by knowing the addition facts (sums) through $9 + 9$, or the multiplication facts (products) through 9×9, the sums or products of numbers of any size may be found without counting.

8. Reading numbers in the Hindu notation. — For convenience in reading, the number expression to be read is usually marked off by commas into **periods** of three figures each, beginning, in the whole number, at the right. Each period is read as if it stood alone, and then the name of the period

is given, except in reading the ones' period. That is, in reading 3465, one does not say "3 thousand 465 ones", but "3 thousand 465".

The names of the first six periods, reading from right to left, are:

ones, *thousands*, *millions*, *billions*, *trillions*, and *quadrillions*.

The word "and" should not be used in reading a whole number. Thus, 17,843,708 should be read "seventeen million, eight hundred forty-three thousand, seven hundred eight".

EXERCISES

Write in the Hindu notation:

1. XXVII.	**5.** XCIX.	**9.** MDCCXXV.
2. XXXIV.	**6.** CCXIV.	**10.** MMMCDXXXIII.
3. CXXVI.	**7.** CDXLV.	**11.** MMDCCCLXVII.
4. XLVII.	**8.** MDCLXIV.	**12.** MMCDLXXIV.

Write in the Roman notation:

13. 35.	**17.** 249.	**21.** 1739.	**25.** 10,527.
14. 47.	**18.** 567.	**22.** 2424.	**26.** 32,305.
15. 56.	**19.** 934.	**23.** 3762.	**27.** 27,624.
16. 94.	**20.** 763.	**24.** 1308.	**28.** 19,139.

Read the following:

29. 35,250.	**34.** 800,080.	**39.** 3,465,832,426.
30. 40,807.	**35.** 106,009.	**40.** 5,072,720,805.
31. 39,046.	**36.** 607,201.	**41.** 17,080,906,085.
32. 30,946.	**37.** 5,607,003.	**42.** 1,705,090,800,017.
33. 190,007.	**38.** 7,085,240.	**43.** 7,075,301,096,705.

44. Write from dictation the numbers from Exercise 29 to Exercise 43, inclusive.

45. Write and read a number of five periods. One of six periods.

46. Find in your reading, in or out of school, numbers expressed in thousands; in millions; in billions.

47. Can you find any use for trillions or quadrillions?

ADDITION

9. **Meaning of addition.** — **Addition** is the process of finding, without counting, a number equal in value to two or more other numbers. The two or more numbers which are added are called the **addends.** The number equal in value to all the addends is called the **sum.**

10. **The forty-five primary facts of addition.** — There are only forty-five possible combinations of the nine one-figured numbers taken two at a time. The sum of these forty-five combinations must be found through **counting,** but, by memorizing these sums, the sums of all other numbers may be found by **addition.** Therefore these forty-five sums are called the **forty-five primary facts of addition.**

A DRILL TABLE UPON THE PRIMARY COMBINATIONS

	A	B	C	D	E	F	G	H	I
1.	2 8	4 5	2 9	2 3	1 7	3 3	6 9	1 3	2 5
2.	3 9	1 1	5 5	4 6	3 7	5 6	1 9	8 9	4 8
3.	2 2	5 7	1 6	7 7	2 4	1 2	6 8	3 6	7 9
4.	5 9	3 8	6 6	3 4	5 8	4 7	2 6	8 8	1 4
5.	1 8	4 4	2 7	6 7	1 5	3 5	7 8	4 9	9 9

11. Derived facts needed in addition. — There are certain facts easily derived from the forty-five primary facts that are essential to rapid work in addition. Thus, to add the column in the margin, adding down, $9 + 8$ is the only use made of the "primary facts". The next combination is $17 + 6$, the next is $23 + 5$, and the next is $28 + 7$. While all of these are easily derived from the "primary facts", such combinations need to be drilled upon until the sums can be called automatically. The following are types of the derived facts that need special attention:

$$\begin{array}{r} 9 \\ 8 \\ 6 \\ 5 \\ \underline{7} \end{array}$$

7 6	17 6	47 6	37 6	67 6	87 6	57 6

Since 6 added to any number ending in 7 gives a sum ending in 3, such drills are called "Adding by endings".

A DRILL TABLE UPON ADDING BY ENDINGS (I)

A	3 6	13 6	23 6	53 6	43 6	73 6	63 6	93 6	33 6
B	2 6	12 6	42 6	32 6	72 6	52 6	62 6	22 6	82 6
C	4 5	14 5	44 5	74 5	94 5	64 5	84 5	24 5	54 5
D	5 3	15 3	55 3	75 3	25 3	85 3	95 3	45 3	65 3
E	3 4	13 4	43 4	73 4	63 4	23 4	93 4	33 4	83 4

Note.—Such drills should be made and used frequently for all of the forty-five combinations. The following table contains some of the facts derived from those whose sums exceed 10.

A DRILL TABLE UPON ADDING BY ENDINGS (II)

A	5 6	15 6	25 6	45 6	75 6	95 6	65 6	85 6	55 6	35 6
B	7 8	17 8	97 8	77 8	27 8	57 8	87 8	37 8	67 8	47 8
C	9 6	19 6	59 6	29 6	69 6	99 6	39 6	79 6	89 6	49 6
D	4 9	14 9	24 9	54 9	34 9	64 9	44 9	74 9	94 9	84 9
E	6 7	16 7	46 7	36 7	56 7	76 7	26 7	66 7	96 7	86 7
F	8 7	18 7	28 7	48 7	38 7	58 7	88 7	98 7	78 7	68 7
G	8 9	18 9	38 9	28 9	48 9	88 9	58 9	78 9	68 9	98 9
H	7 5	17 5	27 5	37 5	97 5	47 5	57 5	77 5	67 5	87 5

12. Adding single columns. — Since all addition depends upon adding single columns, daily practice upon such tables as those given below is very important.

	A	B	C	D	E	F	G	H	I	J	K	L	M
I	9	7	3	5	6	4	2	3	7	6	6	4	5
	2	8	6	6	9	7	9	8	7	4	7	8	9
	6	6	9	7	8	6	9	7	6	8	3	7	7
	8	9	8	9	3	9	6	6	9	9	4	9	6
	4	4	4	8	7	8	8	9	8	3	9	6	4
	3	3	7	4	2	4	4	3	1	2	6	1	7
	2	2	3	3	4	2	3	2	7	4	8	7	8
II	8	3	5	7	9	1	7	8	6	9	3	4	2
	9	7	7	6	7	4	2	3	9	6	8	3	5
	2	9	6	3	6	6	1	6	9	8	4	7	6
	4	6	3	4	8	9	9	3	2	8	6	7	5
	6	8	9	9	4	8	2	9	6	3	8	4	5
	7	4	7	8	7	4	7	8	9	6	7	3	4
	8	6	6	2	2	7	8	8	3	9	3	6	7
	9	7	8	7	7	6	8	2	1	7	6	9	3
	3	8	4	2	6	5	7	4	6	8	2	4	3
	2	9	3	6	8	2	8	7	6	3	8	9	7
	1	3	2	5	4	6	6	8	3	4	9	6	3
	4	7	6	4	9	9	7	6	8	4	7	6	3
	6	6	7	3	1	8	8	9	9	9	8	7	4
	7	9	9	2	3	4	3	9	6	8	7	4	2

Begin at the top and add down :

	1	2	3	4	5	6	7	8	9	10
A	54	46	59	34	87	52	76	26	64	94
	7	8	3	9	6	7	5	7	9	8
	6	9	6	6	9	6	4	4	7	7
	8	7	8	8	3	9	9	8	8	6
	4	6	4	4	8	8	8	3	4	4
	2	3	7	3	7	3	7	2	3	3
	3	4	2	2	4	4	2	7	6	5
B	89	58	47	43	29	49	57	88	72	96
	9	7	8	9	7	6	6	7	5	7
	6	9	6	6	6	8	9	6	9	9
	4	4	2	8	9	7	8	5	6	8
	7	6	3	8	8	3	7	9	8	5
	3	3	4	7	4	4	4	8	4	7
	2	2	9	9	5	2	5	6	2	4
C	76	84	49	53	74	69	85	96	88	79
	8	7	3	7	8	9	9	8	3	2
	8	6	6	9	9	3	9	7	8	8
	6	9	9	8	6	7	6	6	9	6
	4	8	8	6	5	6	8	9	6	8
	9	4	7	7	4	9	3	8	2	4
	6	3	2	9	9	3	5	4	8	3

13. Checking the work in addition. — All computation in arithmetic should be carefully checked. In practical life one always checks his work, no matter how expert he is in computation.

There are two general methods of checking addition :

(1) *Adding a second time in reverse order.*
(2) *Casting out the nines.*

The first method is the one most used by accountants. They form the habit of first adding from bottom up, then from top down. Where the columns are long the sum of each column is recorded as shown below.

CHECK		1ST
	3568	
49	2974	51
42	9362	53
53	4736	42
51	9265	49
53781	7889	53781
	6287	
	5239	
	4461	
	53781	

EXPLANATION. — Beginning at the right, the sums are found and are recorded as shown. Then the second addition usually begins at the left, and the sums are recorded as shown in the "check". If the sums of any two columns do not correspond, the error is found and corrected.

14. Casting out the nines. — It will be shown in § 37, that the remainder arising when dividing a number by 9 is the same as the remainder arising when dividing the sum of the digits by 9. Thus, $34,689 \div 9$ will give the same remainder as that obtained from $(3 + 4 + 6 + 8 + 9) \div 9$. This remainder is called the **excess of nines.**

The check depends upon the fact that

The excess of nines in the sum equals the excess in the sum of the excesses from the addends.

CHECK	
346	4
258	6
793	1
487	1
1884	3

$3 + 6 = 9$, so 4 is the excess of the first addend. $2 + 5 + 8 = 9 + 6$, so 6 is the excess of the second addend. $9+3+7=9+9+1$, so 1 is the excess of the third addend. $4 + 8 + 7 = 9 + 9 + 1$, so 1 is the excess of the fourth addend. In the sum, $1 + 8 + 8 + 4 = 9 + 9 + 3$, so 3 is the excess. But $4 + 6 + 1 + 1 = 3 + 9$, so 3 is also the excess of the sum of the excesses. Hence we conclude that the addition is accurate.

Note. — Checking by casting out the nines will not detect a transposition of figures nor any error not affecting the sum of the digits.

EXERCISES

Add and check by first method:

1.	2.	3.	4.	5.
3468	5763	8463	9684	7463
9763	8421	7961	1863	8765
7864	7963	8476	2679	1969
9698	8476	7463	8465	2847
1769	7961	7698	5766	3256
3784	8437	1846	8434	8396
6799	6792	6989	4695	7342
7369	8463	3462	5768	1988
8463	7849	8754	8116	2864
7968	8172	7621	1769	7648

Add and check by second method:

6.	7.	8.	9.	10.
6534	4362	5763	3465	3465
8765	8461	4611	2846	7281
9798	1789	7651	9762	4563
1467	6754	9875	8846	8712
5767	1768	6381	9381	9630
4381	7235	5763	4268	8145
4365	1763	4565	4356	6435
7644	7845	7692	9630	9468

Add and check by either method:

11.	12.	13.	14.	15.
8496	1678	5964	7846	7846
7964	9634	8538	1963	2463
8763	7684	7634	6849	3452
9784	4391	3896	4271	8462
4362	7684	7842	6796	7684
8765	4896	9768	4354	3998
9636	7836	7467	4368	4287
6481	4638	3896	7462	6784
4276	5476	7469	9746	3568
5181	9367	4678	4381	7624

Write in columns from dictation and add:

16. 3465, 8476, 7398, 4621, 3824, 9763, 8421, 7312

17. 5681, 3796, 8437, 6405, 9371, 6280, 7439, 9842

18. 7856, 2147, 6387, 9048, 3609, 8476, 3291, 8642

19. 4962, 8251, 6374, 3296, 8741, 3438, 9247, 1386

20. 5632, 7682, 7391, 2658, 1946, 2847, 1398, 9076

21. 8642, 3128, 7641, 9782, 8397, 1148, 3081, 7642

22. 4291, 2687, 9346, 8241, 3968, 1984, 7631, 5296

23. 5661, 7219, 8346, 9478, 3219, 5285, 6372, 9218

24. 4296, 8432, 7216, 8431, 1968, 3242, 7681, 9763

25. 5139, 1887, 9346, 8873, 7685, 9241, 3892, 7687

26. 3896, 9248, 7631, 2928, 3271, 8640, 9307, 6413

27. 5271, 4682, 3946, 8575, 4696, 6941, 7387, 5296

28. 6897, 9246, 8341, 1796, 8243, 6894, 7395, 8642

29. 8296, 7641, 1763, 2941, 8762, 9143, 8349, 7687

Test your accuracy by adding these long columns:

30.	31.	32.	33.
346,921	576,357	978,468	854,981
765,314	824,926	752,947	857,643
896,787	954,684	976,843	927,645
765,982	463,759	243,786	397,962
448,539	456,968	746,982	547,763
896,764	438,976	432,398	436,847
468,975	843,768	459,643	847,928
576,435	842,967	848,764	849,369
241,842	365,968	426,847	358,763
842,936	748,346	584,368	927,846
396,854	962,141	138,974	269,843
869,749	276,848	976,467	348,968
346,847	768,943	348,765	854,729
358,696	846,346	569,847	342,968
963,247	768,924	736,192	548,367
246,857	639,546	574,628	396,849
796,843	796,543	846,538	395,687
384,962	983,175	183,978	986,315
542,684	398,574	361,892	968,453
769,746	769,847	983,129	186,329
784,964	968,219	183,467	782,198
198,246	843,962	853,962	347,846
576,981	463,249	968,764	962,459
769,877	176,195	243,846	736,249
734,561	168,237	196,275	824,396
846,392	187,462	875,936	274,847
538,241	489,645	546,875	198,276
193,842	786,938	628,649	765,984
764,963	769,534	146,768	395,647
967,348	738,962	597,649	828,327
769,384	962,738	549,697	278,832

15. Horizontal addition. — In some kinds of accounting it is necessary to add numbers that are written in horizontal lines. By practice it will be found as easy to add numbers written thus as to add numbers written in columns. Care must be taken that only units of like orders are added.

EXERCISES

Add horizontally:

1. 3, 4, 9, 6, 8, 1, 7, 9, 4, 3, 6, 9, 8, 4, 3.
2. 5, 6, 3, 9, 7, 6, 1, 8, 3, 9, 7, 9, 3, 6, 5.
3. 7, 3, 6, 9, 6, 5, 3, 8, 9, 7, 3, 3, 9, 6, 3.
4. 9, 3, 3, 8, 3, 9, 6, 3, 8, 7, 6, 9, 4, 6, 8.
5. 7, 2, 6, 3, 9, 8, 4, 9, 6, 3, 9, 8, 4, 6, 7.
6. 38, 46, 93, 47, 86, 42, 83, 96, 74, 54, 63, 78.
7. 54, 53, 86, 52, 47, 67, 96, 84, 76, 53, 46, 86.
8. 67, 78, 94, 36, 87, 54, 96, 83, 72, 19, 48, 57.
9. 39, 53, 67, 54, 89, 73, 86, 96, 84, 47, 26, 39.
10. 47, 16, 83, 43, 96, 54, 72, 84, 38, 98, 53, 48.
11. 376, 296, 781, 342, 768, 319, 742, 863, 974, 848.
12. 359, 873, 968, 764, 329, 346, 892, 519, 463, 743.
13. 932, 736, 392, 473, 856, 398, 726, 968, 842, 961.
14. 748, 769, 938, 962, 725, 986, 395, 748, 962, 398.
15. 473, 962, 378, 541, 316, 824, 396, 782, 354, 729.
16. 37, 168, 97, 375, 265, 8465, 9768, 934.
17. 386, 947, 38, 75, 235, 768, 1965, 3785.
18. 368, 1763, 965, 175, 38, 9652, 738, 165.
19. 246, 782, 963, 847, 3245, 7854, 96, 84, 16.
20. 175, 368, 92, 17, 565, 3875, 2967, 345.

21. Add by columns and by lines, and check by adding totals:

	a	*b*	*c*	*d*	*e*	*f*	Totals
A	4653	9871	4392	8546	7894	3563	
B	7396	8439	9639	7694	8936	7349	
C	9628	4576	7392	8675	9283	4696	
D	8765	9781	5386	7642	8476	9757	
E	7396	3846	7961	8643	7392	8451	
F	6732	9751	8348	1936	2438	7623	
G	5896	7634	4628	3796	8429	3846	
Totals							

22. Complete the following sales sheet:

Clerk	Monday		Tuesday		Wednesday		Thursday		Friday		Saturday		Total for Week	
E. D. Brown	763	98	359	46	586	25	763	98	642	18	786	42		
R. F. Clark	689	17	736	51	642	96	837	42	796	84	558	62		
G. E. Cole	563	84	561	96	785	19	689	53	682	42	976	85		
A. E. Day	762	95	684	52	546	18	298	56	796	51	843	65		
H. A. Davis	642	72	564	74	618	32	465	43	621	42	706	54		
L. M. Dole	901	51	863	28	517	30	925	16	784	21	428	12		
R. G. Fales	819	28	796	13	821	40	395	62	589	17	612	86		
E. L. Hart	728	13	821	14	693	47	506	86	619	42	916	14		
M. E. Moore	629	17	763	98	726	42	736	14	897	42	369	28		
L. E. Smith	810	92	795	16	693	56	791	42	816	57	892	16		
A. R. Sloan	796	18	689	53	812	42	862	14	961	48	792	65		
B. E. Stone	697	52	761	87	689	75	916	42	798	94	963	87		
E. L. Young	729	58	819	14	762	78	879	65	814	32	897	34		
Daily Totals														

SUBTRACTION

16. Meaning of subtraction. — **Subtraction** is the inverse of addition. That is, the sum of two addends, and one of the two addends are given, and the other addend is to be found. The given sum is called the **minuend**. The given addend is called the **subtrahend**. The addend found is called the **remainder** or **difference**.

Subtraction embodies three ideas: (1) the *remainder idea*, (2) the *difference idea*, and (3) the *addition idea*. The three following questions illustrate these ideas:

1. If 5 pencils are taken from 8 pencils, how many remain? (*i.e.* What is the *remainder?*)

2. If there are 5 pencils in one box and 8 pencils in another, how many more are there in the second than in the first? (*i.e.* What is the *difference?*)

3. If you have 5 pencils, how many more must you get to have 8 pencils? (*i.e.* How many must be *added?*)

Tell which idea is embodied in the following:

1. A farmer raised 240 bu. of potatoes and sold 208 bushels of them. How many bushels did he keep?

2. John weighs 115 lb. and James weighs 127 lb. Who weighs more and how much?

17. The primary facts of subtraction. — Since subtraction is the inverse of addition, the *primary number facts* of subtraction are known from those of addition (see § 10). To every combination in addition, except the doubles, there are two subtraction facts, and to each of the nine doubles there is one subtraction fact. Thus, to $8+5=13$, there are $13-5=8$, and $13-8=5$; and to $8+8=16$ there is $16-8=8$. Therefore there are in all eighty-one subtraction facts.

A DRILL TABLE UPON THE PRIMARY SUBTRACTION FACTS

	A	B	C	D	E	F	G	H	I
1.	5 1	6 4	7 1	7 2	7 4	6 2	8 3	5 2	8 1
2.	6 5	4 2	7 3	6 3	5 3	8 2	3 2	17 9	4 3
3.	9 4	8 6	10 1	10 8	9 1	9 6	10 6	8 7	11 2
4.	10 9	14 5	9 5	13 6	10 7	14 7	11 7	13 4	12 4
5.	12 7	9 8	12 5	11 5	13 5	12 3	9 7	13 7	11 4
6.	11 6	12 6	14 6	16 7	10 5	15 9	11 8	12 8	10 4
7.	8 5	18 9	10 2	9 3	2 1	7 5	6 1	13 9	15 6
8.	3 1	8 4	7 6	9 2	11 3	5 4	17 8	12 9	16 4
9.	14 8	15 8	4 1	10 3	14 9	15 7	16 9	13 8	11 9

18. Methods of subtraction. — There are two general methods of subtraction now in use. The first method shown below, called the **Austrian** or **Addition method,** is rapidly coming into general use.

$$\begin{array}{r} 932,637 \\ \underline{156,392} \\ 776,245 \end{array}$$

FIRST METHOD. — In this method we think 2 and **5** are 7 (writing **5**); 9 and **4** are 13 (writing **4**); 1 (carried from 13, the sum of 4 and 9) and 3 and **2** are 6 (writing **2**); 6 and **6** are 12 (writing **6**); etc.

SECOND METHOD. — In this method we think 2 from 7 leaves **5** (writing **5**); 9 from 13 (using 1 of the 6 in column 3, which makes 10 in column 2) leaves **4** (writing **4**); 3 from 5 (1 of the 6 has been taken) leaves **2** (writing **2**); 6 from 12 (using 1 of the 3 in column 5, which makes 10 in column 4) leaves **6** (writing **6**); etc.

EXERCISES

Subtract quickly:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
98	92	81	45	92	86	83	74	71	86
46	57	36	28	78	47	58	36	49	49

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
75	63	42	63	84	40	38	47	84	59
48	48	19	17	28	26	19	26	43	26

21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
53	64	32	57	84	92	41	74	86	45
17	28	17	29	36	78	16	35	27	19

31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
66	34	44	31	42	61	43	52	64	48
49	18	19	17	29	17	18	17	25	19

41.	42.	43.	44.	45.	46.	47.	48.	49.	50.
301	602	706	805	906	805	701	601	505	501
187	158	149	367	458	766	647	487	386	185

51.	52.	53.	54.	55.	56.	57.	58.	59.
602	703	806	603	5001	4005	5007	6008	7006
285	287	579	196	1683	2867	4689	5689	2987

60.	61.	62.	63.	64.	65.	66.	67.
8001	7001	65000	83000	91111	76000	43000	63000
4698	5696	17968	29396	18465	29789	16798	25768

19. Checking subtraction. — To check subtraction, add the remainder or difference to the subtrahend, and see if the sum equals the minuend. In checking subtraction do not rewrite the numbers, but check as they stand.

For example, in the first exercise below, after writing down the remainder, add it mentally to the subtrahend, watching to see that the sum is 3,465,401.

EXERCISES

Subtract and check:

1.	2.	3.	4.
3,465,401	5,701,105	7,096,801	3,021,708
1,796,847	1,963,748	1,798,936	1,792,849

5.	6.	7.	8.
9,060,781	7,630,500	8,340,096	7,096,381
5,378,964	4,784,096	3,496,897	4,843,968

9.	10.	11.	12.
14,061,708	28,036,502	32,804,106	40,630,907
9,473,962	19,843,965	17,963,849	28,764,938

13.	14.	15.	16.
50,063,017	60,108,720	81,042,001	72,047,005
28,769,843	19,349,853	26,865,392	27,396,248

Test your accuracy in subtraction by the following:

17.

306,097,135,026,713,901
178,189,743,153,169,284

18.

716,008,103,916,087,103
157,139,846,279,843,976

19.

910,034,706,172,906,305
179,847,166,785,132,561

20.

810,030,170,605,710,300
196,056,429,867,809,437

20. Subtracting by adding arithmetical complements. — The **arithmetical complement** of a number is the difference between the number and the power of ten next larger than the number. Thus 3 is the complement of 7 since $10-7=3$; 17 is the complement of 83 since $100-83=17$; etc.

Since
$$85-36=85+100-100-36$$
$$=85+(100-36)-100=85+64-100,$$

it is evident that we may subtract a number by adding its complement if we drop from the sum the power of 10 from which the complement was obtained.

This principle makes it simple to subtract one number from the sum of several others by a single process of addition, as in balancing the following:

Balance	Checks Paid	Deposits	Balance
396.40	283.20	342.96	456.16
265.73	192.84	162.37	235.26

Explanation. — Adding from right to left we have in the first line the sum of the ones is **6**. Adding tens, 9, 17 (from adding to 9, 8 the complement of 2), 21; carrying 1 (1 of the 2 tens in 21 was dropped), and adding hundreds, we have 3, 10, **16**; adding thousands (the ten in 16 was dropped) 4, 6, **15**; adding ten-thousands we have 3, 11, **14**. We write down, after adding each order, the figures in full-faced type.

In the second line we have 7, 13, **16**; 3, 5, **12**; 2, 10, **15**; 6, 7, **13**; 1, 10, **12**.

EXERCISES

Find the balance in each line using arithmetical complements.

	Balance	Checks Paid	Deposits	Balance
1.	$346.81	$297.68	$143.82	. . .
2.	576.43	438.63	346.25	. . .
3.	964.27	724.31	287.38	. . .
4.	348.93	196.38	395.50	. . .
5.	964.22	896.34	483.96	. . .
6.	314.96	125.38	465.80	. . .
7.	792.04	96.42	345.21	. . .
8.	342.84	85.63	236.95	. . .
9.	938.14	125.46	95.60	. . .
10.	890.42	735.19	483.42	. . .
11.	765.48	98.18	395.62	. . .
12.	342.96	97.42	196.84	. . .
13.	792.05	346.81	560.80	. . .
14.	309.17	85.80	190.80	. . .
15.	806.24	190.60	265.85	. . .
16.	973.61	385.42	463.73	. . .
17.	843.42	97.86	196.85	. . .
18.	763.96	119.43	234.25	. . .
19.	834.42	380.40	198.17	. . .
20.	765.25	195.98	380.42	. . .
21.	596.40	372.93	431.28	. . .
22.	807.28	129.46	309.46	. . .
23.	910.42	392.09	176.48	. . .
24.	792.07	209.86	567.56	. . .
25.	819.37	416.98	305.76	. . .

APPLICATIONS OF ADDITION AND SUBTRACTION IN BALANCING ACCOUNTS

21. An account.—An **account** is a record of **debits** (a record of debt or an expression of value received) and **credits** (record of value delivered) between persons having business transactions, or a record of the receipts and expenditures of a particular class of transactions. Among the most common accounts in business are *personal accounts; cash accounts; merchandise accounts; and expense accounts.*

22. To balance an account. — The difference between the sums of the debits and credits of an account is called the **balance.** *To balance an account* is to enter the balance on the lesser side, thus making the total footings of the two sides equal.

The following illustration shows the form of a ledger account between E. L. Holmes & Co., merchants, and Robert L. Ray, debtor:

ROBERT L. RAY

Dr. | *Cr.*

1916					1916				
Jan.	2	Sales	$28	00	Jan.	6	Cash	$20	00
"	7	"	36	00	"	15	"	35	00
"	10	Cash loan	10	00	Feb.	15	"	20	00
"	15	Sales	32	50	Mar.	1	Balance	92	50
Feb.	12	"	18	75					
"	14	"	42	25					
			167	50				167	50

1. Who has bought goods as shown by the above account? Of whom has he bought them?

2. What do the "sales" items of the debit side show? What does the "cash loan" item of the debit side show?

3. What do the "cash" items of the credit side show?

4. What does the "balance" on the credit side show?

5. What would a "balance" on the debit side have meant?

EXERCISES

Arrange each of the following accounts in the form of the account with Robert L. Ray, as shown above. Balance each one, and tell what each balance indicates.

1. John V. Farwell & Co., in account with B. L. Gray: Sales, June 1, brussels carpet, $28; June 8, black silk, $47.84, and Wamsutta cotton, $4.28; June 25, cotton, $7.96, and silk, $40.12. Credits, June 15, cash, $45; June 20, goods returned, $23; June 25, cash, $20. Balance the account, July 1.

2. R. C. Randall & Co., in account with W. R. Brooks: Sales, $38.40; $96.23; $84.68; $34.50; $26.84; $126.18; $342.96. Credits by cash and goods returned, $120; $26.84; $90; $38.50; $84.20. (Supply dates.)

3. Miller Bros., in account with Wm. R. West: Sales, $84.60; $96.80; $64.98; $28.43; $96.84; $38.26; $96.24. Credits by cash and goods returned, $100; $34.60; $75; $26.80. (Supply dates.)

4. J. R. West & Co., in account with James L. Day: Sales, $98.02; $75.38; $42.36; $50.78; $42.05; $57.16; $63.25; $92.14. Credits by cash and goods returned, $200; $46.25; $81.46; $175. (Supply dates.)

5. Simpson & Co., in account with T. L. Hayes: Sales, $196.75; $340.82; $98.75; $401.50; $346.80; $109.20. Credits by cash and goods returned, $113.80; $82.70; $450; $19.88. (Supply dates.)

23. An account with Cash. — When I keep an account with "Cash", I am keeping an account, as it were, with my own pocket-book or cash-box. "Cash" is debtor, that is, owes me, for all that is put in, and "Cash" is credited with all that is taken out.

CASH

Dr. *Cr.*

1916					1916				
May	1	Balance	$100	00	May	13	Mdse. bought	$450	00
"	5	Rent recd.	50	00	"	14	Piano bought	350	00
"	7	Mdse. sold	25	00	"	18	Clothing bought	25	00
"	10	Land sold	725	00	"	22	Balance	75	00
			900	00				900	00
May	22	Balance	75	00					

1. Cash is charged with having received four amounts, which it owes me; that is, for which it is my debtor. How much is on hand at the beginning?

2. What is the total amount Cash has received?

3. When I take out $350 with which to purchase a piano, Cash has paid me back how much of what it owes me?

4. What other amounts has Cash paid me, that is, for what other amounts should Cash be credited?

5. How much more has Cash received than paid out?

6. How much more might I have spent so as to balance the account?

7. For what is Cash debtor at the beginning of the next account?

EXERCISES

Balance the following cash accounts, supplying dates and using the form shown above:

1. Charles Watson has on hand $4.21. He receives at various times $6.24, $7.36, $8.49, $7.34, $6.75. He pays out $8.75, $9.81, $8.39.

2. On Monday morning a merchant begins business with $247.84 on hand. He receives $24.75, $86.91, $84.28, $97.25, $164.29. He pays out $18.99, $37.49, $64.91, $83.15.

3.

Dr.	Cr.
$ 987.65	$629.55
1839.76	83.74
6482.91	968.71
478.85	28.46
698.47	318.93

4.

Dr.	Cr.
$4768.82	$ 468.34
947.61	984.59
847.77	1483.22
3998.64	
8372.91	

5.

Dr.	Cr.
$ 649.81	$ 82.46
8439.87	981.32
648.38	641.25
91.76	239.86
	728.41

6.

Dr.	Cr.
$246.94	$839.75
839.76	646.81
842.94	194.32
327.68	546.78
946.32	937.89

7.

Dr.	Cr.
$698.83	$649.83
376.59	478.88
843.26	694.31
695.98	883.24
831.96	695.64

8.

Dr.	Cr.
$356.78	$132.72
938.12	873.54
45.23	137.92
938.33	689.85
876.23	736.29

9. The following is the open account of E. R. Walker with the First National Bank of Monroe:

E. R. WALKER

Dr. *Cr.*

Jan.	11	Check	$500	00	Jan.	4	Balance	$486	87
"	12	"	57	30	"	10	Deposit	290	00
"	20	"	235	75	"	11	"	198	75
"	22	"	11	80	"	21	"	773	40
"	28	"	97	30	"	25	"	110	00

By inspection, it is evident that the balance will be on the debit side. What does this show? What would a balance on the credit side show? Find the balance.

10. Balance the following and tell what it shows:

On May 1, E. F. Mason had a balance of $4370.28 to his account in the bank. He deposited on May 1, $269; May 6, $165; May 10, $175; May 15, $180.50; May 20, $290. He withdrew by check the following amounts: May 1, $156; May 10, $450; May 13, $125; May 27, $675.50. What was his balance June 1?

Supply dates, rule a form, and make up an account with cash from the following:

11. Dr.	11. Cr.	12. Dr.	12. Cr.
$643.20	$593.08	$1047.69	$547.30
91.26	101.82	398.43	821.65
384.85	207.98	507.97	293.47
763.97	574.61	681.19	439.82
192.54	206.89	298.76	176.92
232.16	198.47	382.57	920.13
516.24	497.16	598.66	125.76
495.76	389.89	279.79	237.28

MULTIPLICATION

24. Meaning of multiplying by a whole number. — Multiplying by a whole number is a short way of finding the sum of a number of equal addends.

Thus, $5 \times \$38 = \$38 + \$38 + \$38 + \$38 + \$38 = \$190$.

5 is the **multiplier**, \$38 the **multiplicand**, and \$190 the **product.**

From the meaning of multiplication the following principles are evident:

1. *The multiplier must be an abstract number; that is, one used without reference to any particular object.*
2. *The multiplicand may be either an abstract number or a concrete number; that is, a number of specified objects.*
3. *The product must be a number like the multiplicand.*

In practical work, the number that will more quickly give the result is used as the multiplier. Thus, if one building lot costs \$850, three lots will cost $3 \times \$850$, or \$2550. Here 3 is the logical multiplier and is used as such. But if one barrel of apples sells for \$3, 850 barrels should sell for $850 \times \$3$, or \$2550. Here \$3 is the logical multiplicand, but in getting the product 3 is used as the multiplier.

In practical work these two problems may be written:

$$3 \times \$850 = \$2550 \qquad\qquad 850 \times \$3 = \$2550$$

$$\begin{array}{r} 850 \\ \underline{3} \\ 2550 \end{array} \qquad\qquad \begin{array}{r} 850 \\ \underline{3} \\ 2550 \end{array}$$

Name the *logical* and the *actual* multiplier:

1. Find the cost of 800 bbl. of apples at \$3.45 per bbl.
2. Find the cost of 756 bbl. of apples at \$4.00 per bbl.
3. Find the cost of 348 bbl. of apples at \$3.65 per bbl.

25. The primary number facts of multiplication. — There are thirty-six **primary number facts** of multiplication which are found by adding equal addends. Thus, to find the "4 times" table, add each of the digits four times.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
4	8	12	16	20	24	28	32	36

Hence, $4 \times 1 = 4$	$4 \times 4 = 16$	$4 \times 7 = 28$
$4 \times 2 = 8$	$4 \times 5 = 20$	$4 \times 8 = 32$
$4 \times 3 = 12$	$4 \times 6 = 24$	$4 \times 9 = 36$

Success in multiplication depends upon ability to call the thirty-six primary facts accurately and rapidly.

Practice calling the products from this table:

	A	B	C	D	E	F	G	H	I
1.	2 2	6 5	9 8	6 2	8 8	9 4	6 3	6 6	9 2
2.	3 3	7 7	7 3	9 5	9 9	6 4	8 3	8 7	5 4
3.	7 5	5 2	9 6	4 4	5 3	8 5	7 4	3 2	6 7
4.	8 2	9 3	5 5	4 2	8 6	4 3	9 7	8 4	7 2

26. Principles underlying long multiplication. — In multiplying by a multiplier of more than one figure, the following principles are used:

PRINCIPLE 1. *A number may be multiplied by the sum of two or more numbers by multiplying it by each of the numbers and adding the products.*

Thus, $25 \times 36 = (20 + 5) \times 36 = 20 \times 36 + 5 \times 36$.

PRINCIPLE 2. *One number may be multiplied by another by multiplying by each of its factors in succession.*

Thus, $6 \times 5 = 2 \times 3 \times 5$; $20 \times 14 = 2 \times 10 \times 14 = 2 \times 140$.

EXAMPLE. Multiply 86 by 27.

$$\begin{array}{r l} 86 & \\ \underline{27} & \\ 602 & = 7 \times 86 \\ \underline{1720} & = 20 \times 86 \text{ or } 2 \times 10 \times 86 \text{ (Prin. 2)} \\ 2322 & = 27 \times 86 \text{ (Prin. 1)} \end{array}$$

A full understanding of these principles often enables one to shorten work in multiplication. This will be seen in some of the short methods discussed in Chapter IV.

27. Checks in multiplication. — There are three checks that may be used in multiplication:

1. *Performing the work a second time, preferably interchanging the multiplier and multiplicand.*
2. *Seeing if the product divided by the multiplier (or multiplicand) gives the multiplicand (or multiplier).*
3. *Casting out the nines.*

Of these methods the first is most common.

28. The check by casting out the nines. — This check depends upon the fact that

The excess of nines in the product equals the excess in the product of the excesses of the multiplier and multiplicand.

ILLUSTRATION	CHECK
34682	5
176	5
208092	25
242774	
34682	
6104032	7

EXPLANATION.—The excess in 34,682 is 5. The excess in 176 is 5. The product of 5 times 5 is 25. The excess in 25 is 7. The excess in 6,104,032 is also 7. Therefore the work is probably correct. What kind of errors are not detected by this method?

EXERCISES

Multiply and check:

1. 385 × 627.	**11.** 768 × 307.	**21.** 840 × 960.
2. 789 × 643.	**12.** 708 × 943.	**22.** 750 × 430.
3. 862 × 947.	**13.** 276 × 709.	**23.** 820 × 760.
4. 375 × 896.	**14.** 846 × 709.	**24.** 930 × 640.
5. 762 × 934.	**15.** 247 × 806.	**25** 804 × 720.
6. 538 × 476.	**16.** 903 × 749.	**26.** 670 × 910.
7. 928 × 647.	**17.** 609 × 354.	**27.** 460 × 380.
8. 329 × 728.	**18.** 872 × 708.	**28.** 720 × 930.
9. 453 × 784.	**19.** 893 × 605.	**29.** 570 × 360.
10. 762 × 847.	**20.** 709 × 834.	**30.** 890 × 740.

Test your accuracy in multiplication by the following:

31. 319,687 × 578,624.	**34.** 785,964 × 683,247.
32. 769,823 × 461,738.	**35.** 684,796 × 386,742.
33. 963,742 × 876,846.	**36.** 968,534 × 278,475.

DIVISION

29. The meaning of division. — **Division** is the inverse of multiplication. That is, having given the product of two numbers and one of them, division is the process of finding the other.

The given product is the **dividend**, the other given number is the **divisor**, and the number to be found is the **quotient**. If the dividend is not exactly divisible by the divisor, the amount by which the dividend is too large for exact division is called the **remainder**.

30. Two interpretations of division. — Since division is the inverse of multiplication, and since multiplication by a whole number is a short process of finding the sums of equal addends, *two* division questions may arise from each multiplication fact.

Thus, since $5 \times 7 = 7 + 7 + 7 + 7 + 7 = 35$, the following questions may arise:

1. How many 7's in 35?
2. One fifth of 35 is how many?

The first of these questions illustrates the **measurement** meaning of division. That is, 35 things are measured by 7 like things, and we find that the seven things are contained in thirty-five things 5 times.

This may be read "35 will contain 7 five times".

The second question illustrates the **partition** meaning of division. That is, 35 things are divided into 5 equal parts, and there are 7 of them in each part.

This may be read "One fifth of 35 is 7", or "35 divided by 5 is 7".

Explain the meaning of each of the following, and show how each may be read:

$$5\overline{)\$130} \atop \$\ 26 \qquad\qquad \$5\overline{)\$130} \atop 26$$

31. Principles of division. — From the meaning of division the following principles are evident:

1. *When the dividend and divisor are like numbers, the quotient is abstract, and shows the number of times the dividend contains the divisor.*

2. *When the divisor is abstract, the quotient is like the dividend, and shows the size of the parts into which the divisor divides the dividend.*

3. *The remainder and dividend are like numbers.*

4. *The dividend is equal to the product of the quotient and divisor plus the remainder.*

32. The process of division. — In explaining the process of division, either of the interpretations in § 30 may be used. In the following, we shall consider the divisor abstract.

EXAMPLE 1. Divide 3248 by 7. (Short division.)

WORK

$$\begin{array}{r} 7\,\underline{)\,3248} \\ 464 \end{array}$$

EXPLANATION. — Dividing any number into parts does not change the unit. Hence $\frac{1}{7}$ of 32 hundreds = 4 hundreds, and 4 hundreds yet undivided. The undivided 4 hundreds and 4 tens = 44 tens. $\frac{1}{7}$ of 44 tens = 6 tens, and 2 tens yet undivided. 2 tens and 8 = 28. $\frac{1}{7}$ of 28 = 4.

EXAMPLE 2. Divide 35,587 by 56. (Long division.)

WORK

$$\begin{array}{r} 635 \\ 56)\overline{35587} \\ \underline{336} \\ 198 \\ \underline{168} \\ 307 \\ \underline{280} \\ 27 \text{ rem.} \end{array}$$

EXPLANATION. — The first left-hand figures which represent a number that will contain 56 are 355. We do not need even to notice the unit (in this case hundreds) represented by it. $\frac{1}{56}$ of 355 of any unit = 6 of that unit, and 19 of them remaining undivided. The 6 is placed above the right-hand figure of the number divided to show that it is of the same unit as the number divided. To "bring down" 8 is equivalent to multiplying the 19 by 10 and adding 8. That is, by bringing down the next figure of the dividend, we are really changing the number remaining after each division to the unit of the next lower order and adding it to the number of that order.

33. Checks in division. — Division may be checked in three ways:

1. *By performing the work a second time.*

2. *By seeing if the product of the quotient and divisor, plus the remainder, equals the dividend.*

3. *By casting out the nines.*

The first way is the one most often used. The second way is perhaps safer. The third is seldom used. For the information of the student, the third way is given in the next section.

34. The check by casting out the nines. — This check depends upon the fact that

The excess in the sum of the product of the excesses of nines in the divisor and quotient, plus the excess in the remainder, equals the excess in the dividend.

ILLUSTRATION

```
      510
 68)34699
    340
      69
      68
      19
```

EXPLANATION. — The excess in 68 is 5. The excess in 510 is 6. The excess in 5×6 is 3. The excess in 19 is 1. $3 + 1 = 4$. The excess in 34,699 is 4. Therefore the work is probably correct. Why "probably" correct?

EXERCISES

Divide and check:

1. 759,470 ÷ 78.	8. 89,175 ÷ 39.	15. 183,974 ÷ 94.
2. 624,789 ÷ 48.	9. 284,603 ÷ 98.	16. 265,371 ÷ 88.
3. 182,347 ÷ 57.	10. 99,134 ÷ 49.	17. 104,288 ÷ 78.
4. 96,343 ÷ 97.	11. 108,264 ÷ 57.	18. 139,267 ÷ 72.
5. 192,462 ÷ 67.	12. 346,271 ÷ 86.	19. 204,306 ÷ 68.
6. 236,475 ÷ 77.	13. 937,441 ÷ 163.	20. 307,961 ÷ 96.
7. 187,931 ÷ 68.	14. 784,267 ÷ 269.	21. 198,001 ÷ 67.

DRILL TABLE FOR DAILY PRACTICE

See how many you can divide in ten minutes:

	A	B	C	D	E	F	DIVISOR
1.	263,816	530,997	465,043	571,377	623,371	398,416	÷ 673
2.	396,516	631,828	605,852	640,996	586,752	708,228	÷ 764
3.	732,615	828,852	773,364	677,127	345,066	418,761	÷ 867
4.	450,072	354,756	517,752	404,952	542,004	328,248	÷ 564
5.	403,172	462,264	545,292	296,956	632,808	440,572	÷ 748

35. Factors. — The **factors** of a given number are those numbers which multiplied together make the given number.

36. Prime factors. — If a number has no factor except itself and 1, it is a **prime number**. If the factors of a number are themselves prime numbers, they are called **prime factors.**

EXERCISES

Give at sight two factors that make:

1. 15; 24; 36. **5.** 38; 42; 49. **9.** 55; 88; 132.
2. 16; 35; 54. **6.** 27; 63; 72. **10.** 70; 90; 781.
3. 18; 21; 48. **7.** 39; 44; 96. **11.** 99; 91; 105.
4. 22; 28; 50. **8.** 30; 64; 81. **12.** 80; 108; 120.

13. Find the prime factors of 3654.

WORK

$$\begin{array}{r|l} 2 & 3654 \\ \hline 3 & 1827 \\ \hline 3 & 609 \\ \hline 7 & 203 \\ \hline & 29 \end{array}$$

EXPLANATION. — A prime factor (2) is found by trial, and when found it is divided out. The quotient (1827) is subjected to the same process, then the next quotient (609) and so on, until a quotient (29) is obtained that is a prime number. The prime factors of 3654, then, are 2, 3, 7, 29.

Find the prime factors of the following:

14. 5625.	**16.** 4410.	**18.** 5346.	**20.** 6930.
15. 6425.	**17.** 7335.	**19.** 3927.	**21.** 14,670.

37. Tests of divisibility. — There are no general methods of determining factors of numbers. There are, however, the following tests for *special* factors, due to our *decimal-place-value system* of notation, that should be known.

1. *Any number is divisible by* 2 *or* 5 *if the right-hand digit is divisible by* 2 *or* 5 *or if it is* 0, *and not otherwise.*

This is seen from the fact that any number may be considered a multiple of 10, plus its right-hand digit. Thus $3845 = 384 \times 10 + 5$.

Now 10, and therefore any number of times 10, is divisible by 2 or 5. So if the last digit of a number is divisible by 2 or 5, or is 0, the number itself is divisible by 2 or 5, and not otherwise.

2. *Any number is divisible by* 4 *or* 25 *if the number represented by the last two digits is divisible by* 4 *or* 25, *and not otherwise.*

The reasoning is similar to that above. Thus

$$35{,}685 = 356 \times 100 + 85, \text{ etc.}$$

The reasoning may be extended to tests for divisibility by 8 and 125; by 16 and 625; etc.

3. *Any number is divisible by* 3 *or* 9 *if the sum of the digits is divisible by* 3 *or* 9, *and not otherwise.*

The following illustrates this fact:

$$\begin{aligned} 34{,}572 &= 30{,}000 + 4000 + 500 + 70 + 2 \\ &= [(3 \times 9999) + 3] + [(4 \times 999) + 4] + [(5 \times 99) + 5] \\ &\qquad + [(7 \times 9) + 7] + 2 \\ &= [(3 \times 9999) + (4 \times 999) + (5 \times 99) + (7 \times 9)] \\ &\qquad + [3 + 4 + 5 + 7 + 2]. \end{aligned}$$

In the last equality, the first bracket is evidently divisible by both 3 and 9. If the last bracket, which consists of the sum of the digits of the given number, is also divisible by either 3 or 9, then the whole quantity, 34,572, is divisible by 8 or 9, and not otherwise.

From this example it is seen also that:

The remainder arising from dividing a number by nine is the remainder obtained from dividing the sum of the digits by nine.

4. *Any number is divisible by* 11 *if the difference between the sums of the odd- and even-placed digits is divisible by* 11, *and not otherwise.*

The following illustrates this fact.

$$
\begin{aligned}
34{,}572 &= 30{,}000 + 4000 + 500 + 70 + 2 \\
&= [(3 \times 9999) + 3] + [(4 \times 1001) - 4] + [(5 \times 99) + 5] + [(7 \times 11) - 7] + 2 \\
&= [(3 \times 9999) + (4 \times 1001) + (5 \times 99) + (7 \times 11)] \\
&\quad + [3 - 4 + 5 - 7 + 2].
\end{aligned}
$$

The first bracket in the last equality is divisible by 11, so that the divisibility of the last bracket by 11 determines the divisibility of the given number. From this illustration the principle stated above is evident.

NOTE. — There is but little advantage in using the test for 11, for a trial division can be made as quickly as the test can be applied.

EXERCISES

Test the following for divisibility by 2, 4, 5, 25, 3, *and* 9:

1. 46,834.	**7.** 46,530.	**13.** 904,602.	**19.** 711,864.
2. 76,845.	**8.** 57,825.	**14.** 247,825.	**20.** 172,125.
3. 89,674.	**9.** 846,384.	**15.** 436,500.	**21.** 134,640.
4. 76,984.	**10.** 762,396.	**16.** 546,334.	**22.** 222,750.
5. 46,075.	**11.** 842,436.	**17.** 580,428.	**23.** 439,680.
6. 97,630.	**12.** 719,163.	**18.** 763,047.	**24.** 104,732.

Write down a 6-figured number that will contain the following factors:

25.	4.	**27.**	25.	**29.**	18.	**31.**	36.	**33.**	15.	**35.**	75.
26.	9.	**28.**	6.	**30.**	12.	**32.**	50.	**34.**	45.	**36.**	225.

38. Two fundamental principles of factoring. — In finding factors common to two or more numbers for purposes of cancellation, or for reducing ratios or fractions to lowest terms, the following principles will often save much work:

1. *A factor of a number is a factor of any multiple of that number.*

Thus, 5 is a factor of 15, and also of 30, 45, 60, 90, etc.

2. *A common factor of two numbers is a factor of their sum or difference, or of the sum or difference of any of their multiples.*

Thus, 4 is a common factor of 12 and 32, so it is a factor of $12 + 32$, or 44, also of $32 - 12$, or 20, also of $24 + 32$, or 56, etc.

EXAMPLE 1. Find the factors common to 3636 and 1764.

WORK

$$\begin{array}{r} 3636 \\ \underline{3528} \\ 108 \end{array}$$

$108 = 2 \times 2 \times 3 \times 3 \times 3.$

EXPLANATION. — $3636 - 2 \times 1764 = 108$. Now any factors common to the given numbers must be factors of 108. Hence there are no prime factors but 2's and 3's. By the first and third tests, 4 and 9 are factors. 3 is the only other possible factor, but when 9 is divided out of 3636, the quotient is 404, which, by the third test, does not contain 3. Hence the common factors are 2, 2, 3, and 3.

EXAMPLE 2. Is there a factor common to 3515 and 3416?

WORK

$$\begin{array}{r} 3515 \\ \underline{3416} \\ 99 \end{array}$$

$99 = 3 \times 3 \times 11.$

EXPLANATION. — $3515 - 3416 = 99 = 3 \times 3 \times 11$. The only possible factors must be factors of 99. That is, must be 3, 3, or 11. By the third test, 3 is not a factor. By the fourth test, 11 is not a factor. Since 3 and 11 are the only possible prime factors, there are no factors common to 3515 and 3416.

39. Greatest common factor. — The greatest common factor, commonly called the **greatest common divisor** (G. C. D.), of two or more numbers may be found by the principles given above. The only practical application is that of reducing ratios of large numbers, expressed as fractions, to their lowest terms.

EXAMPLE 1. Find the G. C. D. of 188 and 329.

WORK

188——329
188——141
47——141
47 = G. C. D.

EXPLANATION. — A factor common to 188 and 329 is also common to 188 and 141 (141 = 329 − 188); and also to 141 and 47 (47 = 188 − 141). But 47 is the largest factor common to 141 and 47 and hence to 188 and 329.

EXAMPLE 2. Find the G. C. D. of 177, 295, and 413.

WORK

177——295——413
177——118——118
59——118—— 0
59 = G. C. D.

EXPLANATION. — The numbers are written down in order of size and the differences taken. By inspection, 59 is the G. C. D. of 59 and 118, and hence of 177, 295, and 413.

EXERCISES

Find the G. C. D. of:

1. 148, 259, 370.	5. 332, 498, 581.	9. 480, 815, 978.
2. 142, 213, 355.	6. 296, 444, 481.	10. 452, 565, 678.
3. 118, 295, 354.	7. 388, 485, 970.	11. 295, 413, 472.
4. 201, 335, 536.	8. 246, 328, 369.	12. 394, 788, 985.

40. Cancellation. — Problems frequently arise that require a series of multiplications and divisions. The computation is usually greatly shortened by canceling the common factors from dividend and divisor before performing the multiplication and division.

EXAMPLE. — Divide $48 \times 56 \times 72 \times 54$ by $36 \times 78 \times 108$.

WORK

$$\frac{\overset{8}{\cancel{48}} \times 56 \times \overset{2}{\cancel{72}} \times \cancel{54}}{\cancel{36} \times \underset{13}{\cancel{78}} \times \underset{2}{\cancel{108}}} = \frac{8 \times 56}{13} = \frac{448}{13} = 34\tfrac{6}{13}.$$

EXERCISES

Simplify :

1. $\dfrac{5 \times 6 \times 7 \times 24 \times 36}{4 \times 8 \times 14 \times 18}$.

2. $\dfrac{42 \times 72 \times 210 \times 90}{21 \times 15 \times 7 \times 10}$.

3. $\dfrac{48 \times 84 \times 25 \times 144}{12 \times 21 \times 15 \times 36}$.

4. $\dfrac{14 \times 16 \times 36 \times 84}{7 \times 72 \times 12 \times 4}$.

5. $\dfrac{6 \times 24 \times 42 \times 8}{7 \times 12 \times 21 \times 2}$.

6. $\dfrac{18 \times 15 \times 56 \times 16}{5 \times 12 \times 4 \times 8}$.

7. $\dfrac{27 \times 35 \times 14 \times 63}{9 \times 7 \times 21 \times 28}$.

8. $\dfrac{33 \times 42 \times 25 \times 49}{7 \times 5 \times 11 \times 15}$.

9. $\dfrac{36 \times 48 \times 56 \times 35}{96 \times 32 \times 14 \times 2}$.

10. $\dfrac{63 \times 24 \times 33 \times 56}{14 \times 9 \times 12 \times 11}$.

11. $\dfrac{175 \times 28 \times 72 \times 18}{25 \times 14 \times 24 \times 9}$.

12. $\dfrac{220 \times 48 \times 60 \times 49}{8 \times 12 \times 13 \times 110}$.

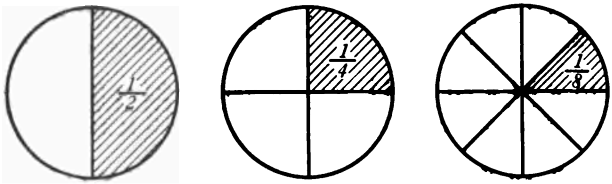
13. $\dfrac{500 \times 128 \times 42 \times 108}{12 \times 125 \times 512}$.

14. $\dfrac{175 \times 300 \times 288 \times 13}{39 \times 25 \times 36 \times 20}$.

CHAPTER II

COMMON FRACTIONS

41. Definition of a fraction. — A **fraction** is usually defined as one or more of the equal parts into which some whole has been divided.



Thus, if some whole is divided into four equal parts, each part is a fourth (written $\frac{1}{4}$) of the whole. Three of these parts are three fourths (written $\frac{3}{4}$) of the whole.

42. The notation of a fraction. — The two numbers used to express a fraction are called its **terms.** The lower term shows the number of parts into which the whole has been divided. It thus names the fraction, and hence is called the **denominator.** The upper term shows how many of the parts, or **fractional units,** compose the fraction, and is called the **numerator.**

43. Other uses of a fraction. — Besides being used to name a part of a whole, a fraction is also used to show the relation of one number to another. Thus, the relation of 7 to 9 is expressed by saying that 7 is $\frac{7}{9}$ of 9. This conforms with the definition given, for 9 is considered the whole. Each of

the nine single things that make up 9 is $\frac{1}{9}$ of the whole, so 7 of them are $\frac{7}{9}$ of the whole; that is, $\frac{7}{9}$ of 9.

$$\underbrace{\overset{\frac{1}{9}}{\bullet}\quad\overset{\frac{1}{9}}{\bullet}\quad\overset{\frac{1}{9}}{\bullet}\quad\overset{\frac{1}{9}}{\bullet}\quad\overset{\frac{1}{9}}{\bullet}\quad\overset{\frac{1}{9}}{\bullet}\quad\overset{\frac{1}{9}}{\bullet}}_{\frac{7}{9}}\quad\overset{\frac{1}{9}}{\bullet}\quad\overset{\frac{1}{9}}{\bullet}$$

A fraction is used also to express a division. Thus, $5 \div 6 = \frac{5}{6}$, $17 \div 24 = \frac{17}{24}$, $35 \div 41 = \frac{35}{41}$, etc. Thus, by the use of fractions the division of one number by another is always possible.

EXERCISES

1. Which is larger, $\frac{2}{3}$ or $\frac{2}{5}$? Why?

2. Name the largest possible fractional unit.

3. Name a very small fractional unit.

4. Name the fractional unit and tell how many are shown:

$$\frac{5}{6},\ \frac{7}{8},\ \frac{9}{10},\ \frac{3}{11},\ \frac{5}{16},\ \frac{3}{17},\ \frac{8}{11},\ \frac{5}{13},\ \frac{16}{21},\ \frac{18}{31}.$$

5. Name a fractional unit half as large as:

$$\frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6},\ \frac{1}{7},\ \frac{1}{8},\ \frac{1}{9},\ \frac{1}{10},\ \frac{1}{11},\ \frac{1}{12}.$$

6. Name a fractional unit one third as large as:

$$\frac{1}{2},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{7},\ \frac{1}{8},\ \frac{1}{10},\ \frac{1}{15},\ \frac{1}{18},\ \frac{1}{20},\ \frac{1}{30}.$$

7. Express the relations of:

3 to 5, 4 to 9, 5 to 7, 6 to 11, 7 to 18, 18 to 25.

8. Express as fractions:

$7 \div 8$, $9 \div 11$, $4 \div 15$, $16 \div 21$, $21 \div 31$, $15 \div 46$.

44. Improper fractions. — When a whole is divided into *three* equal parts, each is a *third* of the whole; when into *four*, each is a *fourth* of the whole; when into *five*, each is a

fifth of the whole; etc. It is evident, then, that any whole equals $\frac{2}{2}$ of it, $\frac{3}{3}$ of it, $\frac{4}{4}$ of it, $\frac{5}{5}$ of it, etc. That is,

$$1=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=\frac{6}{6}=\frac{7}{7}=\frac{8}{8}=\text{etc.}$$

When the numerator is equal to or greater than the denominator, the fraction is called an **improper fraction**. Thus, $\frac{5}{5}$, $\frac{7}{6}$, $\frac{11}{8}$, $\frac{19}{16}$, etc., are improper fractions.

45. Improper fractions changed to whole or mixed numbers. — Since $\frac{6}{6}=1$, there are as many 1's in $\frac{13}{6}$ as there are $\frac{6}{6}$'s in $\frac{13}{6}$. But 13 of any unit will contain 6 of that unit 2 times and 1 unit remains undivided. Hence, $\frac{13}{6}=2\frac{1}{6}$.

Such numbers as $2\frac{1}{6}$, made up of a whole number and a fraction, are **mixed numbers**.

Improper fractions may be reduced more briefly than by the reduction given above, by observing that:

Since the denominator shows the number of parts into which the whole is divided, it shows the number of such units that make one whole. So dividing the numerator by the denominator shows the number of wholes to which an improper fraction is equal.

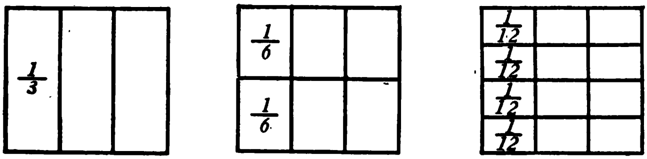
EXERCISES

Change to whole or mixed numbers:

1. $\frac{17}{3}$.	**6.** $\frac{56}{11}$.	**11.** $\frac{64}{7}$.	**16.** $\frac{83}{6}$.
2. $\frac{19}{6}$.	**7.** $\frac{43}{8}$.	**12.** $\frac{93}{11}$.	**17.** $\frac{92}{7}$.
3. $\frac{24}{5}$.	**8.** $\frac{26}{7}$.	**13.** $\frac{64}{9}$.	**18.** $\frac{47}{11}$.
4. $\frac{19}{5}$.	**9.** $\frac{57}{9}$.	**14.** $\frac{95}{8}$.	**19.** $\frac{86}{13}$.
5. $\frac{34}{7}$.	**10.** $\frac{83}{9}$.	**15.** $\frac{79}{4}$.	**20.** $\frac{29}{3}$.

46. Reduction of fractions. — By the following diagram it is seen that dividing a fractional unit into two equal parts gives two *new* units each *half* as large as the old, but denoted by a denominator *twice* as large. Thus, $\frac{1}{3}$ will make $\frac{2}{6}$,

$\frac{2}{3}$ will make $\frac{4}{6}$, etc. If a fractional unit is divided into 4 parts, there will be four times as many *new* units, but each will be *one fourth* as large as the old, and denoted by a denominator *4 times* as large. Thus, $\frac{1}{3} = \frac{4}{12}$; $\frac{2}{3} = \frac{8}{12}$; etc.



Thus it is seen that :

Multiplying or dividing both terms of a fraction by the same number changes the form of the fraction but not its value.

EXERCISES

Reduce to lowest terms:

1. $\frac{24}{36}$.	4. $\frac{36}{84}$.	7. $\frac{65}{91}$.	10. $\frac{105}{150}$.	13. $\frac{38}{95}$.
2. $\frac{72}{84}$.	5. $\frac{54}{81}$.	8. $\frac{42}{77}$.	11. $\frac{48}{80}$.	14. $\frac{51}{136}$.
3. $\frac{96}{124}$.	6. $\frac{49}{63}$.	9. $\frac{84}{108}$.	12. $\frac{85}{119}$.	15. $\frac{69}{115}$.

47. Changing two or more fractions to a common denominator. — Since only like units can be added or subtracted, fractions cannot be added or subtracted until they are changed to a common denominator. Since both terms of a fraction may be multiplied or divided by the same number without changing its value, the problem of changing two or more fractions to a common denominator depends upon finding numbers by which to multiply the several denominators in order to get like products.

Thus, to change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to a common denominator, we see by inspection that since each denominator is a factor of 12, multiplying both terms of the first fraction by 4, of the second by 3, and of the third by 2, we get $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$.

EXERCISES

Change to common denominators and add:

1. $\frac{3}{4}, \frac{7}{8}, \frac{9}{16}$.	**5.** $\frac{5}{9}, \frac{17}{18}, \frac{29}{36}$.	**9.** $\frac{5}{16}, \frac{11}{48}, \frac{5}{8}$.
2. $\frac{4}{7}, \frac{9}{14}, \frac{17}{21}$.	**6.** $\frac{17}{24}, \frac{35}{48}, \frac{1}{12}$.	**10.** $\frac{2}{3}, \frac{4}{5}, \frac{5}{6}$.
3. $\frac{5}{8}, \frac{9}{16}, \frac{7}{32}$.	**7.** $\frac{13}{21}, \frac{39}{42}, \frac{5}{7}$.	**11.** $\frac{2}{5}, \frac{7}{10}, \frac{23}{30}$.
4. $\frac{11}{15}, \frac{17}{30}, \frac{49}{60}$.	**8.** $\frac{17}{28}, \frac{39}{42}, \frac{5}{7}$.	**12.** $\frac{5}{9}, \frac{29}{45}, \frac{4}{5}$.

Change to common denominators and subtract:

13. $\frac{2}{3} - \frac{3}{5}$.	**16.** $\frac{17}{18} - \frac{16}{27}$.	**19.** $\frac{3}{4} - \frac{2}{5}$.
14. $\frac{5}{8} - \frac{3}{16}$.	**17.** $\frac{14}{15} - \frac{7}{45}$.	**20.** $\frac{6}{7} - \frac{5}{8}$.
15. $\frac{14}{15} - \frac{5}{6}$.	**18.** $\frac{5}{8} - \frac{11}{24}$.	**21.** $\frac{7}{8} - \frac{5}{9}$.

48. Adding mixed numbers. — Mixed numbers are added by adding the whole numbers and fractions separately.

EXERCISES

1. Add $24\frac{1}{2}$, $17\frac{3}{4}$, $16\frac{5}{8}$, $14\frac{13}{16}$, $14\frac{7}{8}$.

WORK

	16
$24\frac{1}{2}$	8
$17\frac{3}{4}$	12
$16\frac{5}{8}$	10
$14\frac{13}{16}$	13
$14\frac{7}{8}$	14
$88\frac{9}{16}$	57

EXPLANATION. — 16 is the common denominator. It is written above for reference. The numerators only are written. Their sum is 57, the number of sixteenths. $\frac{57}{16} = 3\frac{9}{16}$. $\frac{9}{16}$ is written under the fractions and 3 is carried.

2. Add $13\frac{2}{3}$, $16\frac{5}{6}$, $17\frac{1}{2}$, $18\frac{3}{4}$, and $19\frac{1}{6}$.
3. Add $27\frac{1}{4}$, $13\frac{9}{16}$, $28\frac{7}{8}$, $32\frac{1}{2}$, $56\frac{1}{4}$, and $28\frac{3}{16}$.

4.	5.	6.	7.	8.	9.
$15\frac{3}{8}$	$13\frac{3}{4}$	$24\frac{2}{3}$	$16\frac{7}{8}$	$34\frac{1}{5}$	$32\frac{2}{3}$
$26\frac{3}{4}$	$16\frac{5}{6}$	$17\frac{1}{6}$	$14\frac{3}{4}$	$16\frac{7}{15}$	$28\frac{2}{9}$
$16\frac{1}{2}$	$12\frac{7}{8}$	$22\frac{5}{9}$	$17\frac{5}{16}$	$28\frac{2}{3}$	$16\frac{3}{4}$
$19\frac{1}{8}$	$13\frac{11}{24}$	$16\frac{1}{2}$	$23\frac{1}{2}$	$17\frac{1}{2}$	$17\frac{1}{2}$
$23\frac{5}{16}$	$12\frac{2}{3}$	$34\frac{7}{18}$	$19\frac{1}{4}$	$16\frac{11}{30}$	$43\frac{5}{18}$

10.	11.	12.	13.	14.	15.
$12\frac{1}{2}$	$21\frac{15}{16}$	$16\frac{3}{4}$	$32\frac{5}{12}$	$82\frac{1}{3}$	$22\frac{3}{4}$
$16\frac{3}{10}$	$19\frac{1}{2}$	$27\frac{7}{8}$	$84\frac{2}{3}$	$24\frac{3}{4}$	$86\frac{1}{5}$
$18\frac{3}{4}$	$27\frac{3}{8}$	$43\frac{1}{2}$	$48\frac{5}{6}$	$86\frac{5}{8}$	$84\frac{14}{15}$
$26\frac{7}{8}$	$25\frac{1}{2}$	$83\frac{5}{8}$	$34\frac{3}{4}$	$32\frac{2}{3}$	$42\frac{1}{2}$
$11\frac{1}{4}$	$19\frac{11}{16}$	$92\frac{3}{16}$	$52\frac{2}{3}$	$97\frac{7}{8}$	$43\frac{5}{12}$
$13\frac{5}{8}$	$17\frac{5}{8}$	$27\frac{3}{4}$	$69\frac{11}{12}$	$42\frac{11}{12}$	$23\frac{1}{4}$

49. Adding special fractions. — The student should always be on the alert for combinations that will save work. Two types of exercises are shown by the following examples.

1. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$.

Observe that $\frac{3}{4} + \frac{1}{4} = 1$; also that $\frac{5}{8} + \frac{3}{8} = 1$. Hence the sum is $2\frac{1}{2}$.

2. Add $\frac{1}{3}$ and $\frac{1}{4}$.

Observe that since 3 and 4 have no common factor and the numerators are each 1, the *new* numerators will be 4 and 3 respectively.
Hence the sum is $\frac{7}{12}$.

At sight give the sums:

3. $\frac{1}{3} + \frac{1}{5}$.
4. $\frac{1}{4} + \frac{1}{5}$.
5. $\frac{1}{5} + \frac{1}{6}$.
6. $\frac{1}{2} + \frac{1}{7}$.
7. $\frac{1}{8} + \frac{1}{3}$.
8. $\frac{1}{8} + \frac{1}{7}$.
9. $\frac{1}{7} + \frac{1}{4}$.
10. $\frac{1}{5} + \frac{1}{6}$.
11. $\frac{1}{6} + \frac{1}{7}$.
12. $\frac{1}{6} + \frac{1}{11}$.
13. $\frac{1}{3} + \frac{1}{7}$.
14. $\frac{1}{4} + \frac{1}{9}$.
15. $\frac{1}{8} + \frac{1}{11}$.
16. $\frac{1}{11} + \frac{1}{9}$.
17. $\frac{1}{9} + \frac{1}{13}$.
18. $\frac{1}{5} + \frac{1}{9}$.
19. $\frac{1}{7} + \frac{1}{13}$.
20. $\frac{1}{11} + \frac{1}{13}$.
21. $\frac{1}{5} + \frac{1}{11}$.
22. $\frac{1}{5} + \frac{1}{13}$.

23. $\frac{1}{3} + \frac{1}{5} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$.
24. $\frac{1}{4} + \frac{7}{8} + \frac{3}{5} + \frac{1}{8} + \frac{3}{4}$.
25. $\frac{1}{2} + \frac{3}{4} + \frac{3}{8} + \frac{1}{7} + \frac{1}{8}$.
26. $\frac{2}{3} + \frac{5}{6} + \frac{1}{6} + \frac{1}{3} + \frac{2}{9}$.
27. $\frac{5}{6} + \frac{2}{3} + \frac{1}{6} + \frac{1}{2} + \frac{1}{3}$.
28. $\frac{1}{7} + \frac{1}{4} + \frac{3}{8} + \frac{1}{2} + \frac{1}{4}$.
29. $\frac{7}{9} + \frac{2}{3} + \frac{2}{9} + \frac{1}{6} + \frac{1}{2}$.
30. $\frac{1}{3} + \frac{1}{6} + \frac{1}{2} + \frac{1}{4} + \frac{3}{8}$.

50. Subtracting mixed numbers. — If the fraction in the minuend is larger than the one in the subtrahend, the whole numbers and fractions are subtracted separately. If not, 1 must be taken from the whole number in the minuend.

EXERCISES

1. From $34\frac{5}{8}$ take $21\frac{1}{4}$.

WORK

$$\begin{array}{l|l} & \underline{8} \\ 34\frac{5}{8} & 5 \\ \underline{21\frac{1}{4}} & \underline{2} \\ 13\frac{3}{8} & 3 \end{array}$$

NOTE. — It is hardly necessary to write down the new numerators since only two fractions are involved, for they can be held in mind and subtracted.

2. From $46\frac{1}{3}$ take $15\frac{3}{4}$.

WORK

$$\begin{array}{l} 46\frac{1}{3} \\ \underline{15\frac{3}{4}} \\ 30\frac{7}{12} \end{array}$$

EXPLANATION. — The work may be done in two ways: (*a*) $\frac{3}{4}$ may be taken from the 1 taken from 6, leaving $\frac{1}{4}$. Then $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$. Or, (*b*) $1\frac{1}{3} = \frac{4}{3}$; $\frac{4}{3} - \frac{3}{4} = \frac{16}{12} - \frac{9}{12} = \frac{7}{12}$.

3. From $160\frac{3}{4}$ take $98\frac{2}{3}$.
4. From $158\frac{1}{3}$ take $96\frac{3}{4}$.
5. Subtract $146\frac{3}{5}$ from $598\frac{1}{3}$.

Subtract:

6.	7.	8.	9.	10.	11.
$96\frac{2}{3}$	$80\frac{1}{4}$	$75\frac{5}{8}$	$94\frac{1}{4}$	$43\frac{1}{5}$	$94\frac{2}{3}$
$84\frac{1}{4}$	$46\frac{2}{3}$	$26\frac{3}{4}$	$83\frac{5}{6}$	$26\frac{2}{3}$	$81\frac{4}{5}$

12.	13.	14.	15.	16.	17.
$215\frac{1}{2}$	$342\frac{1}{4}$	$297\frac{2}{3}$	$368\frac{1}{3}$	$347\frac{1}{5}$	$321\frac{1}{5}$
$93\frac{5}{6}$	$196\frac{7}{8}$	$125\frac{1}{2}$	$124\frac{5}{12}$	$189\frac{2}{3}$	$96\frac{3}{4}$

18.	19.	20.	21.	22.	23.
$342\frac{1}{3}$	$968\frac{1}{4}$	$215\frac{5}{6}$	$160\frac{2}{3}$	$203\frac{5}{6}$	$106\frac{1}{3}$
$135\frac{5}{12}$	$126\frac{1}{3}$	$39\frac{1}{2}$	$49\frac{11}{12}$	$98\frac{7}{8}$	$94\frac{11}{12}$

24.	25.	26.	27.	28.	29.
$104\frac{1}{5}$	$1546\frac{2}{3}$	$112\frac{1}{5}$	$109\frac{1}{2}$	$1096\frac{3}{8}$	$4095\frac{2}{3}$
$26\frac{2}{3}$	$119\frac{5}{8}$	$46\frac{2}{3}$	$26\frac{11}{12}$	$204\frac{3}{4}$	$738\frac{11}{12}$

30.	31.	32.	33.	34.	35.
$4152\frac{5}{7}$	$4012\frac{11}{16}$	$3059\frac{1}{4}$	$2930\frac{1}{3}$	$734\frac{11}{16}$	$324\frac{1}{4}$
$136\frac{2}{3}$	$107\frac{7}{8}$	$126\frac{5}{8}$	$427\frac{5}{6}$	$246\frac{7}{8}$	$163\frac{3}{5}$

36.	37.	38.	39.	40.	41.
$610\frac{5}{9}$	$341\frac{1}{11}$	$403\frac{1}{12}$	$426\frac{1}{5}$	$117\frac{2}{3}$	$213\frac{5}{6}$
$138\frac{11}{12}$	$103\frac{1}{2}$	$231\frac{1}{5}$	$108\frac{1}{3}$	$96\frac{7}{8}$	$108\frac{3}{4}$

42.	43.	44.	45.	46.	47.
$261\frac{1}{8}$	$230\frac{2}{3}$	$209\frac{1}{3}$	$320\frac{1}{4}$	$380\frac{4}{5}$	$810\frac{1}{4}$
$107\frac{15}{16}$	$103\frac{5}{8}$	$142\frac{3}{4}$	$196\frac{1}{2}$	$139\frac{1}{3}$	$275\frac{2}{3}$

51. Subtracting special fractions. — When the numerators are 1 and the denominators have no common factor, the fractions may be subtracted at sight.

EXERCISES

1. From $\frac{1}{3}$ take $\frac{1}{4}$.

Observe that the first numerator of the changed fractions is 4 and the second 3, and that the common denominator is the product of the given denominators. Hence,

$$\frac{1}{3}-\frac{1}{4}=\frac{4-3}{12} \text{ or } \frac{1}{12}.$$

At sight subtract:

2. $\frac{1}{2}-\frac{1}{5}$.	**9.** $\frac{1}{3}-\frac{1}{8}$.	**16.** $\frac{1}{6}-\frac{1}{11}$.	**23.** $\frac{1}{5}-\frac{1}{14}$.
3. $\frac{1}{4}-\frac{1}{5}$.	**10.** $\frac{1}{5}-\frac{1}{8}$.	**17.** $\frac{1}{7}-\frac{1}{9}$.	**24.** $\frac{1}{6}-\frac{1}{13}$.
4. $\frac{1}{5}-\frac{1}{7}$.	**11.** $\frac{1}{4}-\frac{1}{9}$.	**18.** $\frac{1}{7}-\frac{1}{8}$.	**25.** $\frac{1}{7}-\frac{1}{15}$.
5. $\frac{1}{5}-\frac{1}{6}$.	**12.** $\frac{1}{4}-\frac{1}{11}$.	**19.** $\frac{1}{7}-\frac{1}{11}$.	**26.** $\frac{1}{9}-\frac{1}{14}$.
6. $\frac{1}{4}-\frac{1}{7}$.	**13.** $\frac{1}{5}-\frac{1}{9}$.	**20.** $\frac{1}{5}-\frac{1}{12}$.	**27.** $\frac{1}{7}-\frac{1}{16}$.
7. $\frac{1}{3}-\frac{1}{7}$.	**14.** $\frac{1}{5}-\frac{1}{11}$.	**21.** $\frac{1}{3}-\frac{1}{11}$.	**28.** $\frac{1}{5}-\frac{1}{16}$.
8. $\frac{1}{3}-\frac{1}{5}$.	**15.** $\frac{1}{6}-\frac{1}{7}$.	**22.** $\frac{1}{4}-\frac{1}{15}$.	**29.** $\frac{1}{9}-\frac{1}{16}$.

52. Adding and subtracting fractions of large terms. — In all exercises that have been given in the preceding sections, the student was expected to find the least common denominators by inspection. In fact, most fractions that must be added or subtracted in any of the real problems that occur in life may be changed to like fractions by inspection. However, fractions may arise in which the change cannot be so easily made. The change requires finding a number known as the *least common multiple* of the denominators.

53. The least common multiple. — A **multiple** of a number is a number that is some integral number (whole number) of times the given number. That is, it is divisible by the given number without a remainder. Thus, 14, 21, 35, 42, etc., are all multiples of 7.

A **common multiple** of two or more numbers is a multiple of each of them. And the **least common multiple** (L. C. M.) of two or more numbers is the least multiple common to each of them. Thus, 6, 9, 12, 15, 18, 21, 24, etc., are all multiples of 3. And 8, 12, 16, 20, 24, etc., are all multiples of 4. Of the multiples of 3 and 4 written, both 12 and 24 are *common multiples*, while 12 is the *least common multiple.*

EXERCISES

1. Find the L.C.M. of 60, 72, and 108.

WORK

$$60 = 2 \times 2 \times 3 \times 5.$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3.$$
$$108 = 2 \times 2 \times 3 \times 3 \times 3.$$
$$108 \times 2 \times 5 = 1080, \text{ L.C.M.}$$

EXPLANATION. — To contain 108, the largest number, the L.C.M. must have 108 as a factor. But to contain 72 there must be three 2's, or one more 2 than the number in 108. Also to contain 60, there must be a factor 5 which is not in 108. Hence $108 \times 2 \times 5$ contains all of the factors in all three numbers, hence is divisible by each of the numbers.

Another form in common use is given below.

WORK

$$\begin{array}{r|rrr}
2 & 60, & 72, & 108 \\ \hline
2 & 30, & 36, & 54 \\ \hline
3 & 15, & 18, & 27 \\ \hline
3 & 5, & 6, & 9 \\ \hline
 & 5 & 2 & 3
\end{array}$$

EXPLANATION. — Arranging the numbers in line, any factor of two or more is divided out. The process is continued until there are no common factors in the quotients. The product of the divisors and the last quotients is the L. C. M., for it contains every factor as many times as it is found in any of the numbers.

$$2 \times 2 \times 3 \times 3 \times 5 \times 2 \times 3 = 1080, \text{ L. C. M.}$$

Find the least common multiple of:

2. 18, 27, and 54.
3. 16, 18, and 72.
4. 14, 42, and 35.
5. 12, 18, and 48.
6. 18, 24, and 84.
7. 46, 92, and 128.
8. 35, 125, and 225.
9. 72, 96, and 114.
10. 64, 84, and 144.
11. 32, 92, and 160.

12. Add $\frac{25}{66}$ and $\frac{31}{78}$.

WORK

$66 = 2 \times 3 \times 11.$

$78 = 2 \times 3 \times 13.$

$78 \times 11 = 858$, L. C. M.

$$\frac{25}{66} = \frac{25 \times 13}{66 \times 13} = \frac{325}{858}.$$

$$\frac{31}{78} = \frac{31 \times 11}{78 \times 11} = \frac{341}{858}.$$

$$\text{Sum} = \frac{666}{858} = \frac{111}{143}.$$

EXPLANATION — The L. C. M. of the denominators is 858. This is 11×78 and 13×66. Hence, multiplying both terms of $\frac{25}{66}$ by 13 and both terms of $\frac{31}{78}$ by 11 gives fractions with the common denominator 858.

Add:

13. $\frac{7}{15} + \frac{4}{21}$.
14. $\frac{31}{33} + \frac{17}{18}$.
15. $\frac{7}{24} + \frac{11}{21}$.
16. $\frac{24}{48} + \frac{39}{32}$.
17. $\frac{17}{84} + \frac{19}{60} + \frac{121}{140}$.
18. $\frac{49}{160} + \frac{53}{128} + \frac{17}{64}$.
19. $\frac{65}{144} + \frac{61}{132} + \frac{61}{72}$.
20. $\frac{56}{81} + \frac{35}{126} + \frac{135}{189}$.
21. $\frac{19}{68} + \frac{37}{102} + \frac{125}{136}$.
22. $\frac{53}{60} + \frac{19}{72} + \frac{187}{360}$.
23. $\frac{65}{72} + \frac{111}{120} + \frac{143}{180}$.
24. $\frac{17}{35} + \frac{108}{125} + \frac{116}{225}$.

Subtract:

25. $\frac{62}{72} - \frac{107}{128}$	**28.** $\frac{9}{16} - \frac{17}{56}$.	**31.** $\frac{17}{33} - \frac{16}{77}$.
26. $\frac{17}{36} - \frac{7}{60}$.	**29.** $\frac{23}{24} - \frac{101}{300}$.	**32.** $\frac{40}{81} - \frac{37}{90}$.
27. $\frac{7}{8} - \frac{11}{156}$.	**30.** $\frac{16}{81} - \frac{11}{126}$.	**33.** $\frac{81}{84} - \frac{17}{96}$.

MULTIPLICATION OF FRACTIONS

54. Multiplying a fraction by a whole number. — Whether a whole number or a fraction is multiplied by a whole number, the meaning of the multiplication is the same.

Thus, $4 \times \$2 = \$2 + \$2 + \$2 + \$2 = \$8$,

and $4 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{8}{5} = 1\frac{3}{5}$.

Hence a fraction is multiplied by a whole number by multiplying its numerator by the whole number.

EXERCISES

At sight give the products:

1. $5 \times \frac{7}{8}$.	**6.** $5 \times \frac{3}{7}$.	**11.** $9 \times \frac{3}{11}$.	**16.** $9 \times \frac{2}{7}$.
2. $3 \times \frac{3}{5}$.	**7.** $6 \times \frac{3}{5}$.	**12.** $6 \times \frac{4}{10}$.	**17.** $10 \times \frac{5}{9}$.
3. $7 \times \frac{2}{3}$.	**8.** $8 \times \frac{5}{7}$.	**13.** $7 \times \frac{2}{9}$.	**18.** $6 \times \frac{4}{5}$.
4. $6 \times \frac{4}{5}$.	**9.** $4 \times \frac{5}{6}$.	**14.** $6 \times \frac{5}{11}$.	**19.** $5 \times \frac{2}{9}$.
5. $9 \times \frac{7}{8}$.	**10.** $6 \times \frac{3}{7}$.	**15.** $8 \times \frac{3}{5}$.	**20.** $6 \times \frac{5}{13}$.

55. Multiplying a whole number by a fraction. — Here the meaning of multiplication changes. Thus, $\frac{3}{4} \times 7$ means $\frac{3}{4}$ of 7, which means to divide 7 by 4 and multiply the quotient by 3. But $7 \div 4 = \frac{7}{4}$, and $3 \times \frac{7}{4} = \frac{21}{4}$.

Observe that the result is the same as when a fraction is multiplied by a whole number. That is, the numerator of the product is the product of the numerator of the multiplier and the whole number.

EXERCISES

At sight give:

1. $\frac{3}{4} \times 5$.	**4.** $\frac{2}{3} \times 7$.	**7.** $\frac{5}{6} \times 7$.	**10.** $\frac{3}{7} \times 4$.
2. $\frac{2}{5} \times 4$.	**5.** $\frac{5}{6} \times 5$.	**8.** $\frac{3}{8} \times 5$.	**11.** $\frac{2}{9} \times 5$.
3. $\frac{3}{8} \times 7$.	**6.** $\frac{3}{5} \times 7$.	**9.** $\frac{2}{7} \times 9$.	**12.** $\frac{5}{9} \times 5$.

56. Multiplying a fraction by a fraction. — The product of $\frac{2}{3} \times \frac{3}{4}$ means $\frac{2}{3}$ of $\frac{3}{4}$. To find $\frac{2}{3}$ of anything is to divide it into 3 equal parts and take two of them. That is, it means to divide by 3 and multiply by 2. But fourths divided by 3 give twelfths. Hence, $\frac{2}{3} \times \frac{3}{4} = 2 \times \frac{3}{12} = \frac{6}{12} = \frac{1}{2}$. That is,

To multiply a fraction by a fraction, take the product of the numerators for the numerator of the product and the product of the denominators for the denominator of the product.

Work is saved by canceling factors that occur in both terms before multiplying. Thus,

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}.$$

EXERCISES

Find:

1. $\frac{3}{4} \times \frac{12}{17}$.	**4.** $\frac{5}{6} \times \frac{9}{10}$.	**7.** $\frac{8}{9} \times \frac{9}{32}$.	**10.** $\frac{11}{12} \times \frac{36}{55}$.
2. $\frac{2}{5} \times \frac{15}{16}$.	**5.** $\frac{7}{8} \times \frac{16}{22}$.	**8.** $\frac{7}{9} \times \frac{27}{35}$.	**11.** $\frac{9}{13} \times \frac{26}{27}$.
3. $\frac{2}{3} \times \frac{9}{10}$.	**6.** $\frac{5}{7} \times \frac{14}{15}$.	**9.** $\frac{5}{6} \times \frac{24}{25}$.	**12.** $\frac{8}{11} \times \frac{22}{25}$.

At sight give:

13. $4 \times \frac{2}{3}$.	**16.** $3 \times \frac{5}{6}$.	**19.** $\frac{3}{4} \times 3$.	**22.** $\frac{3}{4} \times 6$.
14. $2 \times \frac{3}{4}$.	**17.** $4 \times \frac{5}{6}$.	**20.** $\frac{2}{3} \times 9$.	**23.** $\frac{2}{3} \times 7$.
15. $5 \times \frac{2}{7}$.	**18.** $3 \times \frac{5}{8}$.	**21.** $\frac{3}{4} \times 5$.	**24.** $\frac{3}{4} \times 8$.

25. $\frac{4}{5} \times 15$.	**30.** $\frac{5}{6} \times \frac{7}{8}$.	**35.** $\frac{2}{5} \times \frac{10}{11}$.	**40.** $\frac{5}{7} \times \frac{7}{10}$.
26. $\frac{4}{5} \times 20$.	**31.** $\frac{2}{3} \times \frac{5}{6}$.	**36.** $\frac{7}{8} \times \frac{16}{15}$.	**41.** $\frac{2}{9} \times \frac{9}{10}$.
27. $\frac{2}{3} \times 12$.	**32.** $\frac{3}{5} \times \frac{5}{9}$.	**37.** $\frac{5}{6} \times \frac{11}{15}$.	**42.** $\frac{2}{7} \times \frac{7}{8}$.
28. $\frac{3}{4} \times 9$.	**33.** $\frac{3}{4} \times \frac{8}{9}$.	**38.** $\frac{7}{8} \times \frac{18}{14}$.	**43.** $\frac{1}{9} \times \frac{5}{6}$.
29. $\frac{2}{3} \times \frac{3}{4}$.	**34.** $\frac{5}{6} \times \frac{12}{7}$.	**39.** $\frac{5}{6} \times \frac{9}{10}$.	**44.** $\frac{2}{3} \times \frac{8}{9}$.

57. Multiplying mixed numbers. — Multiplication of mixed numbers is evidently an application of the three classes of multiplication already explained. There are two common methods of procedure.

EXERCISES

1. Find the product of $24\frac{3}{4} \times 15\frac{3}{5}$.

WORK

$$\begin{array}{rr}
24\frac{3}{4} & \\
\underline{15\frac{3}{5}} & \underline{20} \\
\frac{9}{20} & 9 \\
14\frac{2}{5} & 8 \\
11\frac{1}{4} & 5 \\
120 & \\
\underline{24} & \\
386\frac{1}{10} & \overline{22}
\end{array}$$

There are four steps:

1. $\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$.
2. $\frac{3}{5} \times 24 = 14\frac{2}{5}$.
3. $15 \times \frac{3}{4} = 11\frac{1}{4}$.
4. $15 \times 24 = 360$.

A *second method* is to change both mixed numbers to improper fractions.

Thus, $24\frac{3}{4} = \frac{99}{4}$; $15\frac{3}{5} = \frac{78}{5}$.

$$\frac{99}{\underset{2}{\cancel{4}}} \times \frac{\overset{39}{\cancel{78}}}{5} = \frac{3861}{10} = 386\frac{1}{10}.$$

Find the products of:

2. $35\frac{2}{3} \times 15\frac{3}{4}$.	**4.** $48\frac{1}{2} \times 16\frac{2}{3}$.	**6.** $34\frac{2}{5} \times 16\frac{3}{4}$.
3. $16\frac{7}{8} \times 14\frac{2}{5}$.	**5.** $38\frac{2}{5} \times 16\frac{5}{8}$.	**7.** $16\frac{2}{7} \times 13\frac{2}{3}$.

8. $19\frac{2}{5} \times 46\frac{3}{4}$.
9. $17\frac{2}{3} \times 16\frac{2}{3}$.
10. $56\frac{7}{8} \times 5\frac{1}{2}$.
11. $37\frac{2}{3} \times 7\frac{3}{4}$.
12. $5\frac{2}{3} \times 16\frac{7}{8}$.
13. $9\frac{4}{5} \times 17\frac{1}{2}$.
14. $8\frac{5}{6} \times 43\frac{1}{2}$.
15. $9\frac{2}{5} \times 53\frac{1}{3}$.
16. $7\frac{1}{6} \times 53\frac{2}{3}$.
17. $8\frac{2}{5} \times 46\frac{2}{3}$.
18. $9\frac{2}{7} \times 16\frac{1}{3}$.
19. $13\frac{1}{2} \times 17\frac{1}{4}$.
20. $14\frac{2}{3} \times 53\frac{1}{8}$.
21. $19\frac{1}{5} \times 73\frac{5}{8}$.
22. $16\frac{2}{3} \times 17\frac{1}{4}$.
23. $13\frac{3}{4} \times 19\frac{1}{4}$.
24. $16\frac{2}{3} \times 43\frac{2}{5}$.
25. $54\frac{1}{3} \times 19\frac{1}{2}$.

58. Special mixed numbers. — When the fractions are each $\frac{1}{2}$, much of the multiplication can be performed mentally. When both factors are alike, the work can be greatly abridged.

EXERCISES

1. Find $8\frac{1}{2} \times 8\frac{1}{2}$.

WORK

$$\begin{array}{ll} 8\frac{1}{2} & \\ \underline{8\frac{1}{2}} & \\ \frac{1}{4} & (1) \\ 4 & (2) \\ 4 & (3) \\ \underline{64} & (4) \\ 72\frac{1}{4} & \end{array}$$

OBSERVATION. — It will be observed in steps 2 and 3 that $\frac{1}{2}$ of 8 added to $8 \times \frac{1}{2}$ will give 8. This added to 8×8 (step 4) gives 9×8.

Hence, when the whole numbers are alike and the fractions are each $\frac{1}{2}$, the product is one whole number multiplied by the next consecutive number, plus $\frac{1}{4}$.

Thus, $7\frac{1}{2} \times 7\frac{1}{2} = 8 \times 7 + \frac{1}{4}$, or $56\frac{1}{4}$; $3\frac{1}{2} \times 3\frac{1}{2} = 4 \times 3 + \frac{1}{4}$, or $12\frac{1}{4}$; etc.

At sight give:

2. $2\frac{1}{2} \times 2\frac{1}{2}$.
3. $4\frac{1}{2} \times 4\frac{1}{2}$.
4. $5\frac{1}{2} \times 5\frac{1}{2}$.
5. $6\frac{1}{2} \times 6\frac{1}{2}$.
6. $9\frac{1}{2} \times 9\frac{1}{2}$.
7. $10\frac{1}{2} \times 10\frac{1}{2}$.
8. $11\frac{1}{2} \times 11\frac{1}{2}$.
9. $12\frac{1}{2} \times 12\frac{1}{2}$.
10. $13\frac{1}{2} \times 13\frac{1}{2}$.
11. $14\frac{1}{2} \times 14\frac{1}{2}$.
12. $15\frac{1}{2} \times 15\frac{1}{2}$.
13. $16\frac{1}{2} \times 16\frac{1}{2}$.

14. Find $8\frac{1}{2} \times 4\frac{1}{2}$.

WORK

$$\begin{array}{r} 8\frac{1}{2} \\ \underline{4\frac{1}{2}} \\ \frac{1}{4} \\ 4 \\ 2 \\ \underline{32} \\ 38\frac{1}{4} \end{array}$$

OBSERVATION. — It will be observed that $\frac{1}{2}$ of $8 + \frac{1}{2}$ of $4 = \frac{1}{2}$ of 12, or 6. This might have been found mentally and added to 4×8.

Put into words and write out the rule for this special case.

At sight give:

15.	$2\frac{1}{2} \times 8\frac{1}{2}$.	**23.**	$9\frac{1}{2} \times 7\frac{1}{2}$.
16.	$3\frac{1}{2} \times 5\frac{1}{2}$.	**24.**	$4\frac{1}{2} \times 12\frac{1}{2}$.
17.	$7\frac{1}{2} \times 5\frac{1}{2}$.	**25.**	$3\frac{1}{2} \times 11\frac{1}{2}$.
18.	$6\frac{1}{2} \times 4\frac{1}{2}$.	**26.**	$7\frac{1}{2} \times 3\frac{1}{2}$.
19.	$8\frac{1}{2} \times 6\frac{1}{2}$.	**27.**	$6\frac{1}{2} \times 8\frac{1}{2}$.
20.	$7\frac{1}{2} \times 11\frac{1}{2}$.	**28.**	$8\frac{1}{2} \times 10\frac{1}{2}$.
21.	$10\frac{1}{2} \times 6\frac{1}{2}$.	**29.**	$5\frac{1}{2} \times 9\frac{1}{2}$.
22.	$12\frac{1}{2} \times 8\frac{1}{2}$.	**30.**	$6\frac{1}{2} \times 12\frac{1}{2}$.

31. Find $9\frac{1}{2} \times 8\frac{1}{2}$.

WORK

$$\begin{array}{r} 9\frac{1}{2} \\ \underline{8\frac{1}{2}} \\ \frac{1}{4} \\ 4\frac{1}{2} \\ 4 \\ \underline{72} \\ 80\frac{3}{4} \end{array}$$

OBSERVATION. — Show that when the sum of the whole numbers is odd you still add $\frac{1}{2}$ of the sum, as in Exercises 14–30, but that the fraction in the product is $\frac{3}{4}$ or $\frac{1}{2} + \frac{1}{4}$, instead of $\frac{1}{4}$.

Put into words and write out the rule for this special case.

At sight give:

32.	$7\frac{1}{2} \times 6\frac{1}{2}$.	**42.**	$18\frac{1}{2} \times 5\frac{1}{2}$.
33.	$5\frac{1}{2} \times 8\frac{1}{2}$.	**43.**	$20\frac{1}{2} \times 5\frac{1}{2}$.
34.	$6\frac{1}{2} \times 9\frac{1}{2}$.	**44.**	$7\frac{1}{2} \times 10\frac{1}{2}$.
35.	$4\frac{1}{2} \times 11\frac{1}{2}$.	**45.**	$9\frac{1}{2} \times 14\frac{1}{2}$.
36.	$5\frac{1}{2} \times 10\frac{1}{2}$.	**46.**	$5\frac{1}{2} \times 18\frac{1}{2}$.
37.	$8\frac{1}{2} \times 7\frac{1}{2}$.	**47.**	$6\frac{1}{2} \times 21\frac{1}{2}$.
38.	$10\frac{1}{2} \times 9\frac{1}{2}$.	**48.**	$6\frac{1}{2} \times 15\frac{1}{2}$.
39.	$16\frac{1}{2} \times 5\frac{1}{2}$.	**49.**	$9\frac{1}{2} \times 12\frac{1}{2}$.
40.	$14\frac{1}{2} \times 5\frac{1}{2}$.	**50.**	$8\frac{1}{2} \times 25\frac{1}{2}$.
41.	$16\frac{1}{2} \times 7\frac{1}{2}$.	**51.**	$8\frac{1}{2} \times 31\frac{1}{2}$.

52. Find $6\frac{2}{3} \times 12\frac{2}{3}$.

WORK

$$\begin{array}{r} 12\frac{2}{3} \\ \underline{6\frac{2}{3}} \\ \frac{4}{9} \\ 8 \\ 4 \\ \underline{72} \\ 84\frac{4}{9} \end{array}$$

OBSERVATION. — Observe that $\frac{2}{3}$ of 12 plus $\frac{2}{3}$ of 6 is $\frac{2}{3}$ of $(12 + 6)$, and show how to shorten the work when the fractions are alike.

That is, state a rule for finding such products.

At sight give:

53.	$4\frac{2}{3} \times 8\frac{2}{3}$.	**60.**	$14\frac{1}{4} \times 6\frac{1}{4}$.
54.	$3\frac{3}{4} \times 5\frac{3}{4}$.	**61.**	$9\frac{1}{3} \times 6\frac{1}{3}$.
55.	$8\frac{2}{5} \times 7\frac{2}{5}$.	**62.**	$7\frac{1}{4} \times 9\frac{1}{4}$.
56.	$7\frac{3}{4} \times 9\frac{3}{4}$.	**63.**	$6\frac{1}{5} \times 9\frac{1}{5}$.
57.	$5\frac{2}{3} \times 7\frac{2}{3}$.	**64.**	$12\frac{1}{4} \times 8\frac{1}{4}$.
58.	$9\frac{3}{4} \times 11\frac{3}{4}$.	**65.**	$14\frac{1}{3} \times 10\frac{1}{3}$.
59.	$16\frac{2}{3} \times 5\frac{2}{3}$.	**66.**	$16\frac{1}{5} \times 14\frac{1}{5}$.

67. Find $17\frac{3}{5} \times 47\frac{3}{5}$.

WORK

$$\begin{array}{r} 47\frac{3}{5} \\ 17\frac{3}{5} \\ \hline \frac{9}{25} \\ 38\frac{2}{5} \\ 329 \\ 47 \\ \hline 837\frac{19}{25} \end{array}$$

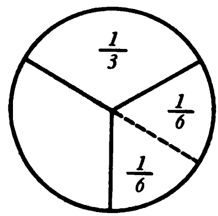
EXPLANATION. — Here the 47 and 17 are added and $\frac{3}{5}$ of the sum taken. This saves one of the four general steps.

Find:

68.	$96\frac{1}{3} \times 84\frac{1}{3}$.	**75.**	$54\frac{3}{5} \times 46\frac{3}{5}$.
69.	$53\frac{1}{4} \times 68\frac{1}{4}$.	**76.**	$17\frac{2}{3} \times 25\frac{2}{3}$.
70.	$43\frac{1}{5} \times 67\frac{1}{5}$.	**77.**	$26\frac{2}{3} \times 18\frac{2}{3}$.
71.	$16\frac{1}{7} \times 19\frac{1}{7}$.	**78.**	$45\frac{2}{5} \times 25\frac{2}{5}$.
72.	$46\frac{2}{3} \times 16\frac{2}{3}$.	**79.**	$56\frac{3}{4} \times 24\frac{3}{4}$.
73.	$52\frac{3}{4} \times 18\frac{3}{4}$.	**80.**	$62\frac{2}{5} \times 18\frac{2}{5}$.
74.	$42\frac{5}{6} \times 16\frac{5}{6}$.	**81.**	$42\frac{1}{7} \times 28\frac{1}{7}$.

DIVISION OF FRACTIONS

59. Dividing a fraction by a whole number. — To divide a fraction by a whole number means to separate it into a number of equal parts (see § 30). For example, to divide $\frac{1}{3}$ by 2 means to divide it into 2 equal parts, as shown in the figure, thus giving sixths.



Hence a fraction is divided by a whole number by multiplying the denominator by that number.

EXERCISES

At sight give:

1.	$\frac{1}{4} \div 2$.	**5.**	$\frac{1}{5} \div 3$.	**9.**	$\frac{1}{5} \div 5$.	**13.**	$\frac{3}{5} \div 2$.	**17.**	$\frac{3}{4} \div 5$.
2.	$\frac{1}{3} \div 3$.	**6.**	$\frac{1}{2} \div 4$.	**10.**	$\frac{1}{6} \div 3$.	**14.**	$\frac{2}{7} \div 3$.	**18.**	$\frac{3}{8} \div 2$.
3.	$\frac{1}{5} \div 2$.	**7.**	$\frac{1}{3} \div 5$.	**11.**	$\frac{2}{3} \div 3$.	**15.**	$\frac{2}{3} \div 5$.	**19.**	$\frac{2}{3} \div 7$.
4.	$\frac{1}{6} \div 2$.	**8.**	$\frac{1}{4} \div 3$.	**12.**	$\frac{3}{4} \div 2$.	**16.**	$\frac{3}{5} \div 4$.	**20.**	$\frac{5}{6} \div 2$.

21. Divide $\frac{6}{7}$ by 3.

WORK

$$\frac{6}{7} \div 3 = \frac{6}{3 \times 7} = \frac{2}{7}.$$

Hence, dividing the numerator by a number divides the fraction by that number.

This same truth is seen from another consideration. For, just as $6 ÷ 3 = $2, so $\frac{6}{7} \div 3 = \frac{2}{7}$, or 6 of any unit $\div 3 = 2$ of that unit.

At sight give :

22. $\frac{8}{9} \div 2$.	**26.** $\frac{12}{13} \div 4$.	**30.** $\frac{16}{17} \div 8$.	**34.** $\frac{15}{17} \div 3$.
23. $\frac{6}{7} \div 3$.	**27.** $\frac{8}{9} \div 4$.	**31.** $\frac{18}{19} \div 3$.	**35.** $\frac{24}{25} \div 6$.
24. $\frac{15}{16} \div 5$.	**28.** $\frac{6}{11} \div 2$.	**32.** $\frac{15}{16} \div 3$.	**36.** $\frac{21}{25} \div 3$.
25. $\frac{14}{15} \div 7$.	**29.** $\frac{10}{11} \div 5$.	**33.** $\frac{21}{22} \div 7$.	**37.** $\frac{28}{33} \div 4$.

38. Divide $\frac{8}{9}$ by 6.

WORK

$$\frac{8}{9} \div 6 = \frac{\overset{4}{\cancel{8}}}{9 \times \underset{3}{\cancel{6}}} = \frac{4}{27}.$$

Thus common factors may be canceled before multiplying.

Find :

39. $\frac{8}{11} \div 12$.	**43.** $\frac{32}{33} \div 18$.	**47.** $\frac{14}{15} \div 21$.	**51.** $\frac{18}{19} \div 12$.
40. $\frac{12}{13} \div 8$.	**44.** $\frac{9}{10} \div 6$.	**48.** $\frac{21}{22} \div 14$.	**52.** $\frac{27}{28} \div 18$.
41. $\frac{16}{17} \div 6$.	**45.** $\frac{10}{11} \div 15$.	**49.** $\frac{12}{17} \div 8$.	**53.** $\frac{21}{23} \div 19$.
42. $\frac{24}{25} \div 16$.	**46.** $\frac{12}{13} \div 9$.	**50.** $\frac{15}{17} \div 10$.	**54.** $\frac{28}{29} \div 21$.

60. Dividing a fraction by a fraction. — To divide one fraction by another is to see how many times the first will contain the second. This kind of division has the same meaning as 16 qt. $\div$ 4 qt. $= 4$. For, just as 16 qt. $\div$ 4 qt. $= 4$, so $\frac{16}{21} \div \frac{4}{21} = 4$, or 16 of any unit will contain 4 of that unit 4 times. This, then, is what was called in § 30 *measurement.* Evidently, then, both dividend and divisor must be expressed in like units.

EXERCISES

At sight give :

1. $\frac{4}{5} \div \frac{2}{5}$.	**5.** $\frac{9}{11} \div \frac{3}{11}$.	**9.** $\frac{12}{13} \div \frac{4}{13}$.	**13.** $\frac{7}{10} \div \frac{3}{10}$.
2. $\frac{6}{7} \div \frac{2}{7}$.	**6.** $\frac{10}{11} \div \frac{5}{11}$.	**10.** $\frac{9}{10} \div \frac{3}{10}$.	**14.** $\frac{5}{11} \div \frac{2}{11}$.
3. $\frac{8}{9} \div \frac{4}{9}$.	**7.** $\frac{12}{13} \div \frac{3}{13}$.	**11.** $\frac{8}{9} \div \frac{5}{9}$.	**15.** $\frac{11}{8} \div \frac{3}{8}$.
4. $\frac{8}{11} \div \frac{2}{11}$.	**8.** $\frac{14}{15} \div \frac{2}{15}$.	**12.** $\frac{9}{10} \div \frac{4}{10}$.	**16.** $\frac{7}{9} \div \frac{8}{9}$.

17. $\frac{10}{13} \div \frac{3}{13}$.	**20.** $\frac{12}{13} \div \frac{7}{13}$.	**23.** $\frac{2}{3} \div \frac{1}{2}$.	**26.** $\frac{7}{8} \div \frac{1}{4}$.
18. $\frac{12}{13} \div \frac{6}{13}$.	**21.** $\frac{19}{20} \div \frac{8}{20}$.	**24.** $\frac{4}{5} \div \frac{1}{2}$.	**27.** $\frac{5}{6} \div \frac{1}{3}$.
19. $\frac{16}{17} \div \frac{5}{17}$.	**22.** $\frac{1}{2} \div \frac{1}{4}$.	**25.** $\frac{6}{7} \div \frac{1}{3}$.	**28.** $\frac{8}{9} \div \frac{1}{3}$.

61. Dividing by inverting the divisor. — By observing all of the steps involved in changing fractions to like units, a method of shortening the work may be found. Thus, to divide $\frac{3}{4}$ by $\frac{5}{7}$, we first change to twenty-eighths by multiplying both terms of $\frac{3}{4}$ by 7, and both terms of $\frac{5}{7}$ by 4, so that for $\frac{3}{4}$ we have $\frac{3 \times 7}{28}$, and for $\frac{5}{7}$ we have $\frac{4 \times 5}{28}$. But it is the numerator 3×7 divided by the numerator 4×5 that is used. The quotient expressed in fractional form is $\frac{3 \times 7}{4 \times 5}$, which is the product of $\frac{3}{4} \times \frac{7}{5}$. Hence, $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20} = 1\frac{1}{20}$. That is,

To divide one fraction by another, invert the divisor and multiply.

EXERCISES

Find:

1. $\frac{2}{3} \div \frac{2}{4}$.	**9.** $\frac{2}{3} \div \frac{5}{9}$.	**17.** $\frac{5}{7} \div \frac{1}{3}$.	**25.** $\frac{4}{5} \div \frac{2}{7}$.
2. $\frac{3}{5} \div \frac{2}{3}$.	**10.** $\frac{3}{4} \div \frac{2}{7}$.	**18.** $\frac{14}{15} \div \frac{9}{20}$.	**26.** $\frac{7}{8} \div \frac{3}{5}$.
3. $\frac{4}{5} \div \frac{5}{6}$.	**11.** $\frac{5}{6} \div \frac{5}{8}$.	**19.** $\frac{5}{9} \div \frac{11}{12}$.	**27.** $\frac{4}{7} \div \frac{5}{14}$.
4. $\frac{4}{7} \div \frac{5}{6}$.	**12.** $\frac{7}{8} \div \frac{1}{6}$.	**20.** $\frac{2}{7} \div \frac{3}{8}$.	**28.** $\frac{7}{9} \div \frac{5}{12}$.
5. $\frac{6}{7} \div \frac{5}{9}$.	**13.** $\frac{4}{5} \div \frac{6}{35}$.	**21.** $\frac{1}{6} \div \frac{7}{24}$.	**29.** $\frac{5}{8} \div \frac{7}{12}$.
6. $\frac{2}{3} \div \frac{8}{9}$.	**14.** $\frac{8}{9} \div \frac{3}{10}$.	**22.** $\frac{6}{35} \div \frac{4}{15}$.	**30.** $\frac{3}{10} \div \frac{9}{25}$.
7. $\frac{3}{4} \div \frac{9}{10}$.	**15.** $\frac{7}{10} \div \frac{9}{25}$.	**23.** $\frac{3}{10} \div \frac{9}{25}$.	**31.** $\frac{6}{5} \div \frac{7}{15}$.
8. $\frac{3}{10} \div \frac{2}{5}$.	**16.** $\frac{11}{12} \div \frac{3}{16}$.	**24.** $\frac{3}{16} \div \frac{5}{12}$.	**32.** $\frac{4}{9} \div \frac{7}{18}$.

62. Dividing a mixed number by a mixed number. — Evidently both dividend and divisor may be changed to improper fractions and the resulting fractions divided as above.

Another method is to multiply both dividend and divisor by a number that will give whole numbers. Thus, $12\frac{1}{2} \div 2\frac{3}{4} = (4 \times 12\frac{1}{2}) \div (4 \times 2\frac{3}{4})$, or $50 \div 11$.

EXERCISES

1. Find $345\frac{1}{2} \div 16\frac{3}{4}$.

WORK

$$\begin{array}{r} 345\frac{1}{2} \div 16\frac{3}{4} \\ 4 \quad\quad 4 \\ \hline = 1382 \div 67 \end{array} \qquad \begin{array}{r} 20\frac{42}{67} \\ 67\overline{)1382} \\ 134 \\ \hline 42 \end{array}$$

Find:

2. $362\frac{3}{4} \div 13\frac{1}{2}$.	6. $984\frac{5}{6} \div 26\frac{1}{3}$.	10. $8462 \div 17\frac{2}{3}$.
3. $875\frac{3}{5} \div 21\frac{2}{3}$.	7. $1054\frac{3}{8} \div 17\frac{1}{2}$.	11. $1980 \div 15\frac{3}{4}$.
4. $495\frac{3}{4} \div 16\frac{7}{8}$.	8. $1176\frac{2}{3} \div 14\frac{5}{9}$.	12. $1642 \div 17\frac{3}{4}$.
5. $630\frac{7}{8} \div 15\frac{3}{4}$.	9. $1263\frac{3}{4} \div 19\frac{2}{3}$.	13. $1580 \div 17\frac{2}{3}$.

63. Dividing a mixed number by a whole number. — This differs from division of whole numbers only in expressing the remainder as a fractional part of the divisor.

EXERCISES

1. Divide $3896\frac{2}{3}$ by 7.

WORK

$$\begin{array}{l} 7\underline{)3896\frac{2}{3}} \\ \quad 556,\ 4\frac{2}{3} \text{ rem.} \end{array}$$

$4\frac{2}{3} \div 7 = 14 \div 21 = \frac{2}{3}$.

Hence, $3896\frac{2}{3} \div 7 = 556\frac{2}{3}$.

EXPLANATION. — The remainder $4\frac{2}{3}$ is expressed as a fractional part of 7 by dividing by 7.

Multiplying both $4\frac{2}{3}$ and 7 by 3, the denominator of the fraction, gives $14 \div 21$, or $\frac{2}{3}$.

Find:

2. $3846\frac{2}{5} \div 8$.	8. $3462\frac{1}{2} \div 9$.	14. $384\frac{3}{5} \div 15$.
3. $7613\frac{1}{2} \div 5$.	9. $678\frac{2}{3} \div 8$.	15. $867\frac{7}{8} \div 18$.
4. $6184\frac{1}{3} \div 5$.	10. $769\frac{1}{7} \div 9$.	16. $936\frac{4}{7} \div 24$.
5. $7163\frac{3}{4} \div 6$.	11. $846\frac{5}{8} \div 7$.	17. $837\frac{5}{9} \div 19$.
6. $1986\frac{2}{3} \div 7$.	12. $769\frac{3}{8} \div 9$.	18. $869\frac{5}{7} \div 26$.
7. $2769\frac{2}{5} \div 6$.	13. $682\frac{3}{4} \div 11$.	19. $980\frac{7}{9} \div 35$.

64. Complex fractions. — A **complex fraction** is merely an indicated division, expressed in the form of a fraction, where one or both of the terms are fractions or mixed numbers. They arise in expressing the work to be done in the solution of a problem. The following are complex fractions:

$$\frac{3\frac{1}{2}}{7},\ \frac{3}{4\frac{1}{2}},\ \frac{3\frac{1}{4}}{5\frac{2}{3}},\ \frac{\frac{3}{7}}{5},\ \frac{\frac{3}{8}}{5\frac{1}{2}},\ \frac{\frac{5}{9}}{\frac{7}{8}}.$$

Such fractions are simplified by performing the indicated divisions. It is often more simple to multiply both terms of the complex fraction by the least common denominator of the fractions composing the terms than to perform the division.

Thus, $\frac{3\frac{1}{2}}{4} = \frac{2 \times 3\frac{1}{2}}{2 \times 4} = \frac{7}{8}$; $\frac{2\frac{2}{5}}{3\frac{2}{3}} = \frac{15 \times 2\frac{2}{5}}{15 \times 3\frac{2}{3}} = \frac{36}{55}$; etc.

NOTE. — The multiplications and divisions must be performed before the additions and subtractions.

EXERCISES

Simplify:

1. $\frac{3\frac{1}{4}}{5}$.
2. $\frac{6\frac{1}{2}}{7\frac{1}{3}}$.
3. $\frac{5\frac{2}{3}}{8\frac{3}{4}}$.
4. $\frac{6\frac{2}{3}}{7\frac{1}{2}}$.
5. $\frac{12}{13\frac{3}{4}}$.
6. $\frac{15}{17\frac{1}{2}}$.
7. $\frac{12\frac{1}{2}}{30}$.
8. $\frac{\frac{3}{4}}{2\frac{2}{3}}$.
9. $\frac{6\frac{3}{4}}{15}$.
10. $\frac{5\frac{1}{2}}{\frac{2}{7}}$.
11. $\frac{6\frac{1}{2}}{\frac{3}{4}}$.
12. $\frac{\frac{7}{8}}{9\frac{1}{2}}$.
13. $\frac{(\frac{3}{5} + \frac{2}{7}) \times (\frac{2}{3} - \frac{1}{4})}{2\frac{1}{2} \times \frac{7}{8} + 4\frac{1}{3} - 2}$.
14. $\frac{4\frac{3}{8} - 1\frac{3}{4}}{4\frac{2}{7} \times 5\frac{3}{5}} \times 6\frac{1}{2}$.
15. $\frac{1}{3} \times \frac{3\frac{1}{7}}{4} + \frac{2\frac{1}{3}}{\frac{1}{14}}$.
16. $\frac{18\frac{1}{2} \times 7\frac{2}{5}}{8\frac{3}{4} - 6\frac{1}{5}} \times 2\frac{1}{2}$.
17. $\frac{17\frac{5}{12} - 9\frac{3}{4} + 4\frac{5}{7}}{\frac{5}{9} \times 9\frac{2}{7}}$.
18. $\frac{3\frac{3}{4} - \frac{2}{7} \times 4\frac{1}{5}}{\frac{5}{18} \times \frac{27}{20}}$.
19. $\frac{2\frac{1}{3} + 4\frac{1}{4}}{1 + \frac{2}{3} + \frac{1}{2}}$.
20. $\frac{2\frac{2}{3}}{\frac{3}{4} - \frac{1}{8} \div \frac{1}{2}}$.

21. $\dfrac{4\frac{1}{2} \times 3\frac{2}{3}}{4\frac{1}{2} + 3\frac{2}{3}} \times \dfrac{5}{6}.$

22. $\dfrac{\frac{3}{8} + \frac{13}{16} + 1\frac{3}{4}}{2 - 1\frac{1}{4} + \frac{15}{16}}.$

23. $\dfrac{\frac{2}{5} + 3\frac{3}{5} + 4\frac{1}{2}}{4\frac{4}{5} \times \frac{2}{3}}.$

24. $\dfrac{(2\frac{1}{2} + \frac{3}{5}) \div \frac{2}{3} \times \frac{3}{5}}{2\frac{1}{2} - \frac{2}{5} \times 1\frac{2}{3}}.$

25. $\dfrac{2 \times (\frac{4}{5} - \frac{9}{28} \div 2\frac{4}{7} - \frac{5}{12} \times \frac{21}{20})}{1 \div (9 - 2 \times 2)}.$

26. $\dfrac{3\frac{1}{2} + 2\frac{1}{3} - 1\frac{1}{4}}{\frac{3}{2} \times \frac{10}{3}} \div 1\frac{3}{8}.$

27. $\dfrac{\frac{3}{5} + \frac{1}{2} \times \frac{2}{3}}{\frac{1}{8} + \frac{5}{7} \times \frac{7}{9}} \times \dfrac{\frac{2}{3}}{4}.$

28. $\dfrac{(\frac{3}{7} \times 2\frac{1}{3}) \div (\frac{3}{5} \times 7\frac{1}{2}) + \frac{3}{4} \times \frac{8}{9}}{\frac{1}{2} \times (\frac{3}{4} - \frac{9}{16}) \times (1 - \frac{5}{9})}.$

29. $\dfrac{\frac{7}{16} + 1\frac{1}{2} \times \frac{2}{5} + \frac{1}{8}}{1 \div (\frac{5}{6} - \frac{1}{15})}.$

30. $\dfrac{\frac{3}{4} - \frac{2}{3}}{\frac{7}{8} - \frac{3}{4}} + \frac{4}{5} + \frac{3}{5}.$

31. $\dfrac{\frac{9}{10} \div (\frac{1}{6} + \frac{3}{10} + \frac{11}{15})}{1\frac{5}{9} \times 3\frac{3}{4} - 5\frac{1}{4} \div 1\frac{2}{5}}.$

32. $\dfrac{2\frac{1}{2} - 1\frac{2}{3} + \frac{5}{6}}{\frac{2}{5} \times (\frac{3}{4} - \frac{1}{2})} \div \dfrac{2}{3}.$

33. $\dfrac{4\frac{3}{8} \times \frac{9}{14} + 1\frac{4}{5}}{4\frac{5}{7} \div \frac{11}{28}}.$

34. $\dfrac{2\frac{1}{3} + \frac{5}{9} \times (12\frac{3}{5} + 11\frac{2}{3})}{4\frac{7}{8} - 8\frac{2}{3} \times \frac{1}{2} + 1\frac{1}{9}}.$

35. $\dfrac{(\frac{4}{15} + 1\frac{1}{3}) \div (3\frac{1}{9} \times \frac{4}{7})}{1 - \frac{2}{3} \times \frac{3}{20}}.$

36. $\dfrac{\frac{2}{3} \times (3\frac{1}{2} - 1\frac{2}{3})}{\frac{3}{4} \div 1\frac{1}{2}} \times \dfrac{\frac{2}{3}}{5}.$

65. Use of fractions. — Problems are solved by knowing the relation of what is wanted to what is given. Often this relation is a fraction. Hence the ability to express the needed relation as a fraction is of fundamental importance. The study of fractions, then, is necessary to develop the power to see and use fractional relations in the solution of problems. This will be seen in many of the problems that follow.

PROBLEMS INVOLVING FRACTIONS

Solve without a pencil

1. If I pay \$.60 for $\frac{5}{6}$ of a yard of cloth, how much is that per yard?

2. If $\frac{3}{4}$ of a yard of lace cost \$.72, how much will $3\frac{1}{2}$ yards cost?

3. I filled my coal bin $\frac{7}{8}$ full at a cost of \$105. At the end of the season it was $\frac{1}{4}$ full. How much was the remaining coal worth?

4. If a manufacturer's profit is $\frac{1}{8}$ as much as the cost when selling an article for \$180, what did it cost him to manufacture it?

5. A manufacturer finds that the total cost to make and sell an article is \$10. At what price should he list it to make a profit of $\frac{1}{6}$ of the list price?

6. At what should a manufacturer list an article costing \$96 to make a profit equal to $\frac{1}{4}$ of the list price? Equal to $\frac{1}{4}$ of the cost?

7. A merchant received $\frac{6}{7}$ of the cost of a whole shipment of goods for only $\frac{3}{4}$ of them. If the remaining $\frac{1}{4}$ are sold at the same rate, what part of the cost will he make?

8. In problem 7, what part of the selling price is the merchant's profit?

9. A man sells $\frac{2}{3}$ of his farm for what $\frac{4}{5}$ of it cost. What part of the cost is he making? What part of the selling price is he making?

10. If $2\frac{1}{2}$ acres produce $212\frac{1}{2}$ bushels of potatoes, how much will $7\frac{1}{2}$ acres produce at the same rate?

11. If a boy rides his bicycle $10\frac{5}{6}$ miles in $1\frac{2}{3}$ hours, how far can he ride in $3\frac{1}{3}$ hours at the same rate?

12. The cost of an article was increased from 16¢ to 20¢. The increase was what part of the former cost? Of the increased cost?

13. Using the data given in problem 12, what will be the increased cost of an order formerly costing \$28?

14. Using the same data, what would an order formerly have cost that now costs \$75?

15. After marking down an overcoat $\frac{1}{3}$, it sold for \$16. At what price was it marked?

16. A merchant marked goods down $\frac{1}{3}$ and still made $\frac{1}{5}$ of the cost. They were marked to gain what part of the cost? What part of the first marked price?

17. A tank is $\frac{2}{3}$ full. After 1340 gallons have been drawn off it is still $\frac{1}{6}$ full. Find the capacity of the tank.

18. If the profit equals $\frac{1}{6}$ of the selling price, what part of the cost does it equal?

19. If the profit equals $\frac{1}{4}$ of the cost, what part of the selling price does it equal?

20. After spending $\frac{1}{4}$ of his money for a suit, and $\frac{1}{6}$ of it for an overcoat, a boy had \$21 left. How much did each article cost?

21. A merchant paid \$450 for cloth which he sells at \$2 per yard. If this gives him a profit equal to $\frac{1}{3}$ of the cost, how many yards did he buy?

22. A boy spent $\frac{5}{8}$ of his money for a bicycle, and had \$15 left. How much had he at first?

23. A merchant paid \$4500 for some goods which he sold so as to make a profit equal to $\frac{1}{6}$ of the sales. Find his gain.

24. If 5 tons of hay cost \$87.50, how much will 2 tons cost at the same rate?

25. A farmer sold $\frac{1}{3}$ of his crop of apples to one man, and $\frac{1}{4}$ of the crop to another. If he then had 150 bushels left, how many bushels did he sell?

26. If $\frac{3}{5}$ of the enrollment in a certain high school are girls and there are 480 boys, how many girls are there?

27. After the price of an article was lowered $\frac{1}{5}$, it cost \$120. What was the price before it was lowered?

28. A man earning $175 per month spends $35 of it for rent and $50 for food and $75 for all other expenses, and places the rest in the savings bank. What part of his salary is used for each item?

29. If the same part of a man's income as in problem 28 is used for each item when he is earning $225 monthly, how much will be used for each?

30. A boy sold his bicycle for $\frac{8}{9}$ of the cost. If it cost him $45, what did he get for it? If he had got $32 for it, as $\frac{8}{9}$ of the cost, how much must it have cost him?

31. Two men own a business. At first each owned $\frac{1}{2}$ of it. Later one sold $\frac{1}{3}$ of his share to the other. What was the relation then of their interests in the business?

32. A purchasing agent got a commission of $\frac{1}{12}$ of the cost of the goods purchased. To earn $240 per month, how much must he purchase?

33. A commission merchant gets $\frac{1}{40}$ of the price received from the sales of a certain article. How much must he sell to make $12 per day?

34. If an agent gets $\frac{1}{25}$ of the amount of his sales, how much must he sell to make $40 per week?

35. An agent gets $\frac{2}{5}$ of the sales of a certain article. To make $150 per month, what must his sales be?

36. In a certain factory employing 350 men, 70 are laid off. Those laid off are what part of the whole force? If the force is reduced again by laying off the same fractional part of those remaining, how many are still employed?

37. A man bought a farm and paid $\frac{2}{5}$ of the price at the time of purchase and had $1800 yet to pay. How much did he pay at the time of purchase?

38. I paid my builder $\frac{1}{4}$ of the contract price of a house at one time and $\frac{1}{5}$ of it at another, and still have \$3300 to pay. What was the contract price?

39. If John has only $\frac{3}{5}$ as much money as James, and together they have \$6.40, how much has each?

PROBLEMS INVOLVING FRACTIONS

Use a pencil only when necessary

1. If a farmer raised 330 bushels of potatoes from $2\frac{3}{4}$ acres, how much should he get from $5\frac{1}{4}$ acres that yield the same per acre?

2. A merchant received an invoice of goods costing \$9250 which he marked $\frac{3}{5}$ above cost, but they were sold for $\frac{2}{3}$ of the marked price. Find the gain.

3. If a planer makes a cut $\frac{3}{4}$ of an inch wide, how many strokes will it have to make in planing a casting $10\frac{1}{8}$ inches wide?

4. A car of wheat containing 93,730 pounds is loaded $\frac{1}{12}$ over its permitted capacity. How much should be removed?

5. If you use an average of $\frac{1}{15}$ of a ton of coal a day, how many days will $8\frac{1}{3}$ tons last?

6. A manufacturer listed his goods at $\frac{5}{8}$ more than the cost, but discounted them $\frac{1}{3}$ of the list price. Find the cost and list price of an article upon which he made \$26.

7. At how much above cost must a manufacturer list goods so that he will make a profit equal to $\frac{1}{4}$ of the cost after discounting the list price $\frac{1}{4}$? On this plan, at what must he list goods costing \$45 to manufacture?

8. If it costs \$160 to manufacture an article, at what shall it be listed to make a profit equal to $\frac{1}{4}$ of the cost when discounting the list price $\frac{1}{6}$?

9. At $11\frac{1}{4}$ cents per pound, a bale of cotton was worth \$55.80. What would it have been worth at $13\frac{3}{8}$ cents per pound?

10. In making jute twine the material cost $8\frac{3}{5}$ cents per pound, the spinning $\frac{7}{8}$ cents per pound, and the twisting $\frac{15}{16}$ cents per pound. If the total cost of a lot of twine was \$16,660, how much did each of the three items cost?

11. If an automobile runs $\frac{3}{4}$ of a mile in $\frac{4}{5}$ of a minute, what is the rate per hour?

12. At an election 510 votes were cast. If $\frac{2}{3}$ of the number cast for one candidate was equal to $\frac{3}{4}$ of the number cast for the other, find the number cast for each.

13. When $\frac{5}{8}$ of a bill of goods sell for the cost of the whole bill, what part of the cost is gained?

14. A boy made 9 cents by buying apples at the rate of 3 for 5¢ and selling them at the rate of 4 for 7¢. How many did he sell?

15. Gunpowder is made of 15 parts saltpetre, 2 parts sulphur, and 3 parts charcoal. Give the fractional part of each in the whole.

16. When goods costing \$450 sell for \$650, the profit equals what part of the cost? What part of the selling price?

17. If goods cost \$1340 to manufacture, at what must the manufacturer sell them to make $\frac{1}{6}$ of the selling price?

18. If goods are listed to sell at a gain equal to $\frac{1}{6}$ of the cost, what is the cost of goods listed at \$1631?

19. How much is made on goods costing \$1728 if sold so as to make $\frac{1}{5}$ of the selling price?

20. How much is made on goods listed at \$1345 if the gain equals $\frac{1}{4}$ of the cost?

21. At 28 ¢ per pound find the cost of $4\frac{1}{2}$ lb. of beef and $5\frac{3}{4}$ lb. of lamb.

22. Allowing $1\frac{3}{4}$ lb. of meat for each of a camping party, how much must be ordered for a party of 18?

23. When the price of sugar increases from $4\frac{3}{4}$ ¢ per pound to $8\frac{1}{2}$ ¢ per pound, what is the increased cost to a family that uses 575 lb. per year?

24. When the average cost of meats increase from $18\frac{1}{2}$ ¢ per pound to $23\frac{1}{4}$ ¢ per pound, what is the increase in cost to an institution that uses 150 lb. daily for 185 da. each year?

25. If the labor cost is $\frac{1}{3}$ of the entire cost of an article, how much will an increase of $\frac{1}{4}$ in wages increase the entire cost?

26. A family of five found that the average cost per month of three items of food were as follows: bread $6.15, meat $24.60, fruit and vegetables $14.50. If prices remain the the same, what should they expect to spend for each item when two are away?

27. If there is an average increase of $\frac{1}{6}$ in price, what should be the average cost of each item in Problem 26 when one is away?

28. A family of six found that in 10 months they used $12\frac{3}{4}$ bu. of potatoes. At the same rate, how many bushels will an institution of 350 use in 8 months?

29. If $13\frac{1}{2}$ acres of corn yield $630\frac{1}{2}$ bu., find the average yield per acre. At the same rate find the yield of 54 acres. Check the last result by finding it in two ways.

30. The weight of 8 dressed fowls was as follows: $5\frac{3}{4}$ lb., $6\frac{1}{8}$ lb., $4\frac{7}{8}$ lb., $5\frac{1}{2}$ lb., $3\frac{7}{8}$ lb., $4\frac{7}{8}$ lb., $5\frac{1}{4}$ lb., and $5\frac{5}{8}$ lb. Find the average weight per fowl.

CHAPTER III

DECIMAL FRACTIONS

66. A decimal notation. — In our system of writing numbers, 10 units of any order make 1 unit of the next higher order, and for that reason our system of writing numbers is called a **decimal system**, as stated in § 6.

67. A decimal fraction. — If we extend our notation to the right of ones' place, the first order to the right is *tenths*, the next *hundredths*, the next *thousandths*, etc. All orders, then, to the right of ones' place are fractions, and since they have a decimal relation to ones, they are called **decimal fractions.**

68. A mixed decimal. — A number composed of a whole number and a decimal fraction is a **mixed decimal.**

In reading a mixed decimal the word *and* should be used between the whole number and the fraction. Thus, 19.047 should be read "nineteen and forty-seven thousandths".

EXERCISES

Read:

1. .38.	**5.** 25.04.	**9.** .324.
2. .465.	**6.** 38.125.	**10.** 300.024.
3. .075.	**7.** 56.045.	**11.** .576.
4. .1685.	**8.** 52.052.	**12.** 500.076.

ADDITION AND SUBTRACTION

69. Adding and subtracting decimals. — Since only *like* units can be added or subtracted, the decimal points are written under each other in addition and subtraction in order to bring like units in the same column.

EXERCISES

Add:

1. 96.875	2. 348.6	3. 48.16	4. .065
13.94	96.85	9.043	7.84
6.072	105.046	154.68	9.63
19.8	34.98	9.76	19.8
6.42	16.7	19.403	4.683

5. 6.843, 84.052, .06, 12.008, .096.

6. 248.05, 47.68, .9003, 2.9684, 26.0806.

Subtract:

7. 46.08	9. 40.93	11. 142.06	13. 63.012
19.684	17.842	87.938	46.98

8. 13.403	10. 130.9	12. 54.096	14. 42.5
8.97	96.843	4.18	16.873

15. 8.06 from 21.754. 16. 290.008 from 468.7.

MULTIPLICATION

70. Multiplying a decimal by 10, 100, etc. — Since in our decimal place-value notation each figure of any order represents a value 10 times as great as it would in the order to the right of it, every time the decimal point is moved one order to the right, the number is multiplied by 10, for the effect is the same as though every figure had been moved one order to the left.

EXERCISES

Multiply by 10, 100, *and* 1000:

1. 3.45.	5. 17.3.	9. 9.1.	13. 8.46.
2. 2.68.	6. 68.7.	10. 8.4.	14. 9.081.
3. 435.2.	7. 96.8.	11. 6.7.	15. .035.
4. 684.3.	8. 84.3.	12. 7.38.	16. .006.

71. Multiplying by .1, .01, etc. — To multiply a number by .1 is to find one tenth of it. That is, it is found by dividing by 10. Likewise, to multiply by .01 is to divide by 100. Since division is the inverse of multiplication, from § 70 it follows that every time a decimal point is moved one order to the left, the number is multiplied by .1

EXERCISES

Multiply by .1, *and* .01:

1. 38.2.	5. 96.48.	9. 3.8.	13. 3.7.
2. 146.1.	6. 1340.	10. 15.4.	14. .6.
3. 3.5.	7. 34.25.	11. 340.	15. .35.
4. 348.	8. 26.	12. 7.2.	16. .07.

72. Multiplying a decimal by a whole number. — Since multiplying by a whole number may be considered a short form of addition, the unit of the product must be the same as the unit of the multiplicand. That is, multiplying a decimal by a whole number gives a product with the same number of decimal places as there are in the multiplicand.

EXERCISES

1. 4×3.98.	3. 5×93.7.	5. 9×1.756.
2. 6×7.83.	4. 8×4.36.	6. 24×8.46.

7. 32×9.27.	10. $47 \times .893$.	13. $93 \times .074$.
8. $64 \times .843$.	11. $48 \times .096$.	14. $83 \times .097$.
9. $89 \times .736$.	12. $87 \times .043$.	15. $39 \times .084$.

73. Multiplying a decimal by a decimal. — Since to multiply by any fraction is to perform both a division and a multiplication, the number of decimal places in the product depends upon those of both multiplier and multiplicand.

EXERCISES

1. Multiply 3.46 by .9.

WORK

$$\begin{array}{r} 3.46 \\ .9 \\ \hline 3.114 \end{array}$$

EXPLANATION. — To multiply by .9 is to find $\frac{9}{10}$ of a number. This requires a division by 10 and a multiplication by 9. Suppose that the multiplication is done first. 9×3.46 gives 31.14. But dividing by 10 moves the point one place to the left (§ 71), and gives 3.114. Hence, *the number of decimal places in the product is the sum of the numbers of decimal places in the factors.*

Multiply:

2. $.8 \times 9.6$.	7. 1.6×8.5.	12. $4.5 \times .862$.
3. $.7 \times .34$.	8. 1.7×9.6.	13. $2.41 \times .65$.
4. $.9 \times 17.2$.	9. 2.4×7.6.	14. $4.82 \times .85$.
5. $.8 \times 9.63$.	10. $8.3 \times .481$.	15. $6.24 \times .953$.
6. $.6 \times 7.34$.	11. $9.6 \times .534$.	16. $5.8 \times .4508$.

74. Abridged process of multiplication. — When several decimals occur in both factors, the product may contain more decimals than are required in the particular problem. In such cases all figures need not be kept. It is most convenient to begin the multiplication by the *highest* order first.

EXERCISES

1. Find the product of 34.265 and 3.1416 to the second decimal place.

ABRIDGED WORK

$$\begin{array}{r} 34.265 \\ 3.1416 \\ \hline 102.795 \\ 3.4265 \\ 1.3706 \\ 342 \\ 205 \\ \hline 107.65 \end{array}$$

EXPLANATION. — First multiply by 3, then by 1, etc. Since the result is to be the nearest second decimal, four decimals are kept in the partial products in order to get accurately the number to "carry" to the column whose sum is desired. If great care is used in expressing the right-hand figure of each partial product, only one more decimal place than the number wanted in the complete product is needed.

Find the products, approximated to tenths, of the following:

2. 34.6×38.42.
3. 9.362×84.52.
4. 62.96×3.141.
5. 8.695×6.843.
6. 36.81×49.26.
7. 26.83×3.1416.
8. 879.6×1.732.
9. 89.07×1.414.
10. 32.61×4.208.
11. 396.2×16.84.

DIVISION

75. Dividing a decimal by a whole number. — It was stated in § 30 that dividing by a whole number means *partition* and that the quotient is like the thing divided. That is, $\$8 \div 2 = \$4$; 8 ft. $\div 2 = 4$ ft.; $.8 \div 2 = .4$; $.08 \div 2 = .04$; etc.

EXERCISES

1. Divide 183.38 by 53.

WORK

```
      3.46
53)183.38
   159
    243
    212
     318
     318
```

EXPLANATION. — Since the divisor is a whole number, each quotient is of the same unit as the number divided. Hence if each quotient figure is placed directly over the number divided, the decimal point in the quotient must be placed over the decimal point of the dividend.

In the illustration, 183 ones divided by 53 gives a quotient of 3 ones, with 24 ones (the remainder) undivided. Begin the quotient, then, by putting 3 over the ones' place. "Bring down" 3, and you have 243 tenths divided by 53, which gives 4 tenths, with a remainder of 81 tenths undivided. Put 4 into the quotient, over the tenths' place. "Bring down" 8 and proceed as before.

Divide:

2. $83.507 \div 113$.
3. $731.79 \div 173$.
4. $5.7629 \div 143$.
5. $1778.7 \div 121$.
6. $19.740 \div 105$.
7. $288.41 \div 151$.
8. $14.688 \div 108$.
9. $84.3336 \div 7208$.
10. $769.986 \div 7938$.
11. $72.5508 \div 8637$.
12. $1701.00 \div 2700$.
13. $91{,}285.8 \div 706$.
14. $345.144 \div 394$.
15. $88.3666 \div 889$.
16. $3.29832 \div 509$.
17. $7.31430 \div 315$.
18. $0.39346 \div 103$.
19. $0.19781 \div 131$.
20. $300.352 \div 608$.
21. $1402.01 \div 893$.
22. $0.01728 \div 144$.
23. $1.99225 \div 325$.

76. Dividing a decimal by a decimal. — In order that division may mean *partition*, so as to point off as above, the divisor must be a whole number. Since any quotient remains the same if both dividend and divisor are multiplied by the same number, any decimal divisor may be changed to a whole number before division if the same change in the position of the decimal point is made in the dividend.

EXERCISES

1. Divide 4.797 by 2.46.

WORK

```
          1.95
2.46‸)4.79‸70
      2 46
      2 33 7
      2 21 4
        12 30
        12 30
```

POINTS TO OBSERVE IN DIVIDING BY A DECIMAL. — 1. Free the divisor of decimals by moving the point to the right of the right-hand figure.

2. Move the point in the dividend as many places to the right as it is moved in the divisor.

3. Place the point in the quotient above the new position of the point in the dividend.

4. Place each quotient figure above the right hand figure of the partial dividend divided.

Divide:

2. $4.41 \div 0.42.$
3. $97.28 \div 3.2.$
4. $752 \div 0.16.$
5. $95.7 \div 2.9.$
6. $25.11 \div 0.27.$
7. $7.011 \div 5.7.$
8. $70.918 \div 0.059.$
9. $8.700 \div 0.087.$
10. $20.976 \div 0.76.$
11. $44.472 \div 1.02.$
12. $0.7866 \div 85.5.$
13. $879.79 \div 0.907.$
14. $472.131 \div 6.27.$
15. $6.2222 \div 58.7.$
16. $1747.2 \div 0.312.$
17. $57.629 \div 0.0143.$
18. $2898.75 \div 0.0125.$
19. $3.9346 \div 1.03.$

77. Abridged process of division. — When a large number of figures occur in division, work may be saved by cutting off a figure from the divisor instead of bringing down one from the dividend. If care is used, this will not seriously affect the quotient.

EXERCISES

1. Find the quotient of 26.843 divided by 3.1416, to the second decimal place.

ABRIDGED WORK

$$\begin{array}{r} 8.54 \\ 3.\cancel{1}\cancel{4}\cancel{1}\cancel{6})\overline{26.84\cancel{3}} \\ \underline{25.13} \\ 1\;71 \\ \underline{1\;57} \\ 14 \\ \underline{12} \end{array}$$

EXPLANATION. — Multiplying a dividend by a number has the same effect upon the quotient as dividing the divisor by that number. Also, to "bring down" a figure is equivalent to multiplying the remainder by 10 and adding that number; and to "cut off" a figure from the divisor is equivalent to subtracting that number and dividing by 10. Then evidently one may "cut off" a figure from the divisor instead of "bringing down" one from the dividend, without having much effect upon the quotient.

Before beginning the work shown above, the quotient is estimated, and found to be a little more than 8. 8 is set down in the quotient, above the ones' place. Now since the completed quotient is to have two decimal places, two places are kept in the dividend and the rest, that is the 3, is cut off. Then "16" is cut off from the divisor since it is evidently not needed now. The "4" of the divisor is cut off before the second division, then the "1" to its left before the third division.

2. Find the quotient of 3529.4617 divided by 3.1416, to the second decimal place.

ABRIDGED METHOD

$$\begin{array}{rl} 1123.46 & \\ 3.\cancel{1}\cancel{4}\cancel{1}\cancel{6})\overline{3529.46\cancel{1}\cancel{7}} & \\ \underline{3141\;6} & (1) \\ 387\;86 & \\ \underline{314\;16} & (2) \\ 73\;70 & \\ \underline{62\;83} & (3) \\ 10\;87 & \\ \underline{9\;42} & (4) \\ 1\;45 & \\ \underline{1\;26} & (5) \\ 19 & \\ \underline{18} & (6) \end{array}$$

EXACT METHOD

$$\begin{array}{r} 1123.46 \\ 3.1416_{\wedge})\overline{3529.4617_{\wedge}00} \\ \underline{3141\;6} \\ 387\;86 \\ \underline{314\;16} \\ 73\;701 \\ \underline{62\;832} \\ 10\;8697 \\ \underline{9\;4248} \\ 1\;44490 \\ \underline{1\;25664} \\ 188260 \\ \underline{188496} \end{array}$$

Explanation. — In this problem, beginning as in Problem 1, the estimated quotient of 1 thousand and a little more tells us to set 1 in the quotient over the thousands' place. As only two decimal places are required in the answer, the "17" of the dividend is immediately cut off. All of the divisor is needed for division steps (1) and (2). Before step 3, the "6" is cut off from the divisor. In performing the multiplication for step (3) it is noticed that if the "6" had not been cut off, we should have had 1 to carry to the 2. Therefore we carry this 1, getting 3 for the first figure in the product (3). To get close, indeed almost exact results, this care must be taken always.

Observe the correspondence of the quotients obtained by the two methods, and the fact that the abridged method requires writing down altogether only 59 figures as against 81 by the exact method.

Find the quotients, approximated to tenths, of the following:

3. $96.285 \div 43.265$.
4. $396.28 \div 3.1416$.
5. $84.305 \div 9.6281$.
6. $1.4260 \div 0.06345$.
7. $32.465 \div 0.13462$.
8. $3 \div 0.0562$.
9. $8 \div 1.165$.
10. $14.6 \div 3.246$.
11. $256 \div 3.1416$.
12. $85 \div 1.732$.
13. $94.06 \div 1.728$.
14. $107.65 \div 6.318$.
15. $142.5 \div 16.75$.
16. $100 \div .625$.
17. $7.5 \div 18.375$.
18. $16.5 \div 0.7854$.

78. Changing common fractions to decimals. — Since a common fraction may be considered an indicated quotient, it may be changed to a decimal by dividing the numerator by the denominator by the rules for division of decimals. A remainder may be expressed as a common fraction after the division is carried as far as desired. This result is a *complex decimal*. If the remainder is dropped, the result is an *incomplete decimal*.

Thus, $\frac{3}{7} = .428\frac{4}{7}$, or .429, the choice between these forms depending upon the requirements of the particular problem under discussion.

EXERCISES

Change to complex decimals of three places:

1. $\frac{5}{9}$.	5. $\frac{17}{30}$.	9. $\frac{3}{22}$	13. $\frac{16}{31}$.
2. $\frac{13}{16}$.	6. $\frac{16}{19}$.	10. $\frac{8}{17}$.	14. $\frac{17}{42}$.
3. $\frac{9}{14}$.	7. $\frac{17}{18}$.	11. $\frac{19}{33}$.	15. $\frac{16}{53}$.
4. $\frac{11}{12}$.	8. $\frac{5}{17}$.	12. $\frac{14}{27}$.	16. $\frac{18}{47}$.

Change to incomplete decimals of three places:

17. $\frac{7}{12}$.	20. $\frac{7}{15}$.	23. $\frac{8}{43}$.	26. $\frac{17}{42}$.
18. $\frac{9}{17}$.	21. $\frac{4}{11}$.	24. $\frac{16}{21}$.	27. $\frac{19}{21}$.
19. $\frac{5}{18}$.	22. $\frac{2}{19}$.	25. $\frac{18}{35}$.	28. $\frac{42}{43}$.

79. Changing decimals to common fractions. — A complete or complex decimal is changed to a common fraction by expressing the denominator and simplifying.

Thus, $.34 = \frac{34}{100} = \frac{17}{50}$; $.24\frac{3}{5} = \frac{24\frac{3}{5}}{100} = \frac{123}{500}$.

The last result is found by multiplying both terms by 5, the denominator of the fraction in the numerator.

EXERCISES

Change to common fractions:

1. .56.	4. .125.	7. $.33\frac{1}{3}$.	10. $.38\frac{1}{3}$.
2. .45.	5. .385.	8. $.46\frac{2}{3}$.	11. $.46\frac{5}{9}$.
3. .36.	6. .425.	9. $.54\frac{2}{7}$.	12. $.34\frac{10}{11}$.

80. Repeating decimals. — In changing common fractions to decimals, or in division of decimals, it often occurs that a remainder in some step of the division contains the same figures as the given dividend or some former remainder. Hence we know that the next figures of the quotient will be a

repetition of those already found. Thus, $\frac{5}{11} = .45\frac{5}{11}$. Therefore, $\frac{5}{11} = .45454545\cdots$. This result is sometimes written $.\dot{4}\dot{5}$. Thus, $.\dot{1}4\dot{6} = .146146146\cdots$. Sometimes only part of the quotient repeats. Thus,

$$\tfrac{1}{6} = .1666\cdots = .1\dot{6}\,;\ \tfrac{5}{12} = .41666\cdots = .41\dot{6}.$$

EXERCISES

Divide until the figures that repeat are found;

1. $\frac{3}{11}$.	**5.** $\frac{13}{24}$.	**9.** $\frac{3}{22}$.	**13.** $\frac{159}{220}$.
2. $\frac{8}{11}$.	**6.** $\frac{11}{15}$.	**10.** $\frac{81}{111}$.	**14.** $\frac{643}{1980}$.
3. $\frac{5}{6}$.	**7.** $\frac{7}{99}$.	**11.** $\frac{113}{360}$.	**15.** $\frac{487}{1650}$.
4. $\frac{7}{12}$.	**8.** $\frac{8}{55}$.	**12.** $\frac{65}{198}$.	**16.** $\frac{116}{495}$.

81. Changing repeating decimals to common fractions. — From the meaning of $.6\dot{2}\dot{3}$,

$$
\begin{array}{lr}
 & 1000 \times .6\dot{2}\dot{3} = 623.\dot{2}\dot{3} \\
\text{and} & 10 \times .6\dot{2}\dot{3} = \quad 6.\dot{2}\dot{3} \\
\hline
\text{Subtracting,} & 990 \times .6\dot{2}\dot{3} = 617. \\
\text{Therefore,} & .6\dot{2}\dot{3} = \frac{617}{990}.
\end{array}
$$

Thus it is seen that the numerator is equal to the given decimal less the non-repeating part, and the denominator has as many 9's as there are figures in the repeating part, and as many zeros as there are figures in the non-repeating part.

Thus, $$.26\dot{4}\dot{7} = \frac{2647 - 26}{9900} = \frac{2621}{9900}.$$

EXERCISES

Change to common fractions:

1. $.\dot{5}$. **2.** $.\dot{6}$. **3.** $.1\dot{6}$. **4.** $.5\dot{7}$. **5.** $.\dot{7}\dot{2}$. **6.** $.\dot{8}\dot{4}$.

7. $.2\dot{1}$. **8.** $.1\dot{5}\dot{6}$. **9.** $.2\dot{8}\dot{7}$. **10.** $.3\dot{2}\dot{8}$. **11.** $.\dot{2}9\dot{7}$. **12.** $.\dot{0}1\dot{6}$.

82. Fundamental processes with repeating decimals. — Only when the greatest accuracy is desired are repeating decimals used as such. They are usually used as incomplete decimals giving but approximate results. However, to use them so as to get *exact* results, they must be changed to common fractions before the processes are performed. The results may then be changed back to decimals or repeating decimals, as the case may require.

Thus,

$$.4\dot{6} + .\dot{5} = \tfrac{42}{90} + \tfrac{5}{9} = \tfrac{92}{90} = 1.0\dot{2}.$$

$$.\dot{5} - .4\dot{6} = \tfrac{5}{9} - \tfrac{42}{90} = \tfrac{8}{90} = .0\dot{8}.$$

$$.4\dot{6} \times .\dot{5} = \tfrac{42}{90} \times \tfrac{5}{9} = \tfrac{7}{27} = .\dot{2}5\dot{9}.$$

$$.4\dot{6} \div .\dot{5} = \tfrac{42}{90} \times \tfrac{9}{5} = \tfrac{21}{25} = .84.$$

PROBLEMS INVOLVING DECIMALS

1. A cubic foot of water weighs 62.5 pounds. Steel weighs 7.8 times as much as water. Find the weight of a cubic foot of steel.

2. Milk weighs 1.032 times as much as water. If a gallon of milk weighs 8.6 lb., find the weight of a gallon of water.

3. A cubic foot of a certain limestone weighs 187.5 pounds. It is how many times as heavy as the same volume of water? (See problem 1.)

4. The total amount of digestible nutrients in 1 pound of corn is .789 of a pound. In 1 pound of oats, the amount is .608 of a pound. How many pounds more of digestible nutrients are there in a ton of corn than in a ton of oats?

5. Timothy hay has .476 lb. of digestible nutrients to the pound, while alfalfa has .516 lb. When timothy hay sells for $22 per ton, for what should alfalfa sell?

6. Using the data of Problem 5, for what should timothy hay sell when alfalfa is $25 per ton?

7. It is estimated that a working man needs .24 lb. of protein daily. If he gets it all from beef loin which contains .156 lb. of protein to the pound, how much beef will he need daily?

8. A quart of oysters contains .106 lb. of protein. Using the data of Problem 7, how many quarts daily would be needed to supply the needed protein for a working man?

9. Using the results of Problems 7 and 8, compare the cost of oysters at 50 ¢ per quart with loin of beef at 22 ¢ per pound. Carry the result to the third decimal place.

NUTRIENTS IN 1 POUND OF SOME COMMON FOODS

Food	Protein	Fat	Carbohydrates	Refuse and Water
Beef	.156 lb.	.166 lb.	none	.658 lb.
Pork	.131 lb.	.232 lb.	none	.615 lb.
Fowl	.138 lb.	.217 lb.	none	.630 lb.
Whole milk	.032 lb.	.038 lb.	.051 lb.	.870 lb.
Wheat bread	.078 lb.	.012 lb.	.520 lb.	.353 lb.
Dried beans	.175 lb.	.016 lb.	.578 lb.	.126 lb.
Raw eggs	.127 lb.	.088 lb.	none	.767 lb.

10. If a man needs 1.12 lb. of carbohydrates daily, how many pounds of wheat bread would he need when he gets it wholly from bread? How many pounds of beans when he gets it wholly from beans?

11. Find how much of each of the articles in the table is required to yield as much protein as a quart (2 lb.) of milk.

12. When beef is selling at 22 ¢ per pound, find the cost of a pound of nutrients furnished by beef.

13. When beans are $5\frac{1}{2}$ ¢ per pound, find the cost of a pound of nutrients furnished by beans.

14. Make and solve problems from the data in the table given on the preceding page.

15. A cubic foot contains 7.4805 gallons. How many gallons will a cistern contain if the volume is 128.46 cu. ft.?

16. If a cistern contains 203.98 gallons, how many cubic feet in the volume?

17. A standard bushel is 2150.42 cubic inches. A cubic foot contains 1728 cubic inches. A cubic foot is what decimal part of a bushel?

18. A ton of field-cured corn fodder contains 127.6 lb. of nutrient, while a ton of clover hay contains 886.2 lb. When clover hay is worth $18 per ton, how much is corn fodder worth?

19. The diameter of a circle multiplied by 3.1416 gives the circumference of the circle. Find to four decimal places by what the circumference must be multipled to give the diameter.

20. The weight of a certain piece of copper wire 24.75 inches long was 3.25 oz. Find to three decimal places the weight of 1 in.

21. In Problem 20, find to three decimal places how many inches of the copper wire it will take to weigh 1 ounce?

22. In a certain locality, the average rainfall for the month of August was as follows: 2.52 in., 3.01 in., 2.75 in., 1.98 in., 2.63 in., and 2.41 in. Find to two decimal places the average during the six years.

23. Experiments showed the following weights of a cubic foot of anthracite coal taken from different cars during the winter: 62.85 lb., 63.51 lb., 62.95 lb., 63.54 lb., 64.05 lb., 63.68 lb., 64.12 lb., 63.48 lb. Find the average weight.

CHAPTER IV

SHORT METHODS OF COMPUTATION

83. Efficiency in computation. — Efficiency in computation depends upon two things: (*a*) ability to recall number facts quickly and accurately; and (*b*) ability to see number relations that will save figures. The first of these depends upon the drills given in the preceding chapters. Some examples to show the advantage of the second of these requirements are given in this chapter.

It is not intended that the student should memorize all of the special methods given here. He should learn to use those which will help him most in computation. The chapter, however, will show the great advantage of seeing number relations that save work, and should encourage the student to look for relations that will aid him in any computation he has to perform.

The explanations given are very brief, for it is intended that the student should study the given illustrations and derive a rule or show the reasons for the rule when given. This will aid greatly in developing power to see short methods of computation.

84. Multiplying by powers of 10. — Always multiply by a power of 10 by annexing zeros to a whole number or by moving the decimal point in decimals. Thus, $10 \times 17 = 170$; $100 \times 35 = 3500$; $100 \times 3.654 = 365.4$; $100 \times 4.6 = 460$.

EXERCISES

Give products at sight:

1. 10×85.	7. $100 \times .035$.	13. 1000×3.64.
2. 100×175.	8. 10×17.34.	14. 1000×48.6.
3. 10×3.84.	9. 100×96.2.	15. $1000 \times .375$.
4. 100×16.5.	10. 100×7.654.	16. 1000×6.2.
5. 100×1.85.	11. 1000×6.5.	17. 1000×1.72.
6. 10×3.468.	12. $1000 \times .76$.	18. 1000×84.2.

85. The multiplier nearly some power of 10. — When the multiplier is nearly 10, 100, 1000, etc., much work may be saved. Thus, $9\frac{1}{2} \times 42 = 420 - 21 = 399$; $99 \times 78 = 7800 - 78 = 7722$; $99\frac{3}{4} \times 84 = 8400 - 21 = 8379$.

EXERCISES

Find the products of:

1. 9×72.	8. 99×78.	15. 99×384.	22. 999×175.
2. $9\frac{1}{2} \times 94$.	9. $99\frac{1}{2} \times 86$.	16. 98×276.	23. 998×374.
3. $9\frac{3}{4} \times 96$.	10. $99\frac{2}{3} \times 72$.	17. 97×385.	24. 998×246.
4. $9\frac{4}{5} \times 75$.	11. 98×64.	18. 98×682.	25. 997×681.
5. $9\frac{2}{3} \times 87$.	12. $99\frac{3}{4} \times 72$.	19. 96×125.	26. 997×675.
6. $9\frac{5}{6} \times 84$.	13. $99\frac{4}{5} \times 85$.	20. 98×763.	27. 995×847.
7. $9\frac{7}{8} \times 64$.	14. $99\frac{5}{6} \times 96$.	21. 95×864.	28. 996×346.

86. The product of multiples of powers of 10. — When one or both factors end in zeros, work is saved as follows: Find 500×1700. Since $5 \times 17 = 85$, the product $= 850,000$.

EXERCISES

At sight give the products of:

1. 30×1600.	6. 400×600.	11. 400×170.	16. 900×1600.
2. 40×1500.	7. 700×800.	12. 500×260.	17. 300×2400.
3. 60×1300.	8. 900×600.	13. 800×340.	18. 600×5400.
4. 70×1500.	9. 700×900.	14. 900×630.	19. 200×8600.
5. 80×1400.	10. 600×800.	15. 700×540.	20. 500×7600.

Find the products of:

21. 170×8500.	24. 380×9600.	27. 3800×7400.
22. 360×7400.	25. 740×3900.	28. 9600×5800.
23. 540×8600.	26. 830×7600.	29. 7900×6300.

87. Aliquot parts of 10, 100, and 1000. — The **aliquot part** of any number is a number that is contained in it an integral number of times. Aliquot parts of 10, 100, and 1000 are so important that they should be memorized.

TABLE OF ALIQUOT PARTS

$5 = \frac{1}{2}$ of 10.	$25 = \frac{1}{4}$ of 100.	$33\frac{1}{3} = \frac{1}{3}$ of 100.
$2\frac{1}{2} = \frac{1}{4}$ of 10.	$12\frac{1}{2} = \frac{1}{8}$ of 100.	$16\frac{2}{3} = \frac{1}{6}$ of 100.
$3\frac{1}{3} = \frac{1}{3}$ of 10.	$6\frac{1}{4} = \frac{1}{16}$ of 100.	$8\frac{1}{3} = \frac{1}{12}$ of 100.
$125 = \frac{1}{8}$ of 1000.	$625 = \frac{1}{16}$ of 10,000.	

88. Multiplying by aliquot parts of 10, 100, etc. — Observe the following examples and show how to use aliquot parts in multiplication. $2\frac{1}{2} \times 28 = \frac{280}{4} = 70$; $33\frac{1}{3} \times 84 = \frac{8400}{3} = 2800$; $125 \times 45 = \frac{45000}{8} = 5625$.

EXERCISES

Find the products of:

1. 5×847.	**10.** $12\frac{1}{2} \times 72$.	**19.** 25×368.
2. $2\frac{1}{2} \times 846$.	**11.** $12\frac{1}{2} \times 94$.	**20.** $33\frac{1}{3} \times 984$.
3. $2\frac{1}{2} \times 984$.	**12.** $6\frac{1}{4} \times 48$.	**21.** 25×826.
4. $3\frac{1}{3} \times 726$.	**13.** $33\frac{1}{3} \times 96$.	**22.** $33\frac{1}{3} \times 385$.
5. $3\frac{1}{3} \times 534$.	**14.** $33\frac{1}{3} \times 84$.	**23.** $16\frac{2}{3} \times 934$.
6. $3\frac{1}{3} \times 678$.	**15.** $33\frac{1}{3} \times 75$.	**24.** $12\frac{1}{2} \times 735$.
7. 25×46.	**16.** $16\frac{2}{3} \times 54$.	**25.** 125×765.
8. 25×87.	**17.** $16\frac{2}{3} \times 59$.	**26.** 625×742.
9. 25×59.	**18.** $8\frac{1}{3} \times 72$.	**27.** 625×896.

89. Multiplier nearly some aliquot part. — When a multiplier is a little larger or a little smaller than an aliquot part, the method of § 88 may be used by making proper corrections. Thus, $26 \times 385 = 9625 + 385 = 10{,}010$; $32\frac{1}{3} \times 786 = 26{,}200 - 786 = 25{,}414$. $16\frac{1}{2} \times 495 = 8250 - 82\frac{1}{2} = 8167\frac{1}{2}$; $127 \times 896 = 112{,}000 + 1792 = 113{,}792$.

EXERCISES

Find the products of:

1. 26×248.	**11.** 27×486.	**21.** 124×968.
2. $34\frac{1}{3} \times 216$.	**12.** 27×587.	**22.** 126×842.
3. $13\frac{1}{2} \times 728$.	**13.** 13×965.	**23.** 123×764.
4. $18\frac{2}{3} \times 558$.	**14.** 16×358.	**24.** 127×987.
5. $14\frac{1}{2} \times 656$.	**15.** 12×764.	**25.** 626×458.
6. 24×412.	**16.** 13×972.	**26.** 624×576.
7. $32\frac{1}{3} \times 525$.	**17.** 26×568.	**27.** 627×972.
8. $15\frac{2}{3} \times 324$.	**18.** 27×349.	**28.** 623×846.
9. $11\frac{1}{2} \times 354$.	**19.** 23×625.	**29.** 623×965.
10. 33×945.	**20.** 33×684.	**30.** 627×739.

90. Other methods of using aliquot parts. — A study of the examples given below will enable the student to discover short methods connected with aliquot parts.

$18 = \frac{1}{5}$ of $100 - \frac{1}{10}$ of $\frac{1}{5}$ of 100. Then, $18 \times 645 = 12{,}900 - 1290 = 11{,}610$.

$27\frac{1}{2} = \frac{1}{4}$ of $100 + \frac{1}{10}$ of $\frac{1}{4}$ of 100. Then, $27\frac{1}{2} \times 782 = 19{,}550 + 1955 = 21{,}505$.

$45 = \frac{1}{2}$ of $100 - \frac{1}{10}$ of $\frac{1}{2}$ of 100. Then, $45 \times 685 = 34{,}250 - 3425 = 30{,}825$.

$13\frac{3}{4} = \frac{1}{8}$ of $100 + \frac{1}{10}$ of $\frac{1}{8}$ of 100. Then, $13\frac{3}{4} \times 748 = 9350 + 935 = 10{,}285$.

$36\frac{2}{3} = \frac{1}{3}$ of $100 + \frac{1}{10}$ of $\frac{1}{3}$ of 100. Then, $36\frac{2}{3} \times 954 = 31{,}800 + 3180 = 34{,}980$.

The student can easily make short methods for other factors.

EXERCISES

1. 18×763.	5. 22×796.	9. $27\frac{1}{2} \times 648$.	13. 45×875.
2. 45×865.	6. 45×967.	10. $22\frac{1}{2} \times 862$.	14. 55×984.
3. 55×865.	7. 55×967.	11. $36\frac{2}{3} \times 762$.	15. $36\frac{2}{3} \times 786$.
4. 22×642.	8. 18×748.	12. $13\frac{3}{4} \times 964$.	16. $22\frac{1}{2} \times 956$.

91. Multiplying by special decimals. — There are some special decimals that are more easily used when changed to common fractions.

TABLE

$.5 = \frac{1}{2}$.	$.33\frac{1}{3} = \frac{1}{3}$.	$.14\frac{2}{7} = \frac{1}{7}$.
$.25 = \frac{1}{4}$.	$.16\frac{2}{3} = \frac{1}{6}$.	$.11\frac{1}{9} = \frac{1}{9}$.
$.125 = \frac{1}{8}$.	$.08\frac{1}{3} = \frac{1}{12}$.	$.0625 = \frac{1}{16}$.

EXERCISES

Give at sight:

1. $.25 \times 32$.
2. $.75 \times 84$.
3. 1.25×32.
4. 1.75×16.
5. 2.25×12.
6. 2.5×64.
7. 1.25×36.
8. 2.5×28.
9. 3.5×42.
10. 1.25×64.
11. $.125 \times 96$.
12. $.125 \times 816$.
13. $.33\frac{1}{3} \times 24$.
14. $1.33\frac{1}{3} \times 36$.
15. $.16\frac{2}{3} \times 54$.
16. $1.16\frac{2}{3} \times 24$.
17. $.125 \times 48$.
18. 1.125×88.
19. $.14\frac{2}{7} \times 49$.
20. $1.14\frac{2}{7} \times 63$.
21. $1.11\frac{1}{9} \times 54$.
22. 64 yd. cloth @ $0.25.
23. 16 yd. cloth @ $1.25.
24. 27 lb. coffee @ $$0.33\frac{1}{3}$.
25. 72 lb. meat @ $$0.16\frac{2}{3}$.
26. 48 lb. lard @ $$0.12\frac{1}{2}$.
27. 24 pr. shoes @ $$2.37\frac{1}{2}$.

92. The sum of two products having a common factor. — When two products have a common factor, the common factor may be multiplied by the sum of the *other* factors.

Thus, $3 \times 3.1416 + 7 \times 3.1416 = 10 \times 3.1416 = 31.416$.

EXERCISES

Give at sight:

1 $13 \times 84 + 7 \times 84$.
2. $8 \times 357 + 2 \times 357$.
3. $26 \times 24.8 + 4 \times 24.8$.
4. $14 \times 79.8 + 6 \times 79.8$.
5. $15 \times 74.6 + 5 \times 74.6$.
6. $16 \times 97.8 + 14 \times 97.8$.
7. $18 \times 64.9 + 12 \times 64.9$.
8. $32 \times 68.4 + 18 \times 68.4$.

Find:

9. $17 \times 934 + 48 \times 934$.
0. $63 \times 7.84 + 72 \times 7.84$.
11. $42 \times 5.98 + 78 \times 5.98$.
12. $36 \times 84.9 + 84 \times 84.9$.
13. $53 \times 9.67 + 37 \times 9.67$.
14. $82 \times 96.8 + 78 \times 96.8$.
15. $19 \times 87.6 + 51 \times 87.6$.
16. $23 \times 8.46 + 47 \times 8.46$.
17. $82 \times 7.39 + 48 \times 7.39$.
18. $57 \times 8.16 + 93 \times 8.16$.
19. $65 \times 7.64 + 85 \times 7.64$.
20. $83 \times 4.85 + 57 \times 4.85$.
21. $28 \times 7.29 + 72 \times 7.29$.
22. $36 \times 2.58 + 14 \times 2.58$.

93. Dividing by 10 and powers of 10. — All such divisions should be done by removing zeros or moving the decimal point.

Thus, $3500 \div 100 = 35$; $468.5 \div 100 = 4.685$.

EXERCISES

At sight give:

1. $450 \div 10$.
2. $396 \div 10$.
3. $487 \div 100$.
4. $52.6 \div 100$.
5. $620 \div 100$.
6. $3.4 \div 10$.
7. $5.7 \div 100$.
8. $8.2 \div 100$.
9. $7.6 \div 10$.
10. $.35 \div 100$.
11. $62.8 \div 10$.
12. $638.4 \div 100$.
13. $725.5 \div 100$.
14. $3.78 \div 10$.
15. $57.6 \div 100$.

94. Dividing by multiples of powers of 10. — Before beginning the division, cut off the zeros from the divisor and move the decimal point as many places to the left. Why?

Thus, $456 \div 20 = 45.6 \div 2 = 22.8$; $698 \div 200 = 6.98 \div 2 = 3.49$.

EXERCISES

Give at sight:

1. $789 \div 300$.
2. $846 \div 20$.
3. $98.6 \div 300$.
4. $8.56 \div 20$.
5. $94.8 \div 40$.
6. $72.8 \div 40$.
7. $6.39 \div 30$.
8. $94.8 \div 400$.
9. $61.8 \div 300$.
10. $7.48 \div 400$.
11. $69.3 \div 30$.
12. $96.8 \div 80$.
13. $15.47 \div 70$.
14. $365.4 \div 900$.
15. $726.4 \div 800$.
16. $822.6 \div 600$.
17. $972.6 \div 600$.
18. $817.28 \div 400$.

Find:

19. $176 \div 190$.
20. $84.65 \div 260$.
21. $345.6 \div 180$.
22. $469.8 \div 1700$.
23. $846.35 \div 2400$.
24. $17,685 \div 3400$.
25. $16,965 \div 1900$.
26. $84,386 \div 5790$.
27. $46,834 \div 6300$.
28. $24,635 \div 1700$.
29. $86,917 \div 3400$.
30. $48,106 \div 4600$.
31. $39,986 \div 5700$.
32. $90,643 \div 6700$.
33. $38,965 \div 8100$.

95. Products of factors ending in 5. — By observing that besides 5×5, the product is made up of the product of the tens, plus the product of 5 and the sum of the tens, much work may be saved.

WORK
175
95
1325
153
16625

EXPLANATION. — 17 tens + 9 tens = 26 tens. $260 \times 5 = \frac{1}{2}$ of $2600 = 1300$. So half of the sum of the tens with 25 annexed is the first partial product. 9×17, written in hundreds' place (since it is really 90×170) is the second product. (If the sum is odd, take half the sum and annex 75 instead of 25. Why?)

EXERCISES

Find the products of:

1.	2.	3.	4.	5.	6.
165	175	185	215	325	345
85	55	65	95	85	85

7.	8.	9.	10.	11.	12.
245	165	185	215	325	425
75	95	95	85	75	35

13. 985×65.
14. 75×835.
15. 85×965.
16. 745×95.
17. 635×85.
18. 95×625.
19. 65×345.
20. 85×745.
21. 75×965.
22. 845×85.
23. 135×175.
24. 145×165.
25. 175×185.
26. 195×215.
27. 185×325.

96. Squaring a number ending in 5. — This method can be seen from the following example.

WORK
65
65
4225

EXPLANATION. — Besides 5×5, it is evident that there will be $5 \times 60 + 60 \times 5 + 60 \times 60$. Hence, 70×60, or 7×6 hundred. So 7×6 with 25 annexed gives the product. Thus, $45 \times 45 = 4 \times 5$ hundred $+25$, or 2025. $75 \times 75 = 5625$. That is, multiply the tens by the next consecutive number and annex 25.

EXERCISES

Find the products of:

1. 35×35.
2. 55×55.
3. 95×95.
4. 3.5×3.5.
5. 2.5×2.5.
6. 8.5×8.5.
7. 75×75.
8. 4.5×4.5.
9. 115×115.
10. 125×125.
11. 13.5×13.5.
12. 17.5×17.5.
13. 345×345.
14. 275×275.
15. 165×165.
16. 195×195.
17. 205×205.
18. 32.5×32.5.
19. 415×415.
20. 635×635.
21. 185×185.
22. 315×315.
23. 405×405.
24. 645×645.

97. Dividing by aliquot parts of 10, 100, etc. — The use of aliquot parts in division can be seen from the following examples. Show the reason.

$$3896 \div 12\tfrac{1}{2} = 8 \times 38.96 = 311.68.$$
$$9673 \div 33\tfrac{1}{3} = 3 \times 96.73 = 290.19.$$

EXERCISES

Find the quotients:

1. $1564 \div 25$.
2. $3846 \div 16\frac{2}{3}$.
3. $1963 \div 12\frac{1}{2}$.
4. $7864 \div 125$.
5. $576 \div 8\frac{1}{3}$.
6. $1365 \div 50$.
7. $1645 \div 25$.
8. $7806 \div 12\frac{1}{2}$.
9. $8930 \div 8\frac{1}{3}$.
10. $1965 \div 16\frac{2}{3}$.
11. $8645 \div 625$.
12. $1348 \div 125$.
13. $89.36 \div 8\frac{1}{3}$.
14. $176.35 \div 25$.
15. $93.36 \div 25$.
16. $16.84 \div 33\frac{1}{3}$.
17. $876.3 \div 125$.
18. $76.93 \div 16\frac{2}{3}$.
19. $84.37 \div 12\frac{1}{2}$.
20. $63.42 \div 125$.
21. $187.5 \div 33\frac{1}{3}$.
22. $86.38 \div 16\frac{2}{3}$.
23. $869.3 \div 25$.
24. $869.3 \div 125$.
25. $76.34 \div 12\frac{1}{2}$.
26. $84.6 \div 125$.
27. $408 \div 125$.
28. $836 \div 33\frac{1}{3}$.
29. $7.65 \div 12\frac{1}{2}$.
30. $9.36 \div 33\frac{1}{3}$.

98. Making use of products found or known. — Often a product already found may be used. This is shown in the following examples.

WORK

$$\begin{array}{r} 846 \\ \underline{3\frac{3}{4}} \\ 2538 \\ \underline{634\frac{1}{2}} \\ 3172\frac{1}{2} \end{array}$$

EXPLANATION. — In finding $3\frac{3}{4} \times 846$, 3×846 is first found; then 2538 being 3×846, $\frac{3}{4}$ of $846 = \frac{1}{4}$ of 2538, the partial product already found.

WORK

$$\begin{array}{r} 784 \\ \underline{936} \\ 7056 \\ \underline{28224} \\ 733824 \end{array}$$

EXPLANATION. — In finding 936×784, work is saved by first finding 9×784. When multiplying by 9 first and writing the first product in hundreds' place we get 900×784. We can now get 36×784 by finding 4×7056, which is $4 \times 9 \times 784$, or 36×784.

EXERCISES

Find the products of:

1. $5\frac{5}{8} \times 386$.
2. $7\frac{7}{8} \times 528$.
3. $6\frac{6}{7} \times 387$.
4. $4\frac{4}{9} \times 684$.
5. $5\frac{5}{7} \times 385$.
6. $6\frac{6}{11} \times 846$.
7. $9\frac{9}{11} \times 385$.
8. $3\frac{3}{7} \times 645$.
9. $8\frac{8}{9} \times 837$.
10. $6\frac{6}{13} \times 465$.
11. $5\frac{5}{8} \times 380$.
12. $9\frac{9}{10} \times 846$.
13. $8\frac{8}{15} \times 640$.
14. $9\frac{9}{10} \times 380$.
15. $6\frac{6}{7} \times 843$.
16. $5\frac{5}{9} \times 846$.
17. $7\frac{7}{8} \times 368$.
18. $5\frac{5}{7} \times 392$.

19. Find 459×684 by using but two partial products.

Find by using two partial vroducts:

20. 427×854.
21. 486×975.
22. 357×386.
23. 546×875.
24. 639×763.
25. 729×847.
26. 819×369.
27. 637×564.
28. 936×842.
29. 735×628.
30. 642×365.
31. 749×536.
32. 654×876.
33. 545×643.
34. 756×847.
35. 927×362.
36. 824×368.
37. 624×587.

99. Multipliers containing the digit 1. — When the digit 1 occurs in the multiplier, the multiplicand may be used as the first partial product. We write the work as follows:

Find 81×346.

WORK	*Second method*
346	346
2768	81
28026	28026

In the second method 1×6 is written. Then $8 \times 6 + 4$ is found. 2 is written and 5 carried. $8 \times 4 + 5 + 3$ is found. 0 is written and 4 carried. $8 \times 3 + 4$ is found and written.

Find 19×786.

WORK	*Second method*
786	786
7074	19
14934	14934

In the second method 9×6 is found. 4 is written and 5 carried. $9 \times 8 + 5 + 6$ is found. 3 is written and 8 carried. $9 \times 7 + 8 + 8$ is found. 9 is written and 7 carried. $7 + 7$, or 14 is written.

NOTE. — The second method in each requires more mental work and greater concentration, but by practice it can be used accurately and rapidly.

EXERCISES

1. 71×345.	11. 17×627.	21. 103×847.
2. 61×427.	12. 19×846.	22. 301×796.
3. 81×364.	13. 13×789.	23. 510×846.
4. 41×527.	14. 15×647.	24. 150×357.
5. 51×846.	15. 18×726.	25. 105×642.
6. 91×364.	16. 81×387.	26. 918×623.
7. 71×576.	17. 61×358.	27. 186×492.
8. 18×348.	18. 172×846.	28. 361×578.
9. 19×647.	19. 712×345.	29. 519×346.
10. 16×384.	20. 721×387.	30. 821×576.

100. Multiplying by a fraction nearly unity.—In using such fractions as $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, etc., much work may be saved.

Find $\frac{7}{8} \times 768$.

WORK

$$\begin{array}{r} 768 \\ 96 \\ \hline 672 \end{array}$$

(Subtract $\frac{1}{8}$ of 768.)

Find $\frac{11}{12}$ of 978.

WORK

$$\begin{array}{r} 978 \\ 81\frac{1}{2} \\ \hline 896\frac{1}{2} \end{array}$$

(Subtract $\frac{1}{12}$ of 978.)

EXERCISES

Find the products:

1. $\frac{3}{4} \times 795$. **6.** $\frac{3}{4} \times 728$. **11.** $\frac{11}{12} \times 345$. **16.** $\frac{15}{16} \times 762$.

2. $\frac{2}{3} \times 846$. **7.** $\frac{4}{5} \times 936$. **12.** $\frac{10}{11} \times 784$. **17.** $\frac{13}{14} \times 964$.

3. $\frac{4}{5} \times 387$. **8.** $\frac{6}{7} \times 835$. **13.** $\frac{9}{10} \times 834$. **18.** $\frac{12}{18} \times 784$.

4. $\frac{7}{8} \times 963$. **9.** $\frac{8}{9} \times 567$. **14.** $\frac{13}{14} \times 846$. **19.** $\frac{17}{18} \times 596$.

5. $\frac{5}{6} \times 984$. **10.** $\frac{7}{8} \times 346$. **15.** $\frac{14}{15} \times 968$. **20.** $\frac{18}{19} \times 763$.

101. Supplement method of multiplication. — The **supplement** of a number is the excess over the power of 10 next smaller than the number. Thus the supplement of 108 is 8.

In algebra it is shown that

$$(a+b) \times (a+c) = a^2 + a(b+c) + bc.$$

Then, $103 \times 108 = (100 + 3) \times (100 + 8)$

$$= 10{,}000 + 100 \times 11 + 24 = 100(100 + 11) + 24 = 11{,}124.$$

That is, to one number add the supplement of the other, multiply by the power of 10 used, and add the product of the supplements.

Thus, $105 \times 107 = 11{,}200 + 35 = 11{,}235$. Likewise, $17 \times 18 = 250 + 56 = 306$.

EXERCISES

At sight give:

1. 108×109.	6. 109×105.	11. 16×17.
2. 112×108.	7. 115×103.	12. 18×13.
3. 107×110.	8. 105×114.	13. 14×13.
4. 115×106.	9. 117×105.	14. 19×17.
5. 120×104.	10. 104×118.	15. 16×18.

102. Complement method of multiplication. — The **complement** of a number is the amount it lacks of some power of 10. Thus the complement of 96 is 4.

In algebra it is shown that

$$(a-b) \times (a-c) = a^2 - a(b+c) + bc.$$

Then,

$$92 \times 97 = (100-8) \times (100-3) = 10{,}000 - 100 \times 11 + 24$$
$$= 100(100-11) + 24.$$ State a rule.

Then, $\overset{4}{96} \times \overset{3}{97} = 9312$; $\overset{4}{96} \times \overset{7}{93} = 8928$; $\overset{8}{92} \times \overset{5}{95} = 8740$; $\overset{9}{91} \times \overset{7}{93} = 8463$.

NOTE 1. — The complements are written above for convenience. The results can then be written down at once.

NOTE 2. — The methods of § 101 and § 102 are interesting as two of the many possibilities, but they are seldom used.

EXERCISES

At sight give:

1. 98×93.	6. 98×91.	11. 92×99.
2. 92×96.	7. 96×95.	12. 92×95.
3. 91×97.	8. 95×97.	13. 94×94.
4. 92×94.	9. 94×93.	14. 96×96.
5. 93×95.	10. 91×93.	15. 97×97.

103. Cross multiplication. — In finding the product of any two numbers of two figures each, the second and third products may be written down together.

WORK

$$\begin{array}{r} 43 \\ \underline{36} \\ 1548 \end{array}$$

EXPLANATION. — $6 \times 4 + 1$ is not written down, but is held in mind and added to 3×3 before the result is recorded.

EXERCISES

Write down the product of:

1. 24×38.	**8.** 26×92.	**15.** 46×53.	**22.** 38×57.
2. 36×42.	**9.** 54×26.	**16.** 39×46.	**23.** 64×73.
3. 97×26.	**10.** 85×35.	**17.** 57×63.	**24.** 49×62.
4. 84×38.	**11.** 64×63.	**18.** 54×76.	**25.** 83×19.
5. 42×63.	**12.** 92×84.	**19.** 38×71.	**26.** 76×84.
6. 87×42.	**13.** 56×73.	**20.** 57×64.	**27.** 48×63.
7. 37×48.	**14.** 84×52.	**21.** 82×74.	**28.** 72×47.

104. Two factors whose tens are alike and of which the sum of the ones is ten. — This method is shown by an example.

WORK

$$\begin{array}{r} 76 \\ \underline{74} \\ 5624 \end{array}$$

EXPLANATION. — Besides 4×6, there is $4 \times 70 + 6 \times 70 + 70 \times 70$, or 80×70. Hence, multiply the tens' digit by the next higher number and annex the product of the ones' digits. Thus $38 \times 32 = 1216$; $46 \times 44 = 2024$; $51 \times 59 = 3009$.

NOTE. — When the product of the ones' digits is but a single figure, prefix a zero. Thus in finding 51×59 above we wrote 09 for 9×1.

EXERCISES

Give at sight:

1. 43×47.	**6.** 57×53.	**11.** 65×65.	**16.** 86×84.
2. 62×68.	**7.** 82×88.	**12.** 45×45.	**17.** 26×24.
3. 76×74.	**8.** 75×75.	**13.** 55×55.	**18.** 73×77.
4. 39×31.	**9.** 36×34.	**14.** 85×85.	**19.** 81×89.
5. 44×46.	**10.** 64×66.	**15.** 48×42.	**20.** 67×63.

105. Selecting the order of factors. — Since the product is independent of the order of factors, they should be chosen to facilitate the computation. Thus, $4 \times 17 \times 25 = 4 \times 25 \times 17 = 1700$; $12 \times 40 \times 25 = 40 \times 25 \times 12 = 12{,}000$.

EXERCISES

At sight give:

1. $3 \times 19 \times 20$.
2. $4 \times 86 \times 25$.
3. $25 \times 17 \times 8$.
4. $4 \times 15 \times 60$.
5. $17 \times 35 \times 2$.
6. $16 \times 17 \times 3\frac{1}{8}$.
7. $17 \times 18 \times 5$.
8. $12 \times 46 \times 8\frac{1}{3}$.
9. $19 \times 36 \times 16\frac{2}{3}$.
10. $42 \times 32 \times 12\frac{1}{2}$.
11. $38 \times 16 \times 25$.
12. $15 \times 16\frac{2}{3} \times 90$.
13. $12\frac{1}{2} \times 48 \times 17$.
14. $16\frac{2}{3} \times 54 \times 42$.
15. $38 \times 24 \times 12\frac{1}{2}$.

106. When multiplication and division both occur. — When both division and multiplication occur, common factors should be removed. Thus, $36 \times 487 \div 72 = 487 \div 2 = 243\frac{1}{2}$. If several factors occur in both dividend and divisor, the work should be arranged in the common form of cancellation.

EXERCISES

At sight give:

1. $17 \times 634 \div 34$.
2. $19 \times 764 \div 57$.
3. $46 \times 977 \div 92$.
4. $48 \times 674 \div 24$.
5. $38 \times 742 \div 19$.
6. $42 \times 649 \div 84$.
7. $13 \times 987 \div 39$.
8. $46 \times 321 \div 23$.
9. $81 \times 37 \div 27$.
10. $64 \times 98 \div 16$.
11. $32 \times 861 \div 64$.
12. $15 \times 581 \div 45$.
13. $19 \times 864 \div 76$.
14. $52 \times 128 \div 26$.
15. $18 \times 96 \div 54$.
16. $56 \times 73 \div 14$.
17. $26 \times 89 \div 52$.
18. $34 \times 78 \div 68$.

GENERAL EXERCISES IN SHORT METHODS

Give at sight:

1. 10×45.
2. 10×3.65.
3. 100×3.4.
4. 100×3.65.
5. 200×3.4.
6. 300×1.52.
7. 400×3.25.
8. 200×600.
9. 300×1500.

10. 500×350.
11. 5×175.
12. 5×385.
13. 5×682.
14. $2\frac{1}{2} \times 485$.
15. $2\frac{1}{2} \times 672$.
16. $2\frac{1}{2} \times 763$.
17. $3\frac{1}{3} \times 962$.
18. $3\frac{1}{3} \times 764$.
19. $3\frac{1}{3} \times 846$.
20. $3\frac{1}{3} \times 782$.
21. 25×846.
22. 25×938.
23. 25×684.
24. 25×796.
25. $12\frac{1}{2} \times 186$.
26. $12\frac{1}{2} \times 246$.
27. $12\frac{1}{2} \times 784$.
28. $12\frac{1}{2} \times 724$.
29. $16\frac{2}{3} \times 643$.
30. $16\frac{2}{3} \times 963$.

Find without doing the usual work:

31. 26×936.
32. 24×784.
33. $34\frac{1}{3} \times 649$.
34. $32\frac{1}{3} \times 748$.
35. 27×985.
36. 23×765.
37. 124×782.
38. 126×938.
39. 123×584.
40. 127×632.
41. 18×536.
42. 18×928.
43. 18×864.
44. 45×786.
45. 45×936.
46. 55×874.
47. 55×936.
48. 22×789.
49. 22×638.
50. $27\frac{1}{2} \times 738$.
51. $27\frac{1}{2} \times 848$.
52. $27\frac{1}{2} \times 637$.
53. $22\frac{1}{2} \times 356$.
54. $22\frac{1}{2} \times 789$.
55. $22\frac{1}{2} \times 645$.
56. $36\frac{2}{3} \times 784$.
57. $36\frac{2}{3} \times 564$.
58. $13\frac{3}{4} \times 846$.
59. $13\frac{3}{4} \times 726$.
60. $13\frac{3}{4} \times 850$.

Give at sight:

61. $.25 \times 64$.
62. $.25 \times 72$.
63. $.75 \times 84$.
64. $.75 \times 96$.
65. 1.25×48.
66. 1.25×64.
67. 2.25×160.
68. 2.25×240.
69. 2.5×560.
70. 2.5×640.
71. 85×45.
72. 65×25.
73. 75×35.
74. 45×65.
75. 35×35.
76. 45×45.
77. 55×55.
78. 65×65.
79. 75×75.
80. 8.5×8.5.
81. 44×46.
82. 72×78.
83. 61×69.
84. 84×86.

85. 97×93.	**89.** 6.5×6.5.	**93.** $7\frac{1}{2} \times 7\frac{1}{2}$.
86. 42×48.	**90.** 7.2×7.8.	**94.** $8\frac{1}{2} \times 4\frac{1}{2}$.
87. 3.5×3.5.	**91.** $3\frac{1}{2} \times 3\frac{1}{2}$.	**95.** $10\frac{1}{2} \times 8\frac{1}{2}$.
88. 4.3×4.7.	**92.** $5\frac{1}{2} \times 5\frac{1}{2}$.	**96.** $9\frac{1}{2} \times 5\frac{1}{2}$.

97. $7\frac{1}{2} \times 9\frac{1}{2}$. **98.** $6\frac{1}{2} \times 8\frac{1}{2}$. **99.** $9\frac{1}{2} \times 3\frac{1}{2}$. **100.** 16×18.

101. How much will 42 baskets of apples cost at 48¢ per basket?

102. How much will 24 pounds of lard cost at 18¢ per pound?

103. What is the cost of 75 acres of land at $75 per acre?

104. What are 2400 bushels of wheat worth at $1.25 per bushel?

105. Give the cost of 750 pounds of sugar at $7\frac{1}{2}$¢ per pound.

106. Give the cost of $3\frac{1}{2}$ boxes of oranges at $3.50 per box.

107. How much will $8\frac{1}{2}$ bu. of potatoes cost at 85¢ per bushel?

108. At $22\frac{1}{2}$¢ per pound, how much will $4\frac{1}{2}$ lb. of meat cost?

109. At 45¢ per pound, how much will $4\frac{1}{2}$ lb. of butter cost?

110. Give the cost of $8\frac{1}{2}$ yards of cloth at 85¢ per yard.

111. At $7.50 per ton, how much will $7\frac{1}{2}$ tons of coal cost?

112. How much will 35 barrels of apples cost at $3.50 per barrel?

113. What is the cost of 35,000 ft. of lumber at $35 per 1000 ft.?

CHAPTER V

DENOMINATE NUMBERS

107. Denominate numbers. — A **denominate number** is a number of *standard units of measure*, as 4 feet, 20 pounds, etc.

When the number consists of but one denominate number, as 3 feet, 16 pounds, etc., it is a **simple denominate number.** When it consists of two or more units of measure, as 3 feet 6 inches, 4 gallons 3 pints, etc., it is a **compound denominate number.**

108. Tables of denominate numbers. — Those units that bear definite relations to each other form a **table** of measures. Those tables most used should be known. The following include those in most common use, as well as a few given merely for reference. The latter include *troy* and *apothecaries'* weight, and *foreign money.*

TABLE OF LINEAR MEASURE

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards = 1 rod (rd.)
320 rods = 1 mile (mi.)
1 mi. = 320 rd. = 1760 yd. = 5280 ft.

TABLE OF SQUARE MEASURE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

In measuring roofing, 100 sq. ft. are called a **square**.

TABLE OF CUBIC MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)
$24\frac{3}{4}$ cubic feet = 1 perch (P.)
1 cubic yard = 1 load

NOTE.—The **cord** is used in measuring wood; the **perch** in measuring stone or masonry; and the **load** in measuring earth removed from excavations.

TABLE OF LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
1 gallon = 231 cubic inches

While barrels are made of various sizes, $31\frac{1}{2}$ gallons is considered a standard **barrel**.

TABLE OF DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
1 bushel = 2150.42 cubic inches

NOTE.—The **dry quart** is larger than the **liquid quart**. The dry quart contains 67.2 cu. in., while the liquid quart contains only 57.75 cu. in.

TABLE OF AVOIRDUPOIS WEIGHT

Used in weighing all coarse or heavy articles.

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)

A long ton, used in the United States custom houses in determining duty on imported goods, and sometimes used in selling coal and iron ore at the mines, is 2240 pounds.

TABLE OF TROY WEIGHT

Used by goldsmiths in weighing precious metals and stones.

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweight	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

TABLE OF APOTHECARIES' WEIGHT

Used by physicians and druggists in compounding drugs.

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (lb.)

A COMPARISON OF THE THREE TABLES OF WEIGHT

	Pound	Ounce	Grain
Avoirdupois . . .	7000 gr.	437½ gr.	1 gr.
Troy	5760 gr.	480 gr.	1 gr.
Apothecaries' . . .	5760 gr.	480 gr.	1 gr.

The grain is alike in all. The pound and ounce are alike in Troy and Apothecaries' weight.

MEASUREMENT OF TIME

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
30 (31, 28, 29) days	= 1 month (mo.)
52 weeks	= 1 year (yr.)
12 months	= 1 year
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

NOTE. — The **true solar year** does not have exactly 365 days. The true length of time for the earth to make one complete revolution about the sun is 365 da. 5 hr. 48 min. 49.7 sec., or nearly 365¼ da. Because of this extra ¼ da., we add an extra day every fourth year (those years divisible by 4) except every century not divisible by 400.

The extra day is added to the month of February.

TABLE OF ANGLE MEASURE

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree ($^\circ$)
90 degrees	= 1 right angle

TABLE OF ARC MEASURE

60 seconds ($''$)	= 1 minute ($'$)
60 minutes	= 1 degree ($^\circ$)
90 degrees	= 1 quadrant
360 degrees	= 1 circumference

TABLE OF ENGLISH MONEY

1 pound (£)	= 20 shillings (*s.*)	= \$4.8665
1 shilling	= 12 pence (*d.*)	= \$0.243+

We think ordinarily of a *pound* as *about* \$5, of a *shilling* as *about* 25 cents, and of a *penny* as *about* 2 cents.

TABLE OF FRENCH MONEY

1 franc (fr.) = 100 centimes (C.) = $0.193

We think ordinarily of a *franc* as *about* 20 cents.

TABLE OF GERMAN MONEY

1 mark (M.) = 100 pfennigs (pf.) = $0.238

We think ordinarily of a *mark* as *about* 25 cents, and of a pfennig as $\frac{1}{4}$ of a cent.

109. Reduction of compound numbers. — The reduction of a compound number to a single denominate number is shown by an example.

EXAMPLE. Reduce 5 yd. 2 ft. 10 in. to inches.

SOLUTION

5 yd. = 5 × 3 ft. = 15 ft.
15 ft. + 2 ft. = 17 ft.
17 ft. = 17 × 12 in. = 204 in.
204 in. + 10 in. = 214 in.

The work may be shortened by using the abstract number of units and computing as in the margin at the right.

$$\begin{array}{l} 5\text{—}2\text{—}10 \\ \underline{\;\;3\;\;} \\ 17 \\ \underline{\;12\;} \\ 214 \end{array}$$

A single denominate number may be reduced to larger units as follows:

EXAMPLE. Reduce 195 inches to yards, feet, and inches.

SOLUTION

195 in. ÷ 12 in. = 16 times and 3 inches remaining.
16 feet ÷ 3 feet = 5 times and 1 foot remaining.
Hence 195 in. = 5 yd. 1 ft. 3 in.

The work may be shortened by using abstract numbers and working as in the margin at the right.

$$\begin{array}{r} 12\overline{)195} \\ 3\overline{)16}\text{—}3 \\ 5\text{—}1 \end{array}$$

195 in. = 5 yd. 1 ft. 3 in.

EXERCISES

Reduce:

1. 3 rd. 4 yd. 2 ft. to feet.
2. 14 lb. 12 oz. to ounces.
3. 18 bu. 3 pk. 5 qt. to quarts.
4. 17 gal. 3 qt. 1 pt. to pints.
5. 16 sq. yd. 7 sq. ft. 48 sq. in. to square inches.
6. $16' \, 43''$ to seconds.
7. 3 hr. 53 min. 16 sec. to seconds.
8. 4 mi. 96 rd. to rods.
9. 46 A. 98 sq. rd. to square rods.
10. 5 yd. 2 ft. 9 in. to inches.

11. Reduce $\frac{7}{8}$ of a bushel to lower denominations.

WORK

$\frac{7}{8}$ bu. $= \frac{7}{8} \times 4$ pk. $= 3\frac{1}{2}$ pk.
$\frac{1}{2}$ pk. $= \frac{1}{2} \times 8$ qt. $= 4$ qt.
$\frac{7}{8}$ bu. $= 3$ pk. 4 qt.

Note. — This does not differ from the reduction of a whole number to a lower denomination.

Reduce to lower denominations:

12. $\frac{3}{8}$ T.	15. $\frac{5}{18}$ yd.	18. $\frac{3}{5}$ gal.	21. $\frac{13}{15}$ hr.
13. $\frac{9}{16}$ A.	16. $\frac{11}{16}$ bu.	19. $\frac{8}{9}$ rd.	22. $\frac{15}{16}$ yd.
14. $\frac{7}{8}$ mi.	17. $\frac{17}{18}$ sq. yd.	20. $\frac{5}{8}$ A.	23. $\frac{14}{15}$ yd.

Reduce to higher denominations:

24. 128 pints.	30. 1638 ounces.
25. 365 inches.	31. 9638 square feet.
26. 4360 seconds.	32. 1348 feet.
27. 5896 square inches.	33. 2765 pecks.
28. 5380 minutes.	34. 8345 minutes.
29. 4350 feet.	35. 9642 yards.

36. What part of a mile is 168 rd. 4 yd.?

SOLUTION

168 rd. $= 168 \times 5\frac{1}{2}$ yd. $= 924$ yd. $\frac{928}{1760} = \frac{29}{55}$.
924 yd. $+$ 4 yd. $=$ 928 yd. 168 rd. 4 yd. $= \frac{29}{55}$ mi.
928 yd. $\div$ 1760 yd. $= \frac{928}{1760}$.

37. 9 hr. 36 min. is what part of a day?

38. Reduce 2 pk. 5 qt. to a fraction of a bushel.

39. Reduce 3 qt. 1 pt. to a decimal part of a gallon.

40. What decimal part of a mile is 934 yd. 2 ft.?

110. Addition and subtraction of compound numbers. — The processes of addition and subtraction are the same whether the numbers are whole numbers, fractions, or compound numbers. The processes depend upon the principle that *only like things, or things considered alike can be added or subtracted.*

EXAMPLE 1. Add 8 yd. 2 ft. 8 in., 2 yd. 1 ft. 5 in., 6 yd. 2 ft. 7 in.

WORK

yd.	ft.	in.
3	2	8
2	1	5
6	2	7
13.	0	8

EXPLANATION. — The sum of the first column is 20 in. But 20 in. = 1 ft. 8 in., so, just as in simple addition, 8 in. is written down and 1 ft. carried to the next higher denomination. The sum of the second column, with the 1 ft. carried, = 6 ft. = 2 yd. 0 ft. So 0 is written down and 2 yd. carried.

EXAMPLE 2. From 9 yd. 2 ft. 3 in. subtract 2 yd. 2 ft. 8 in.

WORK

yd.	ft.	in.
9	2	3
2	2	8
6	2	7

EXPLANATION. — Since 8 in. cannot be taken from 3 in., 1 ft. of the 2 ft. is added to 3 in., making 15 in. Then 15 in. − 8 in. = 7 in. Likewise 1 yd. of the 9 yd. is added to the 1 ft. remaining, making 4 ft. Then 4 ft. − 2 ft. = 2 ft.

EXERCISES

1. Add: 6 bu. 3 pk. 2 qt., 9 bu. 2 pk. 7 qt., 7 bu. 1 pk. 5 qt., and 12 bu. 2 pk. 4 qt.

2. Add: 5 lb. 6 oz., 6 lb. 10 oz., 9 lb. 8 oz., 2 lb. 7 oz., and 5 lb. 8 oz.

3. Add: 5 rd. 4 yd. 2 ft., 6 rd. 3 yd. 2 ft., 8 rd. 4 yd. 1 ft., 7 rd. 1 yd. 2 ft., and 9 rd. 5 yd. 2 ft.

4. From 8 yd. 1 ft. 8 in. take 3 yd. 2 ft. 10 in.

5. From 5 bu. 1 pk. 5 qt. take 2 bu. 3 pk. 7 qt.

6. From 70 sq. yd. 4 sq. ft. 30 sq. in. take 50 sq. yd. 6 sq. ft. 50 sq. in.

7. If a room is 6 yd. 2 ft. 8 in. long and 5 yd. 1 ft. 2 in. wide, how far around it?

8. If a garden (rectangular) is 2 rd. 5 yd. 2 ft. wide and 8 rd. 2 yd. 1 ft. long, how many feet of fencing will be needed to inclose it?

9. A dining room is 18 ft. 10 in. by 16 ft. 8 in. If 8 ft. 8 in. are deducted for doors and windows, how many feet of molding for a plate rail are needed?

10. How many feet of weather molding are needed for two windows each 4 ft. 6 in. by 2 ft. 4 in. and one window 5 ft. 2 in. by 2 ft. 10 in.?

111. Multiplication and division of compound numbers. — These processes have the same meaning as in all the former work. Hence the same fundamental principles follow.

EXAMPLE 1. Multiply 34 yd. 2 ft. 8 in. by 5.

WORK

34 yd.	2 ft.	8 in.
		5
174 yd.	1 ft.	4 in.

EXPLANATION. — 5×8 in. = 40 in. = 3 ft. 4 in. Just as in simple multiplication, the 4 in. is written down and the 3 ft. carried to the next denomination. 5×2 ft. + 3 ft. = 13 ft. = 4 yd. 1 ft. The 1 ft. is written and 4 yd. carried. 5×34 yd. + 4 yd. = 174 yd.

EXAMPLE 2. Divide 26 yd. 2 ft. by 4.

WORK	
4)26 yd.	2 ft.
6 yd.	2 ft.

EXPLANATION. — 26 yd. ÷ 4 = 6 yd., with 2 yd. remaining. The 6 yd. is written down. The 2 yd. is reduced to feet and added to 2 ft., making 8 ft. 8 ft. ÷ 4 = 2 ft.

EXERCISES

Multiply:

1. 5 yd. 2 ft. 6 in. by 3.
2. 9 gal. 3 qt. 1 pt. by 6.
3. 5 hr. 13 min. 20 sec. by 5.
4. 16 lb. 10 oz. by 6.
5. 13 bu. 2 pk. 7 qt. by 6.
6. 9 sq. ft. 54 sq. in. by 10.
7. 17 gal. 2 qt. 1 pt. by 7.
8. 8 hr. 46 min. 35 sec. by 5.

Divide:

9. 46 lb. 14 oz. by 4.
10. 84 gal. 2 qt. by 6.
11. 16 hr. 48 min. by 5.
12. 57 bu. 1 pk. 6 qt. by 4.
13. 19 lb. 2 oz. by 10.
14. 46 ft. 8 in. by 9.
15. 49 gal. 3 qt. by 7.
16. 84 min. 38 sec. by 5.

17. If the side of a square field is 38 rd. 3 yd. 2 ft., what is the perimeter (the distance around)?

18. A coat rack is to be made containing 6 hooks equal distances apart. The distance from the first to the last hook is 8 ft. 4 in. How far apart are the hooks to be placed?

SUGGESTION. — Show by a diagram that the divisor is 5 instead of 6.

19. It is 34 ft. 6 in. from the first to the last post of a garden fence containing 5 posts equal distances apart. Find the distance from post to post.

20. If the posts are set 10 ft. apart, how many are needed for a fence 29 rd. 1 ft. 6 in. long?

21. How many posts 12 feet apart are needed for a grape arbor 8 rods long? How many cross pieces will be needed if there are three between each pair of posts?

22. How much belting does it take to drive 25 machines in a factory, if it takes 21 ft. 5 in. for each machine?

23. A row 15 ft. 8 in. long is to be set with 9 plants equal distances from one another. The end plants are to be 2 in. from the ends of the row. How far apart must they be placed?

24. A fence row is 92 ft. 7 in. long. There are to be 12 posts equal distances apart. How far apart must they be set?

25. A man has a lot with a frontage of 26 yd. 2 ft. He wishes to set 5 shade trees along the front, equally spaced, one at each end with three between. How far apart will they be?

THE METRIC SYSTEM OF MEASURES

112. Nature and use of the metric system. — The **metric system** of weights and measures is a *decimal* system, for the number of units of one denomination which make one unit of the next higher is always 10 or a power of 10.

The system is used in nearly all the civilized countries of the world except in the United States and England. While it has been legalized in the United States since 1866, it is but little used except in scientific work.

113. The standard unit. — The **standard unit** of the metric system, from which all other units are derived, is the **meter.** The length of the meter was obtained by surveying a long distance on the meridian through Paris and computing the distance from the equator to the pole. One ten-millionth of this distance from equator to pole was taken for the length of the meter. This meter is 39.3707 inches long.

A slight error was made in fixing the length of the meter, but that does not affect the usefulness of the system.

114. The primary units. — The unit of capacity and the unit of weight are defined from the unit of length, the **meter.** The primary unit of capacity is the **liter,** which is the capacity or volume of a cubical vessel one tenth of a meter on each edge. The primary unit of weight is the **gram,** which is the weight of a volume of distilled water at its greatest density equal to a cube one-hundredth of a meter on an edge.

115. The derived units. — All other units are named from the primary units by using Greek and Latin prefixes. The **Latin prefixes** denote *parts* of the primary unit, and the **Greek prefixes** denote *multiples* of the primary unit.

LATIN PREFIXES

Milli means $\frac{1}{1000}$; thus, millimeter means $\frac{1}{1000}$ of a meter.
Centi means $\frac{1}{100}$; thus, centimeter means $\frac{1}{100}$ of a meter.
Deci means $\frac{1}{10}$; thus, decimeter means $\frac{1}{10}$ of a meter.

GREEK PREFIXES

Deka means 10; thus, dekameter means 10 meters.
Hekto means 100; thus, hektometer means 100 meters.
Kilo means 1000; thus, kilometer means 1000 meters.
Myria means 10,000; thus, myriameter means 10,000 meters.

116. The advantages of the metric system. — The metric system grew out of an attempt by the French government to supply a system of weights and measures that would have a uniform decimal scale of relation and thus facilitate computation. The advantages are clearly:

1. A decimal relation between the units, so that all reductions may be made by annexing zeros or moving the decimal point.

2. The simple relation of the primary units of length, capacity, and weight to each other.

3. The uniform self-defining names of all the derived units.

TABLE OF LINEAR MEASURE

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
	= .3937079 inch
10 **centimeters**	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
	= 39.37079 inches.
10 **meters**	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer** (Km.)
	= 3280.9 feet, or .62137 mile
10 **kilometers**	= 1 myriameter (Mm.)

NOTE. — The units in common use are in **bold-faced type**. The abbreviations of the Latin prefixes begin with small letters and those of the Greek prefixes with capitals.

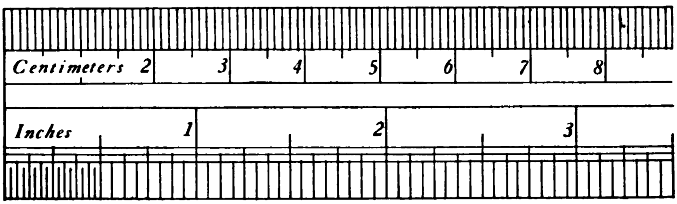


TABLE OF SURFACE MEASURE

The primary unit is the **square meter**.

NOTE. — Since the units of surface measure are squares whose dimensions are the corresponding linear units it takes 100 units of one denomination to make one of the next higher.

100 square millimeters (sq. mm.)	= 1 **square centimeter** (sq. cm.)
	= .155 sq. in.
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 **square meter** (sq. m.)
	= 10.764 sq. ft.
100 square meters	= 1 square dekameter (sq. Dm.)
100 square dekameters	= 1 square hektometer (sq. Hm.)
100 square hektometers	= 1 **square kilometer** (sq. Km.)
	= 247.114 acres

When used in measuring land the square meter is called a **centare** (ca.); the square dekameter an **are** (a.); and the square hektometer a **hektare** (Ha.).

TABLE OF VOLUME

The primary unit is the **cubic centimeter.**

NOTE. — Since the units of volume are cubes whose edges are the corresponding linear units, it takes 10^3 or 1000 units of one denomination to make one of the next higher.

1000 cubic millimeters (cu.mm.)	= 1 **cubic centimeter** (cu.cm.)
	= .06102 cu. in.
1000 cubic centimeters	= 1 cubic decimeter (cu.dm.)
1000 cubic decimeters	= 1 **cubic meter** (cu.m.)
	= 35.314 cu. ft.

In measuring wood the cubic meter is called a **stere** (st.); a **decistere** (dst.) is one tenth of a stere. 1 stere = .2759 cord.

TABLE OF CAPACITY

The primary unit is the **liter** (a cubic decimeter).

10 milliliters (ml.)	= 1 **centiliter** (cl.)	= .6102 cu. in.
10 centiliters	= 1 deciliter (dl.)	
10 deciliters	= 1 **liter** (l.) =	1.0567 liquid quarts, or .908 dry quarts
10 **liters**	= 1 dekaliter (Dl.)	
10 dekaliters	= 1 **hektoliter** (Hl.) =	26.417 gallons, or 2.8377 bushels
10 hektoliters	= 1 kiloliter (Kl.)	

The **liter** is used in measuring liquids and small fruits; the **hektoliter** in measuring grain, vegetables, and liquids in larger quantities.

TABLE OF WEIGHT

The primary unit is the **gram.**

10 milligrams (mg.)	= 1 **centigram** (cg.)	= .15432 grain
10 centigrams	= 1 decigram (dg.)	
10 decigrams	= 1 **gram** (g.)	= 15.432 grains
10 grams	= 1 dekagram (Dg.)	
10 dekagrams	= 1 hektogram (Hg.)	
10 hektograms	= 1 **kilogram** (Kg.)	= 2.20462 pounds
10 kilograms	= 1 myriagram (Mg.)	
10 myriagrams	= 1 quintal (Q.)	
10 quintals	= 1 **metric ton** (T.)	= 2204.621 pounds

The **gram** is the weight of a cubic centimeter, the **kilogram** of a cubic decimeter, and the **metric ton** of a cubic meter, of distilled water at its greatest density.

The *gram* is used in mixing medicines, and in weighing jewels, precious metals, letters, etc. Ordinary articles are weighed by the *kilogram* (commonly called *kilo*) and heavy articles by the *metric ton.*

TABLES OF EQUIVALENTS

The following tables show the legal equivalent values of the units in the English and the Metric system.

LINEAR MEASURE

1 inch	= 2.54 centimeters	1 centimeter	= .3937 of an inch
1 foot	= .3048 of a meter	1 decimeter	= .328 of a foot
1 yard	= .9144 of a meter	1 meter	= 1.0936 yards
1 rod	= 5.0292 meters	1 dekameter	= 1.9884 rods
1 mile	= 1.6093 kilometers	1 kilometer	= .62137 of a mile

SURFACE MEASURE

1 square inch	= 6.452 square centimeters
1 square foot	= .0929 of a square meter
1 square yard	= .8361 of a square meter
1 square rod	= 25.293 square meters
1 acre	= 40.47 ares
1 square mile	= 259 hectares

1 square centimeter	= .155 of a square inch
1 square decimeter	= .1076 of a square foot
1 square meter	= 1.196 square yards
1 are	= 3.954 square rods
1 hectare	= 2.471 acres
1 square kilometer	= .3861 of a square mile

CUBIC MEASURE

1 cubic inch	= 16.387 cubic centimeters
1 cubic foot	= 28.317 cubic decimeters
1 cubic yard	= .7646 of a cubic meter
1 cord	= 3.624 steres
1 cubic centimeter	= .061 of a cubic inch
1 cubic decimeter	= .0353 of a cubic foot
1 cubic meter	= 1.308 cubic yards
1 stere	= .2759 of a cord

MEASURE OF CAPACITY

1 dry quart	= 1.101 liters	1 liter	= .908 of a dry quart
1 liquid quart	= .9463 of a liter	1 liter	= 1.0567 liquid quarts
1 liquid gallon	= .3785 of a decaliter	1 decaliter	= 2.6417 liquid gallons
1 peck	= .881 of a decaliter	1 decaliter	= 1.135 pecks
1 bushel	= .3524 of a hectoliter	1 hectoliter	= 2.8377 bushels

MEASURES OF WEIGHT

1 grain Troy	= .0648 of a gram
1 ounce Troy	= 31.104 grams
1 ounce avoirdupois	= 28.35 grams
1 pound Troy	= .3732 of a kilogram
1 pound avoirdupois	= .4536 of a kilogram
1 ton (short)	= .9072 of a tonneau or ton
1 gram	= 15.432 grains Troy
1 gram	= .03215 of an ounce Troy
1 gram	= .03527 of an ounce avoidupois
1 kilogram	= 2.679 pounds Troy
1 kilogram	= 2.2046 pounds avoirdupois
1 tonneau	= 1.1023 tons (short)

CONVENIENT EQUIVALENT VALUES

1 cu. cm. of water = 1 ml. of water, and weighs 1 gram = 15.432 gr.
1 cu. dm. of water = 1 l. of water, and weighs 1 Kg. = 2.2046 lb.
1 cu. m. of water = 1 Kl. of water, and weighs 1 tonneau = 2204.6 lb.

EXERCISES

At sight reduce:

1. 5 m. to cm.
2. 60 cm. to dm.
3. 800 cm. to m.
4. 500 mm. to cm.
5. 1.5 m. to cm.
6. 385 cm. to m.
7. 500 m. to Dm.
8. 38 dm. to Km.
9. 12000 m. to Km.
10. 25 Km. to m.
11. .37 Km. to m.
12. 3.8 Km. to Dm.
13. Reduce 4.25 sq. cm. to sq. m.; to sq. mm.
14. Reduce 36,284 sq. mm. to sq. m.
15. Reduce 1.5 Kg. to g.; 4896 g. to Kg.
16. Reduce 84.72 Kg. to g.; 384.59 g. to Kg.
17. Change 75 bu. to hektoliters.
18. Change 1300 m. to yards.
19. Change 38.5 sq. rd. to square hektometers.
20. A barrel of flour weighs 196 lb. Express the weight in kilograms.

PROBLEMS INVOLVING METRIC MEASURES

1. An importer bought 2500 l. of oil at $.75 per liter and sold it at $3.20 per gallon. How much was his profit?

2. If American goods cost $2.35 per yard, including transportation, at what price per meter must they be sold in Paris to make a profit equal to $\frac{1}{4}$ of the cost?

3. If goods cost $1.85 per meter, how much is that per yard?

4. The distance between two cities in Germany is given as 320 Km. How many miles is that?

5. A certain specification calls for steel frames 9.365 m. long. Find the length in feet and inches to hundredths of an inch.

6. A bolt of imported silk is marked 68.8 m. What is it worth at $1.35 per yard?

7. If the distance between two places is 85 mi., how far is it in kilometers?

8. A 200-liter cask of oil is purchased in Paris at 1.5 francs per liter, and retailed in New York at $1.95 per gallon. Allowing $12.95 for duties and transportation, how much is gained? (1 franc = $0.193.)

9. An importer bought 348.2 kilos of gum at 28.5 francs per kilo. The duty and transportation charges were $28.75. Find the total cost in United States money.

10. A German liner is given as 338 meters long, and an American liner as 837 feet long. Which is longer and how much? Answer in feet.

11. On a running track there are 12 hurdles each 25 meters apart. How far from the first to the last in yards?

12. A Frenchman flew an aeroplane 85 Km. in $1\frac{1}{4}$ hr. Find the rate in miles per hour.

13. Mt. Everest is approximately 8842 meters high. Find the height in feet. In miles.

14. An importer paid $95.60 for 35.2 meters of velvet. How much per yard was that?

15. Cloth at 74 cents per meter is how much per yard?

16. A certain street in Germany is 18 meters wide. How many feet and inches is that?

17. An importer bought 2 Hl. 10 l. of perfume. How many gallons was that?

18. An importer bought 6 Kl. of olive oil which he put in quart cans. How many cans did it fill?

19. A merchant imported 850 Kg. of cheese. How many pounds did he import?

20. The Roquefort cheese industry at St. Étienne, France, uses annually 349,000 liters of sheep's milk. How many kilograms is this? How many pounds?

21. If it takes 100 liters of cows' milk to make 15 kilos of cheese, how many quarts will it take to make a pound?

22. The distance from Paris to Berlin, via Cologne, is 974.27 kilometers. The time of the express train is $16\frac{3}{4}$ hours. What is its speed in miles per hour?

23. The distance from Paris to Brussels is 310 kilometers, and from Brussels to Berlin is 812.5 kilometers. How many miles longer is a trip from Paris to Berlin via Brussels than by the direct route with distance as given in Problem 22?

24. In going from London to Paris, one may travel by train from London to Dover, 40 miles, at the rate of 25 miles an hour, including stops; from Dover to Calais by steamer, in 80 minutes; and from Calais to Paris, 260 kilometers, at the rate of 38 miles an hour. Not counting transfer time, how long does it take to go from London to Paris?

25. The first class fare on a train from Berlin to Vienna, a distance of 820 kilometers, is 65 marks 60 pfennigs. What is this rate equal to in cents per mile?

LONGITUDE AND TIME

117. **Longitude.** — Longitude is the distance east or west of a *prime meridian.* The prime meridian commonly used is that of Greenwich, near London, England. Sometimes the meridian of Washington, D. C., is used.

118. The measure of longitude. — Since the earth is practically a sphere, distances on its surface are measured along arcs of circles. Hence distances of longitude are measured in *degrees*, *minutes*, and *seconds*. Thus, longitude 30° 48′ 35″ E. indicates that a place is 30° 48′ 35″ east of the prime meridian. If some other place is given as 10° 30′ E., the first place is 20° 18′ 35″ farther east than the second. If the second place had been 10° 30′ W., then the first place would be 41° 18′ 35″ farther east than the second.

119. The relation of longitude to time. — Since the earth makes one complete revolution upon its axis once every 24 hours, every point on the earth's surface moves through one complete circle every 24 hours. That is, every point on the earth's surface moves through 360° every 24 hours.

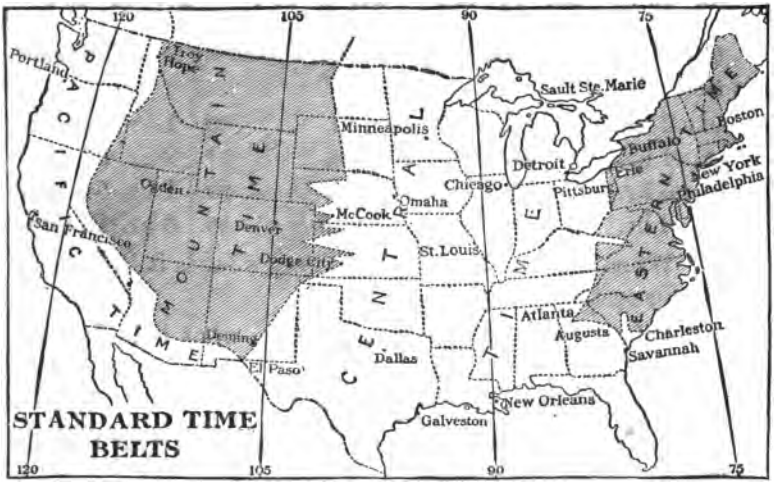
Hence a difference of 1° of longitude makes a difference of 4 minutes of time. And a difference of 15° of longitude makes a difference of 1 hour of time.

Since the revolution of the earth upon its axis is from west to east, sunrise, or noon, or any fixed time of a place comes earlier to that particular place than it does to the places west of it.

120. The local time of a place. — The exact time of a place is determined by the position of the sun, noon being the moment when the sun is on the meridian of the place. This is called **local time**. It is seen, then, that the local time of two places east or west of each other is not the same. If all places kept local time, it is easily seen that much confusion must arise in travel from east to west, or from west to east.

121. Standard Time. — To overcome the confusion arising from local time, the railroads of the United States adopted, in 1883, what is known as **Standard Time**. The country is

divided into four sections. The eastern section takes the local time of the 75th meridian, called **Eastern Time**; the central section takes the local time of the 90th meridian, called **Central Time**; the next section takes the local time of



the 105th meridian, called **Mountain Time**; and the Pacific section takes the time of the 120th meridian, called **Pacific Time.** Thus it is seen that since these meridians are 15 degrees apart, there is a difference of 1 hour of time between the places in adjoining sections.

EXERCISES

At sight give the difference in longitude between:

1. A place 30° 40′ West and one 40° 50′ West.
2. A place 10° 20′ East and one 15° 30′ West.
3. A place 20° 40′ West and one 10° 50′ West.
4. A place 30° 40′ West and one 5° 30′ East.
5. A place 3° 40′ East and one 46° 10′ East.

Give the difference in time corresponding to a difference in longitude of:

6. 30°.	**9.** 50°.	**12.** 10° 30′.	**15.** 30° 40′.
7. 40°.	**10.** 5°.	**13.** 7° 40′.	**16.** 90° 30′.
8. 10°.	**11.** 15° 30′.	**14.** 8° 10′.	**17.** 75° 45′.

Give the difference in longitude corresponding to a difference in time of:

18. 2 hr.	**22.** 3 hr. 40 min.	**26.** 1 hr. 48 min.
19. 4 hr.	**23.** 1 hr. 30 min.	**27.** 2 hr. 32 min.
20. 5 hr.	**24.** 6 hr. 10 min.	**28.** 1 hr. 54 min.
21. 6 hr.	**25.** 10 hr. 15 min.	**29.** 3 hr. 16 min.

30. When it is 8 A.M. Eastern Time, what is it in each of the other sections?

31. When it is 4 P.M. Mountain Time, what is it Eastern Time?

32. In which time belt do you live?

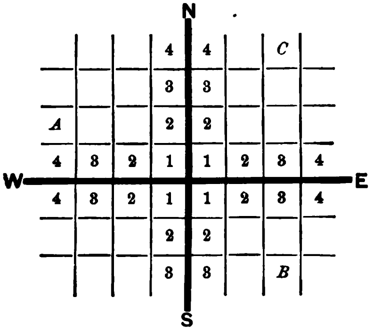
33. Look at your watch or the clock and tell what time it is in each of the other belts.

DIVISION OF LAND

122. Townships and sections. — Most of the land in the Central and Western states has been surveyed and divided into **townships** by systems of parallel lines 6 miles apart running north and south, and east and west. Then a township is 6 miles square, and contains 36 square miles or 23,040 acres.

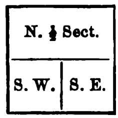
Each township is subdivided into 36 equal squares called **sections.** A section, then, is a mile square and contains 640 acres. Sections are always numbered as in the figure. The sections are divided into halves and quarters; the quarters into halves and quarters, etc.

The rows of townships running north and south are called **ranges**. The townships are described as north or south of some **base line** (marked **WE**) and east or west of some **principal meridian** (marked **NS**). Thus, township *C*, in the diagram, is described as "Township 4 north range 3 east" (T. 4 N. R. 3 E.).



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Township.



Section 15.

EXERCISES

1. A man's farm is described as the southwest quarter of section 15. Locate his land on the figure, and find how many acres he has.

2. A man owns the north half of the southwest quarter of section 25. Locate it, and find what the land is worth at \$65 per acre.

3. If a farm is described as the S. W. $\frac{1}{4}$ N. E. $\frac{1}{4}$ Sec. 16, locate it and find how large it is.

4. Locate N. $\frac{1}{2}$ N. W. $\frac{1}{4}$ S. E. $\frac{1}{4}$ Sec. 12.

5. A tract of land $\frac{1}{2}$ mile square is what part of a section? What is it worth at \$56 per acre?

6. A farm $\frac{1}{4}$ mile square contains how many acres?

7. If a farm is described as N. $\frac{1}{2}$ N. W. $\frac{1}{4}$, Sec. 20, T. 2 S. R. 4 W., locate it and find the value at \$65 per acre.

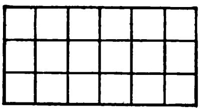
8. Locate E. $\frac{1}{2}$ S. E. $\frac{1}{4}$ Sec. 14, T. 5 N. R. 1 W. and find the value at \$48 per acre.

CHAPTER VI

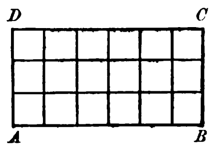
MENSURATION AND SQUARE ROOT

123. The measure of any quantity. — The **numerical** measure of a quantity is the number of times that it contains some ***standard unit of measure.***

124. The area of a surface. — The **area** of a surface is the measure of the surface when the unit of measure is the surface of a chosen square whose side is some linear unit. Thus in the figure, if the surface of one of the small squares is taken as the unit of measure, the area is 18, for the unit square is contained 18 times.



125. The area of a rectangle. — The figure $ABCD$ is called a **rectangle.** It has four straight sides of which the opposite sides are parallel, and the angles are right angles. Two sides at right angles, as AB and BC, are called the **base** and **altitude,** respectively. They are also called the **dimensions** of the rectangle.

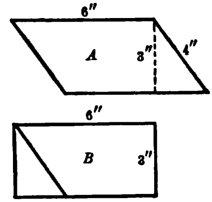


If AB is 6 inches and BC is 4 inches, the rectangle may be divided into 4 rows of 6 sq. in. each. Hence there are 6 sq. in. + 6 sq. in. + 6 sq. in. + 6 sq. in., or 4×6 sq. in. And in general,

The number of square units in the area of any rectangle is the product of the number of linear units in the two dimensions.

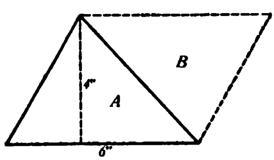
126. The area of a parallelogram. — A **parallelogram** is a figure of four straight sides whose opposite sides are parallel. But the angles need not be right angles as in the rectangle. Any side may be called the **base**, and the perpendicular distance between it and the opposite side the **altitude.**

By drawing the altitude from a corner, as the line marked $3''$ in parallelogram A, a portion of the figure may be cut off. By placing this portion at the other end of the figure, the parallelogram A may be formed into a rectangle B, which has the same base and altitude as the parallelogram. Hence the area of a parallelogram equals the area of a rectangle having the same base and altitude as the parallelogram. That is,



The number of square units in the area of any parallelogram is the product of the number of linear units in its base and the number of linear units in its altitude.

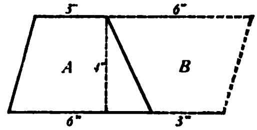
127. The area of a triangle. — A **triangle** is a figure of three straight sides, as figure A. Any side, as the side marked $6''$, may be called the **base,** and the perpendicular distance to that side from the opposite corner, as the distance marked $4''$, the **altitude.**



If another triangle B which is equal to triangle A in shape and size is placed adjoining A, as in the figure, the two triangles form a parallelogram of the same base and altitude as those of triangle A. Hence the area of triangle A is one half that of the parallelogram. Since the area of the parallelogram is 4×6 sq. in., or 24 sq. in., the area of triangle A is one half of 24 sq. in., or 12 sq. in. And in general,

The number of square units in the area of any triangle is one half of the product of the number of linear units in its base and the number of linear units in its altitude.

128. The area of a trapezoid. — A trapezoid is a figure of four straight sides of which two opposite sides are parallel and the other two are not parallel. The parallel sides are called the **bases**, and the perpendicular distance between them the **altitude**. Thus, the sides marked 6″ and 3″ are the bases of trapezoid A, and the line marked 4″ is its altitude.



If another trapezoid B which is equal in size and shape to A is placed adjoining A, as in the figure, the two trapezoids form a parallelogram with the same altitude and with a base equal to the sum of the bases of the trapezoid. Hence the area of trapezoid A is one half that of the parallelogram. Since the area of the parallelogram is 4×9 sq. in., or 36 sq. in., the area of trapezoid A is one half of 36 sq. in., or 18 sq. in. And in general,

The number of square units in the area of any trapezoid is one half of the product of the number of linear units in the altitude and the sum of the number of linear units in the bases.

129. The area of any polygon. — A **polygon** is any figure of three or more straight sides.

The area of any polygon may be found by dividing it into triangles, rectangles, etc., and finding the areas of the several parts, and then finding the sum of these areas.

PROBLEMS

1. How much will it cost to build a cement walk 4 ft. wide and 320 ft. long at 18 ¢ per square foot?

2. What will it cost to pave a street 50 ft. wide and a mile long at $1.25 per square yard?

3. How much will it cost to build a 4-ft. cement walk across the front and along one side of a corner lot 80 ft. by 216 ft. at 16¢ per square foot?

4. How many bricks 8 in. square will it take to cover a court 33 ft. 4 in. long and 16 ft. 8 in. wide?

5. A city lot with a frontage of 85 ft. and a depth of 200 ft. sold for $35 per front foot. How much per square foot was that?

6. Is it cheaper to pay $20 per front foot, or 12 cents per square foot for a city lot having a 60 ft. frontage and a depth of 175 ft.?

7. At $2.70 per running yard, find the cost of covering a floor 18 ft. by 24 ft. with lineoleum 6 ft. wide.

8. A farmer estimates 12 T. of silage to the acre from a field of corn 32 rd. by 48 rd. How much does he expect in all?

9. If a farmer gets 270 bushels of potatoes from a field 12 rd. by 20 rd., how much is the yield per acre?

10. At 8¢ per square yard, how much will it cost to sod a lawn 65 ft. by 186 ft.?

11. Allowing 7 pk. of seed wheat to the acre, how many bushels are needed for a field 35 rd. by 42 rd.?

12. An athletic park 500 ft. by 640 ft. contains how many acres?

13. A 5 ft. walk incloses a rectangular grass plot 98 ft. by 116 ft. Find the area of the walk.

14. How many square feet in the walls and ceiling of a room 22 ft. wide, 26 ft. long, and 10 ft. high?

15. If a roll of wall paper 8 yd. long and 18 in. wide costs 45¢, find the cost of paper for the four walls of a room 16 ft. wide, 22 ft. long, and 9 ft. high, no allowance being made for openings.

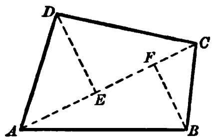
16. Find the cost of plastering the walls and ceiling of a room 15 ft. by 18 ft. and 9 ft. high at 18¢ per square yard, no allowance being made for openings.

17. A lot 360 ft. wide and 420 ft. long is inclosed by a tight board fence $4\frac{1}{2}$ ft. high. At $1.50 per 300 square feet, find the cost of painting the fence, inside and outside.

18. A barn 36 ft. by 54 ft. is 16 ft. high to the eaves. The two gables are each 36 ft. wide and 15 ft. high. At 80¢ per square (100 sq. ft.) find the cost of painting the barn.

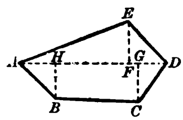
19. How many paving blocks each 6 in. by 12 in. will be needed for paving a street $1\frac{1}{4}$ mi. long and 40 ft. wide?

20. The diagram is that of a field. AC represents 80 rd., BF 30 rd., and DE 35 rd. How many acres in it?



21. Draw a diagram $ABCD$ in which diagonal AC represents 24 in. and the perpendicular to it from D is 12 in. and from B is 10 in. Find the area.

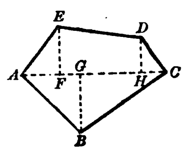
22. This is a diagram of an irregular field. $AH = 20$ rd., $HG = 45$ rd., $GD = 15$ rd., $EF = 24$ rd., $GC = 24$ rd., $BH = 20$ rd. Find the number of acres in it.



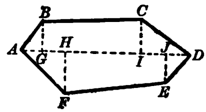
23. Find the area when $AH = 30$ rd., $HG = 70$ rd., $GD = 20$ rd., $EF = 50$ rd., $GC = 40$ rd., and $BH = 45$ rd.

24. A grass plot $90' \times 110'$ is to be surrounded by a 4 ft. walk. A contractor will construct the walk of asphalt for 5¢ per square foot. How much will it cost?

25. Find the area of $ABCDE$, if $AC = 500$ yd.; $BG = 200$ yd.; $AF = 75$ yd.; $AG = 125$ yd.; $AH = 425$ yd.; $EF = 160$ yd.; $DH = 110$ yd.



26. In $ABCDEF$, $AG = 100$ ft.; $GH = 100$ ft.; $HI = 400$ ft.; $IJ = 100$ ft.; $JD = 150$ ft.; $BG = 150$ ft.; $HF = 200$ ft.; $CI = 175$ ft.; and $JE = 150$ ft. Find the area.



THE MEASUREMENT OF THE CIRCLE

130. The relation of circumference to diameter. — If the circumference and diameter of some large circular object, as a dining-room table, or some large wheel, be carefully measured, it will be found that the circumference is about $3\frac{1}{7}$ times the diameter, whatever size of circle is taken. A more exact relation has been found by mathematics to be 3.1416 (π) to the nearest ten thousandth. Hence,

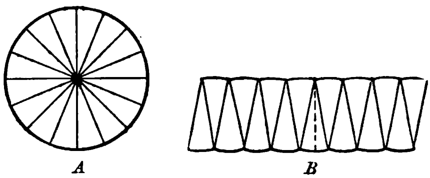
$$\textit{Circumference} = 3.1416 \times \textit{diameter} = 3.1416 \times 2 \times \textit{radius},$$

or

$$C = \pi \times D = 2 \times \pi \times r.$$

NOTE. — It is seen from this that to any change in the diameter of a circle there is a corresponding change in the circumference just 3.1416 times as great.

131. The area of a circle. — Since a circle cannot be measured by dividing it into squares as was done in the case of a rectangle, it must be made as nearly as possible into some figure that can be measured. By cutting a circle A into a large number of equal sectors, as shown in the figure, it may be rearranged into a figure B closely resembling a parallelogram with a base equal to one half of the circumference and an altitude equal to the radius.



If it were a parallelogram of these dimensions, the area would be $\frac{1}{2} C \times R$, and this is the true area as found by geometry.

But $\frac{1}{2}C \times R = \pi R \times R = \pi R^2$. Hence,

The number of square units in the area of any circle is equal to the product of 3.1416 *and the square of the number of linear units in the radius.*

PROBLEMS

1. Find the diameter of a wheel that makes 660 revolutions in going a mile.

2. How many revolutions per mile will a 34-inch automobile wheel make?

3. If a 32-inch automobile wheel is making 225 revolutions per minute, how fast is the machine traveling?

4. The reading of a speedometer of an automobile is controlled by the rate at which the automobile wheels turn. If a speedometer is geared for a 34-inch tire and a 35-inch tire is used, what correction must be made in the reading of the speedometer to give the actual speed or distance?

5. How much belting is required to make a belt to run over two pulleys, each 30 inches in diameter, the distance between their centers being 18 feet?

6. If a 10-inch pulley is making 350 revolutions a minute, how fast is a point on the rim traveling?

NOTE. — This rate is called the *rim speed.*

7. A grindstone of Ohio stone will safely stand a rim speed of 2500 feet per minute. How many revolutions per minute will a 4-foot stone safely stand?

8. A grindstone of Huron stone will safely stand a rim speed of 3600 feet per minute. How many revolutions per minute will a $3\frac{1}{2}$-foot stone safely stand?

9. A 10-inch pulley, running at the rate of 300 revolutions per minute, is to drive another pulley at the rate of 180 revolutions per minute. What diameter must the second pulley have?

10. The main driving pulley of an engine is 9 feet in diameter and is making 96 revolutions per minute. If it is belted to a 36-inch pulley on the main line shaft, at what rate (revolutions per minute) is the latter driven?

11. A running track having two parallel sides and two semicircular ends measures exactly 1320 feet at the inner curb. Two athletes run, one 5 ft. and the other 10 ft. from the inner curb. By how much is the outer man handicapped?

12. Suppose that the earth is a perfect sphere 8000 miles in diameter, and that a closely fitting band or tire could be placed entirely around it at the equator. If this tire were cut and an extra piece 1 ft. long inserted, that is, if the tire were enlarged 1 ft., how much space would there be between the tire and the surface of the earth, if the space were distributed evenly around the earth?

13. Compare the area of a 10-foot circle with that of a 15-foot circle.

14. A contractor agreed to build a circular basin for a fountain at the rate of 65¢ for each square foot in the bottom of the basin. Find the cost of a basin 38 ft. in diameter.

15. A 24-foot basin for a fountain is surrounded by a 5-foot walk. At 24¢ per square foot, how much did the walk cost?

16. When the steam pressure in the boiler of an engine is 96 lb. per square inch, what is the total pressure upon a 12-inch piston?

17. A wash boiler 20 inches long and 12 inches wide has parallel sides and semicircular ends. How many square inches of copper are there in the bottom?

18. If a horse is tethered by a 50-foot rope to the outside corner of an inclosed lot, which is 100 ft. square, over how many square feet can he graze?

19. If the horse is tethered by the same rope to the inner corner of the lot, over how many square feet can he graze?

20. If by the same rope he is tethered to the corner of a barn 30 ft. by 40 ft., over how many square feet can he graze?

SQUARES AND SQUARE ROOT

132. The square of a number.—If a number is the product of two equal numbers, it is called the **square** or **second power** of one of them. Thus, since 6×6 is 36, 36 is the *square* of 6. This is denoted by $6^2 = 36$. The 2 written to the right of and above 6 is called the **exponent** and denotes that 6 is to be multiplied by itself.

133. The square root of a number. — One of the two equal factors that form a number is called the **square root** of the number. Thus 6 is the square root of 36, 7 of 49, 8 of 64, etc. These are written

$$\sqrt{36} = 6, \qquad \sqrt{49} = 7, \qquad \sqrt{64} = 8, \text{ etc.}$$

EXERCISES AND PROBLEMS

1. Give the squares of all numbers from 1 to 9 inclusive.

2. Give the square roots of 64, 81, 100, 121, and 144.

3. Find the side of a square field whose area is 144 sq. rd.

Solution. — Since 144 is the product of the number of linear units in each side, it is the square of the number in *one* side. And since $144 = 12 \times 12$, the length of one side is 12 rods.

4. Find the side of a square whose area is 1225 sq. ft.

SOLUTION

$$\begin{array}{r} 5\,)\underline{1225} \\ 5\,)\underline{245} \\ 7\,)\underline{49} \\ 7 \end{array}$$

$5 \times 7 = 35$, the no. of ft.

EXPLANATION. — By factoring, it is found that $1225 = 5 \times 5 \times 7 \times 7$. Hence 5×7 gives one of the two equal factors that make 1225.

Hence each side is 35 feet.

5. What is the side of a square having the same area as a rectangle 16 ft. wide and 36 ft. long?

SOLUTION. — $\sqrt{16 \times 36} = \sqrt{4 \times 4 \times 6 \times 6} = \sqrt{24 \times 24} = 24$. Hence the side of the square is 24 ft.

6. What is the side of a square having the same area as a rectangle 75 ft. wide and 147 ft. long?

7. Find the side of a square whose area is 9216 sq. rd.

8. A rectangular field twice as long as it is wide contains 20 acres. Find its length and breadth. (Divide into two squares.)

9. If a rectangle containing 864 sq. ft. is $1\frac{1}{2}$ times as long as it is wide, find its dimensions.

10. A square field contains 68.90625 acres. Find how many rods of fence it will take to inclose it.

11. Find the radius of a circle that contains 314.16 sq. ft.

134. Approximate square roots. — From the preceding problems it is seen that if a number is composed of two equal factors, the factors may be found by factoring. Problems arise, however, that require the **approximate square roots** of numbers that are not made up of two equal factors whose exact values can be found. Hence the need arises of a method of finding square root other than by factoring.

Since the method must be the reverse of squaring a number, a careful analysis of the work of squaring a two-figured number will give the process.

135. Squaring a two-figured number. — The general process may be seen by squaring some number such as 47.

WORK

$$\begin{array}{rl}
47 & \\
\underline{47} & \\
329 = & 7^2 + 7 \times 40 \\
1880 = & \underline{\qquad 7 \times 40 + 40^2} \\
\overline{2209} = & \overline{7^2 + 2 \times 7 \times 40 + 40^2}
\end{array}$$

EXPLANATION. — It will be seen by following the work in the order in which it is done that the first step is 7×7, the next 7×40, the next 40×7, and the last 40×40. This being the work in squaring *any* two-figured number, it is seen that,

The square of any two-figured number is the square of the ones' digit plus twice the product of the ones by the value represented by the tens' digit plus the square of the value represented by the tens' digit.

Thus, $64^2 = 4^2 + 2 \times 4 \times 60 + 60^2 = 16 + 480 + 3600 = 4096$.

136. Number of figures in roots and powers compared. —

$1^2 = 1$	$10^2 = 100$	$100^2 = 10{,}000$
$9^2 = 81$	$99^2 = 9801$	$999^2 = 998{,}001$

From the above powers and their roots, it appears that,

The number of periods of two figures each, beginning at ones' place, into which a whole number can be divided equals the number of figures in the square root.

137. Extracting square root. — By use of §§ 135 and 136, the square root of a number may be found. The method is shown by an example.

EXAMPLE. — Find the square root of 2809.

WORK

	$28'09'$ (53
$5^2=$	25
100	309
103	309

EXPLANATION. — It is seen that there are two root figures. The first must be 5, for $50^2 = 2500$ and $60^2 = 3600$, and 2809 lies between the two. Then, of the three addends that make 2809, 2500 or 50^2 is known. Subtracting 2500, 309 remains. This must be the sum of the other two addends, the larger of which is $2 \times 50 \times$ the ones' digit. Hence $309 \div 100$ gives approximately the ones' digit, or 3.

Adding 3 to 100 gives 103, which multiplied by 3 gives 309, thus finding the two remaining addends by one multiplication.

Find:

1. $\sqrt{784}$.	4. $\sqrt{3136}$.	7. $\sqrt{5329}$.	10. $\sqrt{7569}$.
2. $\sqrt{3364}$.	5. $\sqrt{6889}$.	8. $\sqrt{4489}$.	11. $\sqrt{2916}$.
3. $\sqrt{8464}$.	6. $\sqrt{2704}$.	9. $\sqrt{9801}$.	12. $\sqrt{9409}$.

The process is the same for larger numbers, as shown in the following:

13. Find the square root of 2,137,444.

PROCESS

	$2'13'74'44'$ (1462
$1^2=$	1
2	1 13
24	96
28	17 74
286	17 16
292	58 44
2922	58 44

Find the square root of:

14. 283,024.	**20.** 529,984.
15. 299,209.	**21.** 484,416.
16. 404,496.	**22.** 638,401.
17. 556,516.	**23.** 725,904.
18. 755,161.	**24.** 294,849.
19. 6,017,209.	**25.** 1,739,761.

26. The 100th power of 2 is 1,267,650,600,228,229,401, 496,703,205,376. Verify this by extracting square roots, dividing by 2 when necessary to get even powers of 2.

The process of finding the square root of a decimal is shown in the following:

27. Find the square root of .734.

PROCESS

$$\begin{array}{l|l}
 & .'73'40'00' \;\underline{|\,.856^{+}} \\
.8^2 = & .\,64 \\ \hline
1.6 & .\,09\;40 \\
1.65 & \underline{.\,08\;25} \\
1.70 & .\,01\;15\;00 \\
1.706 & \underline{.\,01\;02\;36} \\
 & .\,00\;12\;64
\end{array}$$

EXPLANATION. — Since the square root of .734 can be obtained only approximately, we plan to find it to three decimal places. Hence zeros are added until three full periods of decimal figures are formed. Since the square of tenths is hundredths, to get the first root figure we take the first two figures at the right of the decimal point, or .73, the root of which is .8, nearly. Twice .8, or 1.6, is taken as the first divisor. Each new root figure is determined by division as in the case of integers. The inexactness of the root is expressed by + or − after the last root figure computed.

28. Study the process of extracting the square root of .501 to three decimal places and explain the steps.

$$\begin{array}{l|l}
 & .'50'10'00' \;\underline{|\,.707^{+}} \\
.7^2 = & .\,49 \\ \hline
1.4 & .\,01\;10 \\
1.407 & \underline{.\,00\;98\;49} \\
 & .\,00\;11\;51
\end{array}$$

Find the square root of:

29. .5625. **32.** .783. **35.** 824.9. **38.** 1932.4.

30. .9216. **33.** .89. **36.** .64. **39.** 225.9009.

31. 42.225. **34.** 19.467. **37.** .064. **40.** .8.

Find to two decimal places:

41. 2. **43.** 5. **45.** 10. **47.** 24.

42. 3. **44.** 7. **46.** 18. **48.** 39.

PROBLEMS

1. Find the side of a square whose area is 2764 sq. ft.

2. A square field contains 8 acres. How many rods of fence will it take to inclose it?

3. The area of a circle is 27 sq. ft. Find its radius to tenths of a foot.

4. A water main is 18 inches in diameter. It is to be replaced by one having 3 times the carrying capacity. Find to tenths of an inch the diameter of the new pipe.

SUGGESTION. — To have three times the carrying capacity, the area of a cross section must be three times the area of a cross section of the smaller.

5. The bottom of a silo is to contain 364 square feet. Find to tenths of a foot its diameter.

6. Find the diameter of a circular lot that will contain the same area as one 12 rods square.

7. A circular tray is to contain the same amount of sheet copper as a rectangular one 12 in. by 16 in. Find its diameter.

8. In installing a hot air furnace in a dwelling, one pipe is put in to carry the cold air to the furnace. Three pipes each 10 in. in diameter and three other pipes each 12 in. in diameter are provided to convey the warm air to the several rooms. If the area of a cross section of the cold air pipe is to equal the total area of the cross sections of the warm air pipes, find its diameter.

9. Two families wish to pipe water from a spring to their two houses. If the pipes leading from their houses to the main pipe are each $\frac{1}{2}$ in. in diameter, they wish to know if $\frac{3}{4}$-inch pipe is large enough for the main pipe. Is it? If 1-inch pipe is used for the main pipe, how many families may connect with $\frac{1}{2}$-inch pipe?

10. Two drain pipes each 4 in. in diameter empty into one large one. In order to have the same carrying capacity as the two smaller ones together, what diameter must the larger pipe have? (See suggestion, problem 4.)

11. Eighty tubes each 3 in. in diameter carry the hot air and smoke from the fire box of a boiler through the water, to heat it. If the area of the cross section of the smoke stack should equal .72 of the combined areas of the cross sections of the tubes, what should the diameter of the smoke stack be?

138. Methods of using the table. — Two methods of using the table given on the following page to find approximate roots of numbers larger than 100 are shown as follows.

Find the square root of 7235.

WORK

$$\begin{array}{r} 85.44003 \\ \underline{84.85281} \\ .58722 \\ \underline{.35} \\ 293610 \\ \underline{176166} \\ .2055270 \\ \underline{84.85281} \\ 85.0583370 \end{array}$$

EXPLANATION. — By the tables, $\sqrt{73} = 8.544003$. Hence $\sqrt{7300} = 85.44003$. Also $\sqrt{72} = 8.485281$. Hence $\sqrt{7200} = 84.85281$. The difference is .58722, which is caused by a difference of 100 between the numbers 7300 and 7200. But the given number 7235 is but 35 larger than 7200. Hence .35 of the difference between the roots is added to the root of 7200. This is but a close approximation.

SECOND METHOD

$$\frac{7235 - 7225}{7396 - 7225} = \frac{10}{171} = .0584$$

Root = 85.0584

EXPLANATION. — The given number 7235 lies between two numbers, 7225 and 7396, whose roots are known to be 85 and 86, respectively. The difference between 7235 and 7225 is $\frac{10}{171}$ or .0584 of the difference between the two numbers, 7396 and 7225. Hence the root is approximately that much more than 85, the root of 7225. Observe that the results by the two methods agree to three decimal places.

TABLE OF SQUARES AND SQUARE ROOTS

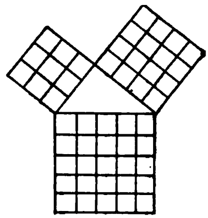
Number	Square	Square Root	Number	Square	Square Root
1	1	1	51	2601	7.141428
2	4	1.414213	52	2704	7.211102
3	9	1.732050	53	2809	7.280109
4	16	2.000000	54	2916	7.348469
5	25	2.236068	55	3025	7.416198
6	36	2.449489	56	3136	7.483314
7	49	2.645751	57	3249	7.549834
8	64	2.828427	58	3364	7.615773
9	81	3.000000	59	3481	7.681145
10	100	3.162277	60	3600	7.745966
11	121	3.316624	61	3721	7.810249
12	144	3.464101	62	3844	7.874007
13	169	3.605551	63	3969	7.937253
14	196	3.741657	64	4096	8.000000
15	225	3.872983	65	4225	8.062257
16	256	4.000000	66	4356	8.124038
17	289	4.123105	67	4489	8.185352
18	324	4.242640	68	4624	8.246211
19	361	4.358898	69	4761	8.306623
20	400	4.472136	70	4900	8.366600
21	441	4.582575	71	5041	8.426149
22	484	4.690415	72	5184	8.485281
23	529	4.795831	73	5329	8.544003
24	576	4.898979	74	5476	8.602325
25	625	5.000000	75	5625	8.660254
26	676	5.099019	76	5776	8.717797
27	729	5.196152	77	5929	8.774964
28	784	5.291502	78	6084	8.831760
29	841	5.385164	79	6241	8.888194
30	900	5.477225	80	6400	8.944271
31	961	5.567764	81	6561	9.000000
32	1024	5.656854	82	6724	9.055385
33	1089	5.744562	83	6889	9.110438
34	1156	5.830951	84	7056	9.165151
35	1225	5.916079	85	7225	9.219544
36	1296	6.000000	86	7396	9.273618
37	1369	6.082762	87	7569	9.327379
38	1444	6.164414	88	7744	9.380831
39	1521	6.244998	89	7921	9.433981
40	1600	6.324555	90	8100	9.486833
41	1681	6.403124	91	8281	9.539392
42	1764	6.480740	92	8464	9.591663
43	1849	6.557438	93	8649	9.643650
44	1936	6.663249	94	8836	9.695359
45	2025	6.708203	95	9025	9.746794
46	2116	6.782330	96	9216	9.797959
47	2209	6.855654	97	9409	9.848857
48	2304	6.928303	98	9604	9.899494
49	2401	7.000000	99	9801	9.949874
50	2500	7.071067	100	10000	10.000000

By the table, find the square roots to nearest hundredth:

1. 4623.	**7.** 938.	**13.** 76.2.	**19.** .3846.
2. 5781.	**8.** 722.	**14.** 84.6.	**20.** .5763.
3. 8746.	**9.** 634.	**15.** 46.7.	**21.** .936
4. 1925.	**10.** 816.	**16.** 75.6.	**22.** .847.
5. 1478.	**11.** 738.	**17.** 35.2.	**23.** .76.
6. 3462.	**12.** 972.	**18.** 28.7.	**24.** .5.

THE PYTHAGOREAN THEOREM

139. The Pythagorean theorem. — A **right triangle** is a triangle of which one angle is a right angle. The side opposite the right angle is called the **hypotenuse**, and the other sides are called the **legs**. By drawing a right triangle whose legs are 3 in. and 4 in. respectively, it will be seen that the hypotenuse is just 5 in., and that the area of the square on the hypotenuse equals the sum of the areas of the squares on the two legs.

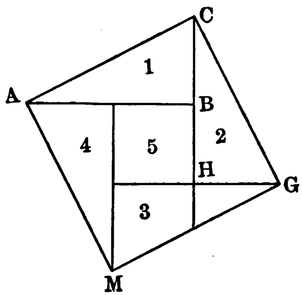


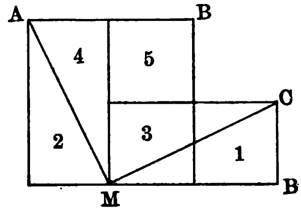
This important truth was proved by Pythagoras about 500 B.C. to be true of any right triangle. That is,

The square on the hypotenuse of any right triangle is equal to the sum of the squares on the other two sides.

NOTE. — Carpenters make use of this fact in laying out the foundation of a building when they want the walls at right angles to each other. Starting at one corner, a line 8 ft. long is taken in one direction along which the foundation is to be laid. Starting from the same corner, another line 6 ft. long is fastened to the end of the first line and moved about until a 10-ft. rod will just reach the outer extremities of the two lines.

The truth of the Pythagorean theorem may be seen by drawing, or cutting from cardboard, figures like the following:





Let ABC be the right triangle. The square on the hypotenuse AC is equal to the four triangles, 1, 2, 3, and 4, and the small square, 5. Now put 1 and 2 in the position of the figure at the right, and the figure is equal to a square on AB and one on CB.

PROBLEMS

1. One leg of a right triangle is 48 feet and the other is 36 feet. What is the hypotenuse?

2. The hypotenuse of a right triangle is 85 feet and one leg is 51 feet. What is the other leg?

3. One leg of a right triangle is 76 feet and the hypotenuse is 95 feet. What is the other leg?

4. What is the diagonal (distance between the opposite corners) of a rectangle 92 ft. long and 69 ft. wide?

5. How long is the diagonal of a 30-foot square?

6. What is the length of the longest straight line that can be drawn on a sheet of paper 16 inches by 20 inches?

7. How far is a place 12 miles east of you from one 18 miles north of you?

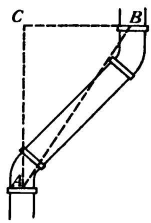
8. What is the distance between the opposite corners of a field 200 rods long and half as wide?

9. If a window is 18 ft. from the ground, how long must a ladder be to reach to the window if the foot of the ladder is placed 6 ft. out from the building?

10. In decorating a room two ribbons are stretched, connecting the opposite corners. If the room is 30 ft. wide and 40 ft. long, how many yards of ribbon does it take?

11. A baseball diamond is 90 ft. square. How long is the throw from first to third base?

12. A derrick is 48 ft. high, and is supported by three steel cables, each reaching from the top of the derrick to a stake in the ground 45 ft. from the foot of the derrick. How much steel cable does it take, allowing 10 ft. for fastening all three cables?



13. A plumber wishes to make an offset in a pipe at A. The offset CB is to be 16 in. and the "travel" AC 24 in. Find the length of the section AB.

14. For reaming round holes .5 of an inch in diameter a square reamer is used. Find the dimensions of the reamer.

15. If the base of an isosceles triangle (a triangle having two equal sides) is 12 inches and its altitude is 10 inches, find the length of its equal sides.

SUGGESTION. — The altitude of an isosceles triangle divides the triangle into two equal right triangles.

16. If the equal sides of an isosceles triangle are each 15 inches and the base 12 inches, what is the altitude?

17. Find the area of an isosceles triangle whose base is 14 inches and whose equal sides are each 12 inches.

18. Find the altitude of a triangle each of whose sides is 24 inches.

19. Find the area of a triangle each of whose sides is 10 inches.

20. How many acres in a field each of whose three sides is 16 rods?

MEASUREMENT OF LUMBER AND TIMBER

140. Board measure. — In measuring lumber, the unit of measure is the **board foot.** A board foot is a board 1 foot square and 1 inch thick, or an equivalent amount of lumber in different shape. Thus, a piece of 2″ by 4″ timber 1 foot long contains $\frac{2}{3}$ of a board foot; a piece of 6″ by 8″ timber 1 foot long contains 4 board feet. Why?

In measuring lumber less than 1 inch thick the computation is the same as if the thickness were 1 inch.

Lumber is usually sold by the thousand board feet. Thus a quotation of "$42 per M" means $42 per 1000 bd. ft.

NOTE. — By lumbermen the term "foot" is used instead of "board foot."

141. Measurement of Timber. — Practical foresters, lumbermen, and others have occasion to compute the amounts of timber in certain kinds of forests. There are over forty rules for such computations. For full discussion see "The Woodman's Handbook," by Henry S. Graves, published by the Government Printing Office, Washington, D.C.

The Doyle Rule is used very extensively throughout the United States for finding the amount of sawed lumber that can be obtained from a log as it comes from the forest. The rule is:

Subtract 4 inches from the diameter of the log at the small end; square one fourth of the remainder; multiply the result by the number of feet in length. The result is the number of board feet.

EXERCISES AND PROBLEMS

NOTE. — Use the Doyle Rule for problems 1, 2, and 3.

1. How many board feet of lumber are there in a log 24 ft. long and 16 in. in diameter at the small end?

2. How much lumber is there in a pile of 850 saw logs averaging 18 ft. in length and 12 in. in diameter at the small end?

3. *Make a chart to show the amount of lumber in logs:*

Diameter in inches	12	13	14	15	16	17	18	19	20	21	22
Length in feet	10	12	14	16	18	20	22	24	26	28	30
Number of bd. ft.											

Find the number of board feet in:

4. A board 1 in. thick, 8 in. wide, and 12 ft. long.

5. A board 1 in. thick, 6 in. wide, and 9 ft. long.

6. A board $\frac{1}{2}$ in. thick, 12 in. wide, and 3 ft. long.

7. A board $\frac{5}{8}$ in. thick, 9 in. wide, and 12 ft. long.

8. A board 2 in. thick, 1 ft. wide, and 10 ft. long.

9. A joist 3 in. by 4 in., and 16 ft. long.

10. A joist 2 in. by 6 in., and 16 ft. long.

11. A beam 4 in. by 8 in., and 14 ft. long.

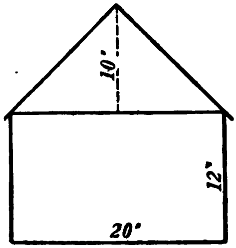
12. A plank $1\frac{1}{2}$ in. by 10 in., and 12 ft. long.

13. Eight 3 in. by 4 in. joists, each 15 ft. long.

14. Twelve joists, each 16 ft. long and 4 in. square.

15. A stick of timber 18 ft. long and 12 in. square.

16. The diagram gives the dimensions of the gable end of a house. Allowing for a window space of 5 × 6 sq. ft., and adding $\frac{1}{8}$ for waste in cutting, how many feet of lumber does it take to "rough side" it (*i.e.* board it over)? What does it cost at $22 per M?



17. Regular siding, or "weather boarding", is 5 inches wide, but being overlapped, lies but $3\frac{3}{4}$ inches to the weather. Hence $\frac{1}{3}$ is added to the number of square feet in estimating the amount of siding needed for a house. What will regular siding cost at $28 per M for the surface described in Ex. 16?

18. How much lumber is required to floor a barn 40 ft. long and 24 ft. wide with 2-inch plank?

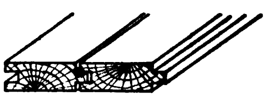
19. How much 1-inch boarding will it take to build a tight board fence 4 ft. high and 166 ft. long between two lots? What will it cost at $28 per M?

20. A contractor, in building a house, bought:

3000 ft. of rough siding at $30 per M;
4000 ft. of weather boarding at $35 per M;
1800 ft. of flooring at $70 per M;
700 ft. of inside finishing at $45 per M;
200 3 in. by 4 in. joists, each 12 ft. long, at $35 per M.

What was the cost?

21. Hardwood flooring is called 3-inch flooring when made from lumber 3 inches wide. There is a waste of $\frac{3}{4}$ of an inch in planing and in making the "tongue and groove." How wide a strip will one "3-inch board" cover?



How much must be added to the area of a floor to allow for the waste in planing and in cutting the "tongue and groove" of 3-inch flooring?

22. Find the cost of 3-inch oak flooring at \$75 per M for three rooms as follows: $12' \times 14'$; $15' \times 16'$; and $18' \times 20'$. (Add $\frac{1}{3}$ of the area for waste in the tongue and groove.)

23. All flooring has the $\frac{3}{4}$-inch waste in planing and in cutting the tongue and groove. What allowance must be made for 6-inch flooring; *i.e.*, what must be added to the surface of the floor to find the board feet of lumber needed?

24. Find the cost of flooring for four bedrooms as follows:

$$11' \times 12';\ 18\tfrac{1}{2}' \times 14';\ 12' \times 13\tfrac{1}{2}';\ 14' \times 16'.$$

Use 6-inch pine flooring (add $\frac{1}{7}$ of area) at \$32 per M.

25. A driveway in a barn is 12 ft. wide and 24 ft. long, and is made of lumber 2 in. thick. Find the cost at \$28 per M.

26. A board walk 4 ft. wide and 65 ft. long is made of lumber 2 in. thick. The boards are nailed to three pieces running lengthwise of the walk. These are each 3 in. thick and 6 in. wide. Find the entire cost of the lumber at \$32 per M.

27. A stack of 18-foot lumber is 6 ft. wide. It has 80 layers of boards 1 in. thick. Find the value of the pile at \$24 per M.

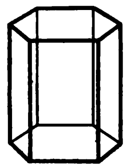
28. A load of lumber 16 ft. long is 3 ft. 2 in. wide. There are 32 layers of boards $\frac{7}{8}$ in. thick. Find the value of the load at \$36 per M.

29. How many board feet of lumber $\frac{3}{4}$ in. thick can be loaded into a car 36 ft. long and $8\frac{1}{2}$ ft. wide, if loaded to a depth of 6 feet?

MEASURE OF VOLUME

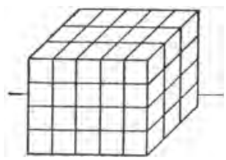
142. The volume of a solid. — The **volume** of a solid is the measure of the solid when the unit of measure is a cube whose edge is a linear unit. Thus, if a solid can be divided into 20 cubes each with an edge of 1 ft., the volume of the solid is 20 cubic feet.

143. Volume of a rectangular prism. — A **prism** is a solid bounded by flat or plane surfaces, two of which are equal polygons in parallel planes, called the **bases**, and the others are parallelograms of which each has sides of the bases as two of its opposite sides. The parallelograms are called the **lateral faces**.



If all the lateral faces are rectangles, the prism is called a **right prism**. Only right prisms will be considered in this book. If the bases, as well as the lateral faces, of a prism are rectangles, it is called a **rectangular prism**.

A **rectangular prism** may be divided into unit cubes. Thus a prism 3 in. by 4 in. by 5 in., as shown in the figure, may be divided into 4 layers of cubes with 3 rows in each layer, and with 5 cubes in each row. Hence there are $4 \times 3 \times 5$, or 60 cubes in all. And in general,

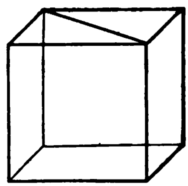


The number of cubic units in any rectangular prism is equal to the product of the numbers of linear units in its three dimensions.

Since each layer of cubes contains as many unit cubes as there are unit squares in one face of the layer, it is seen that:

The number of cubic units in any rectangular prism is equal to the product of the number of square units in the base and the number of linear units in the altitude.

144. Volume of triangular prisms. — If the bases of a prism are triangles, the prism is called a **triangular prism.** It is evident that a triangular right prism is equal to one half of a rectangular prism having the same altitude and having for base a rectangle of which the base of the triangular prism is one half. And in general:



The number of cubic units in any triangular prism is equal to the product of the number of square units in the base and the number of linear units in the altitude.

145. Volume of any right prism. — It is evident that any right prism may be divided into triangular prisms, and hence that the principle in § 144 applies to any right prism.

PROBLEMS

1. The wagon box most generally used in hauling dirt is 9 ft. by 3 ft. by 16 in. How much dirt will make a load when the box is filled level with the top?

2. A square vat is 3 ft. deep, and it holds 75 cu. ft. Find its length and width.

3. A freight car is 36 ft. long and 8 ft. 6 in. wide, inside measure. How many bushels of wheat will it contain if loaded to a depth of 5 ft.? What is the weight of the car of wheat? (A cubic foot is approximately .8 bu. 1 bu. of wheat weighs 60 lb.)

4. The excavation for a house is to be 40 ft. long, 28 ft. wide, and 6 ft. deep. How many cubic yards (loads) of

earth must be removed? What is the cost at 35 cents per cubic yard?

5. A stonemason furnishes the material and lays a front wall 42 ft. long, 8 ft. high, and 16 in. thick at $5.75 per cubic yard. Find the cost.

6. How many solid cords of 16-inch wood can be ricked in a wood house 20 ft. long, 18 ft. wide, and 12 ft. high?

7. A tank 4 ft. 2 in. long, 3 ft. wide, and 5 ft. 6 in. high holds how many gallons? (A gallon = 231 cu. in.)

8. How many tons of ice can be packed in an ice-house 40 ft. by 24 ft., and 16 ft. high, allowing 1 ft. on each side and above and below for sawdust? (1 cu. ft. of ice weighs 56.25 lb.)

9. How many square feet of ice must be cut to fill a car 34 ft. long, $8\frac{1}{2}$ ft. wide, 5 layers deep? What will the car of ice weigh if the ice is 14 in. thick?

10. An ice box will hold a piece of ice 32 in. long, 24 in. wide and 14 in. thick. What will such a piece weigh?

11. A prism has an altitude of 6 in. and its base is a triangle whose sides are each 4 in. long. What is the volume?

12. A regular hexagonal prism (the base has 6 equal sides and equal angles) is 6 in. on its basal edges and is 7 in. high. How many cu. in. does it contain?

13. How many cubic feet of water can a V-shaped gutter discharge (flowing full) in a day, if its depth is 20 in. and its sides meet at right angles, and the water flows 3 ft. per second?

14. In constructing a railroad a cut is made 373 ft. long, 12 ft. deep, 35 ft. wide at the bottom, and 66 ft. wide at the top. How many cubic yards of excavation are made?

15. Find how many tons of coal a bin 20 ft. by 16 ft. by 8 ft. will hold, allowing 63 lb. per cu. ft.

16. How much stone and concrete is required to construct a dam 300 ft. long, 15 ft. high, 10 ft. wide at the bottom and 4 ft. wide at the top?

17. A bar of iron whose cross section is 4 in. square is forged into a bar 24 in. long and with a cross section $2\frac{1}{2}$ in. by $1\frac{1}{2}$ in. What length of bar must be used?

18. In a level town lot 120 ft. long and 70 ft. wide, a cellar is to be dug for a building. The excavation is to be 42 ft. long, 36 ft. wide, and 8 ft. deep. The earth removed is to be used to "fill" the surrounding yard. What will be the depth of the filling?

19. Find the number of cubic yards of filling needed for a railroad embankment 400 ft. long, when its cross section is a trapezoid 10 ft. high, 44 ft. on its lower base, and 14 ft. on its upper base?

20. An ice car 36 ft. long, $8\frac{1}{2}$ ft. wide, and 7 ft. deep will hold how many tons of ice, allowing $56\frac{1}{4}$ lb. per cubic foot?

21. A man wishes to build a coal bin large enough to hold his winter's supply of 15 tons when filled to a depth of but 6 ft. How many square feet in the base? (63 lb. = 1 cu. ft.)

SOLUTION

$$\frac{\overset{5}{\cancel{15}} \times \overset{1000}{\cancel{2000}}}{63 \times \underset{2}{\cancel{6}}} = \frac{5000}{63} = 79^{+}$$

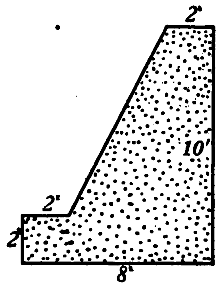
Hence, about 80 sq. ft.

EXPLANATION. — 15×2000 = the number of pounds. $\frac{15 \times 2000}{63}$ = number of cubic feet. Number of cubic feet divided by the number of feet in height = number of square feet in the base.

22. Find the floor area of a coal bin to hold 12 tons when filled to a depth of 5 ft. Give some dimensions of the floor that will make the required area. (63 lb. will fill 1 cu. ft.)

23. A farmer built a watering trough 12 ft. long, 30 in. wide, and 20 in. deep. Allowing $7\frac{1}{2}$ gal. to the cubic foot, how many gallons will it contain?

24. A farmer has a corn crib 12 ft. wide and 16 ft. long. The corn in it is piled 10 ft. high along one side, and slopes off to a depth of 6 ft. along the opposite side. Allowing $\frac{2}{5}$ bu. to a cubic foot of corn "in the ear," find how many bushels the crib contains.



25. The figure shows a cross section of a concrete retaining wall. The wall is 160 ft. long. Find the number of cubic yards of concrete needed to build the wall.

26. We may get some idea of the meaning of the large number of bushels of grain raised each year by finding the size of prism it would form. Find how deep the 700,000,000 bushels of wheat raised in the United States in a recent year would be if stored in a bin one mile square. (1 cu. ft. = $\frac{4}{5}$ bu.)

SOLUTION

$$\frac{5 \times 700{,}000{,}000}{4 \times 5280 \times 5280} = ?$$

EXPLANATION. — $700{,}000{,}000 \div \frac{4}{5}$ = number of cubic feet. This divided by the number of square feet in a mile, which is 5280×5280, will give the depth. Why?

27. Corn in the ear averages about $\frac{2}{5}$ bu. to the cubic foot. Find the depth of the corn raised in one year, 3,000,000,000 bu., if stored in a bin 1 mi. square.

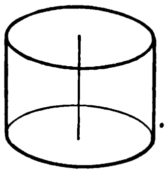
28. In a recent year the United States raised 1,000,000,000 bu. of oats. Find the depth if stored in a bin $\frac{1}{2}$ mi. square.

29. In some recent report, find the production of the various grains and form problems like those given above.

30. The water in an irrigation ditch flows at the rate of $1\frac{1}{2}$ ft. per second. The stream runs $2\frac{1}{2}$ ft. deep, is 6 ft. wide at the surface and 4 ft. wide at the bottom. Find how many cubic feet of water can be delivered in 8 hours.

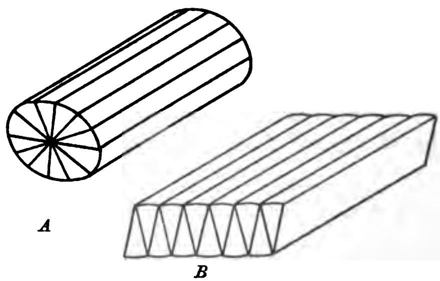
THE SURFACE AND VOLUME OF A CYLINDER

146. The volume of a cylinder. — The figure shows a solid called a **right circular cylinder**. Its surface consists of portions of two planes inclosed by two circles, called the **bases**, and a curved surface, called the **lateral surface**. The straight line joining the centers of the bases is called the **altitude**, or the perpendicular distance between the bases.



While there are other kinds of cylinders, only right circular cylinders are considered in this book.

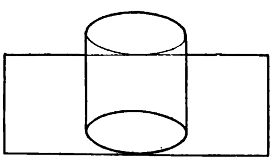
It is evident that a right circular cylinder cannot be divided into a number of cubes as in case of a rectangular prism. However, as shown in the figure, a right circular cylinder A may be cut and the parts rearranged into a solid B closely resembling a rectangular prism. From this we may infer that,



The number of cubic units in any cylinder is equal to the product of the number of square units in the base and the number of linear units in the altitude.

This is shown in geometry to be the true volume.

147. The surface of a cylinder. — By taking a piece of paper and rolling it about a right circular cylinder, as in the figure, it is seen that the **lateral area** is the same as that of a rectangle whose dimensions are the circumference of the base and the altitude of the cylinder. That is,



The number of square units in the area of the lateral surface of a right circular cylinder is the product of the number of linear units in the circumference of the base and the altitude.

The **total area** is the lateral area plus the area of the two bases.

PROBLEMS

1. A cylindrical pail 10 in. in diameter and 12 in. deep will contain how many quarts? (Allow $7\frac{1}{2}$ gal. to the cu. ft.)

2. How many gallons will a hot-water tank 5 ft. long and 16 in. in diameter hold?

3. Find how many gallons a cistern 8 ft. in diameter and 12 ft. deep will hold.

4. Allowing $31\frac{1}{2}$ gal. to a barrel, how many barrels are there in a tank 12 ft. in diameter if filled to a depth of 24 ft.?

5. An oil tank 6 ft. in diameter and $15\frac{1}{4}$ ft. long will hold how many gallons?

6. Find the cost of cementing the wall and bottom of a cistern 7 ft. in diameter and 12 ft. deep at 45¢ per square foot.

7. A boiler has 200 tubes each 8 ft. long and 3 in. in outside diameter, conveying the heat through the water. Find the area of the heating surface of the tubes.

8. How much will it cost to paint a smoke stack 50 ft. high and 4 ft. in diameter at $1\frac{1}{2}$¢ per square foot?

9. A zinc disk 8 in. in diameter, with a hole 1 in. in diameter in it, weighs 8 oz. Find its thickness. Use .26 lb. for the weight of 1 cu. in.

10. A wooden cylinder is cut out on a lathe from a block 6 in. square. What fractional part of the wood is cut away, if the cylinder is made as large as possible?

11. A cylinder 4 ft. 6 in. long and 1 ft. 4 in. in diameter is cut out on a lathe. The surface moves at the rate of 15 ft. a minute. The edge of the cutting tool is $\frac{7}{16}$ in. wide. How long will it take to cut out the whole cylindrical surface?

12. A hollow cast-iron cylinder is to weigh just 100 pounds. It is to be 18 in. long with an inside diameter of 4 in. How thick will it be if the weight of cast-iron is taken as .26 lb. per cubic inch?

13. A brass tube whose inner diameter is 2 in. and outer diameter 3 in. is run from 50 lb. of brass. How long is it? (A cubic inch of brass weighs .303 lb.)

14. The piston of a pump is 8 in. in diameter, and makes a stroke of 20 in. How many gallons of water will it deliver in an hour, if it makes 30 strokes a minute in each direction, delivering water only on the forward stroke?

APPLICATION OF CYLINDERS TO SILOS

A **silo** is cylindrical in form and used to "can" shredded corn fodder.

While the silage is much more compact near the bottom, and while the weight of a cubic foot varies with the size of the silo, a ton requires from 40 to 50 cu. ft. of space.

In the following problems, consider that 50 cu. ft. of silage weighs 1 T.

1. How much surface is there to be painted, not counting the roof, of a silo 16 ft. in diameter and 38 ft. high?

2. At $1.75 per 100 sq. ft., find the cost of painting this silo, not including the roof.

3. How many tons of silage can be stored in a silo 18 ft. in diameter and 42 ft. high?

4. Allowing 40 lb. per day for each head of cattle, how many tons of silage must be stored to provide for a herd of 24 cows for 200 da.?

5. How high must a man build a silo 14 ft. in diameter to furnish the silage needed for the herd in Problem 4?

6. A man who has a herd of 60 cows fills two silos, each 16 ft. in diameter and 30 ft. high. Allowing 40 lb. of silage per day for each cow, how many days will it last?

7. Considering that a crop of corn will yield 12 T. to the acre, how many acres should be planted to fill a silo 14 ft. in diameter and 38 ft. high?

8. A farmer finds that one silo 14 ft. in diameter and 34 ft. high, or two small silos each 10 ft. in diameter and 30 ft. high, will be necessary for his herd of 25 cows. How much more lumber 2 in. thick will be needed for the walls of the two small ones than for the wall of the large one?

9. If each of the silos in Problem 8 has a cement floor costing $12\frac{1}{2}$ ¢ per square foot, how much more will the floors of the two small ones cost than the floor of the large one?

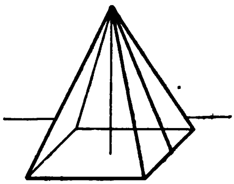
10. Silage exposed to the air for several days spoils; so the diameter of the silo to be used depends upon the number of cattle in the herd. If it requires 10 cattle to keep the silage fresh in a 10-ft. silo, by feeding from the top surface of the silage, how many are required to keep fresh the silage in a 12-ft. silo?

SUGGESTION.—Area of a 10-ft. circle $= 100 \times \frac{\pi}{4}$ sq. ft. Area of a 12-ft. circle $= 144 \times \frac{\pi}{4}$ sq. ft. Hence the larger circle is 1.44 times as large as the smaller. Hence it would require 14 or 15 cattle.

11. If it requires 10 cows to keep fresh the silage in a 10-ft. silo, how many head will be required to keep fresh the silage in a 20-ft. silo? In a 25-ft. silo?

CONES AND PYRAMIDS

148. Pyramids. — A **pyramid** is a solid bounded by a polygon, called the base, and by three or more **triangles, called** the **lateral faces,** which meet at a common point called the **vertex.** If the base is a regular polygon (one having all sides and all angles equal) and all edges meeting at the vertex are equal, the pyramid is a **regular pyramid.**



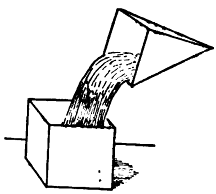
RECTANGULAR PYRAMID

The altitude of one of the equal triangles composing the lateral faces of a regular pyramid, drawn from the vertex of the pyramid, is the **slant height** of the pyramid. The perpendicular distance from the vertex to the base is the **altitude** of a pyramid.

149. The lateral area of a pyramid. — The **lateral area** of a pyramid is the area of its lateral faces. If the pyramid is regular, it is evident that,

The number of square units in the lateral area of a regular pyramid is equal to one half of the product of the number of linear units in the perimeter of the base and the number of linear units in the slant height.

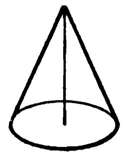
150. The volume of a pyramid. — If a pyramid and a prism having equal altitudes and equal bases be taken, and the pyramid be used as a measure, as in the figure, it is found that the pyramid fills the prism only one third full. Hence,



The number of cubic units in the volume of a pyramid is one third of the product of the number of square units in the base and the number of linear units in the altitude.

151. Cones. — The figure shows a solid called a **right circular cone.** Its surface consists of the portion of a plane surface in-

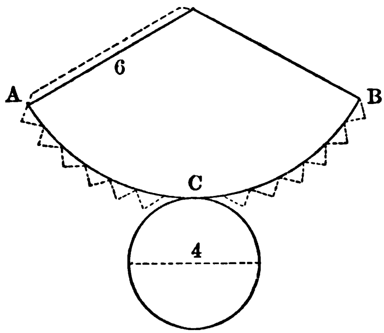
closed by a circle, called the **base**, and of a curved surface, called the **lateral surface**, which tapers uniformly to a point, called the **vertex.** The straight line joining the vertex to the center of the base is the **altitude**, or the perpendicular distance from the vertex to the base. The distance from the vertex to any point in the circumference of the base is called the **slant height.**



RIGHT CIRCULAR CONE

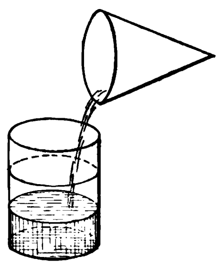
While there are other kinds of cones, only right circular cones will be considered here.

152. The lateral area of a cone. — If a piece of paper is placed about a right circular cone and a pattern of its surface made, the pattern will be like the one in the margin. Thus it is seen that the lateral surface is the sector of a circle whose radius is the slant height and whose arc is the perimeter of the base of the cone. Hence, from the method used in finding the area of a circle, § 131, it is seen that,



The number of square units in the lateral area of a right cone is one half of the product of the number of linear units in the slant height and the number of linear units in the circumference of the base.

153. The volume of a cone. — By the method used in § 150, as suggested in the figure, it may be seen that



The number of cubic units in the volume of a right circular cone is one third of the product of the number of square units in the area of the base and the number of linear units in the altitude.

PROBLEMS

1. If the base of a pyramid contains 120 sq. ft. and the height is 9 ft., how many cubic feet are there in the volume?

2. If the base of a pyramid is 5 ft. square and the height is 9 ft., how many cubic feet are there in the volume?

3. If the slant height of a square pyramid is 10 ft. and each side of the base 6 ft., how many square feet are there in the lateral area?

4. If the slant height of a cone is 12 inches, and the base is 10 inches in diameter, find the lateral area.

5. Find the volume of a cone 9 ft. high whose base is 8 ft. in diameter.

6. How many bushels are there in a conical pile of grain 6 ft. high and 8 ft. across at the bottom, allowing .8 bu. to the cubic foot?

7. In one corner of a bin is a pile of wheat forming a fourth of a cone. The height is 5 ft. and the radius of the base is 7.5 ft. How many bushels are there?

8. A farmer has a pile of apples 4 ft. high and 12 ft. in diameter at the bottom. Allowing 4 bu. to every 5 cu. ft., find how many bushels he has.

9. Find the volume of a pyramid whose slant height is 10 in. and whose base is 12 in. square.

10. A square pyramid has a base 4 ft. square and its altitude is 6 ft. Find the lateral area.

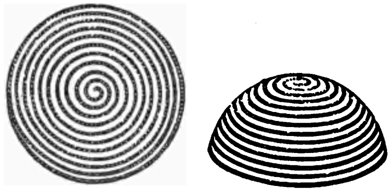
11. The altitude of a pyramid is 8 inches. The base is a triangle each side of which is 6 inches. Find the volume.

12. The slant height of a cone is 15 in. and the diameter of the base is 16 in. Find the volume.

13. A pile of grain in the shape of a cone is 12 ft. across at the bottom and the slant height is 7 ft. How many bushels are there in the pile?

MEASUREMENT OF THE SPHERE

154. The surface of a sphere. — A **sphere** is a solid bounded by a curved surface all points of which are equally distant from a point within called the **center.** A straight line forming the distance from the center to the surface is called the **radius.** Two radii in a straight line form a **diameter.**

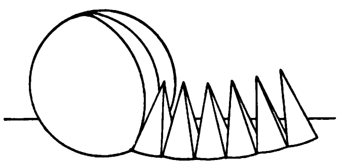


When a sphere is divided by a plane through the center into two hemispheres, the plane surfaces thus exposed are inclosed by **great circles** of the sphere. They have the same radius as the sphere. The relation between the surfaces of a hemisphere and a great circle may be inferred from the following experiment: With a hard cord wind the surface of a hemisphere and of a great circle. If carefully done, it will be found that it requires just twice as much cord to wind the curved surface of the hemisphere as to wind the flat surface of the circle. Hence it will require four times as much to wind the surface of the whole sphere as to wind the surface inclosed by the circle. Hence,

The area of the surface of a sphere equals four times the area of a great circle.

Hence, $S = 4 \times \pi \times R^2$, where R is the radius and S the surface of a sphere.

155. The volume of a sphere. — By drawing three or more planes through the center of a sphere, a portion is cut out which resembles a pyramid with altitude equal to the radius.



The entire sphere may be thus divided into a number of such solids, the sum of the bases of which is the surface of the sphere and the common altitude of which is the radius. Considering the parts approximate pyramids, we see that:

The volume of a sphere equals the volume of a pyramid whose base is the surface of the sphere and whose altitude is the radius.

Hence, volume $= \frac{1}{3} \times 4\ S \times R = \frac{1}{3} \times 4\ \pi R^2 \times R = \frac{4}{3} \times \pi R^3$.

PROBLEMS

1. Find the volume of a sphere whose diameter is 16 ft.

2. Taking the radius as 4000 mi., find the area of the entire surface of the earth.

3. The surface of a tiled dome, in the form of a hemispherical surface, whose diameter is 24 ft., is made of tiles each 1 in. square. How many tiles are required to make it?

4. A gilded dome is in the form of a hemisphere whose diameter is 30 ft. How many square feet are there in its surface?

5. The dome of an astronomical observatory, which is in the form of a hemisphere, is 48 ft. in diameter. How many square feet of tin does it take to cover it?

6. A hemispherical skylight is 16 ft. in diameter. Not allowing for sash, how many square feet of glass are required to make it?

7. Steel weighs 490 lb. to the cubic foot. Find the weight of a steel ball 10 in. in diameter.

8. A bowl in the form of a hemisphere is 6 in. in diameter, inside measure. How many cubic inches will it contain?

9. Find the weight of a cast-iron spherical shell 1 inch thick, with an outside diameter of 9 in. (1 cubic foot of cast iron weighs 450 lb.)

10. A boiler is made in the form of a cylinder 2 ft. in diameter and 4 ft. long, with hemispherical ends. How many gallons will it hold? (1 gal. = 231 cu. in.)

11. A haystack is approximately in the form of a cylinder 16 ft. in diameter and 8 ft. high, surmounted by a hemisphere. Allowing 512 cu. ft. to the ton, find the weight of the stack.

12. A certain brass spherical shell 1 inch thick and 10 inches in external diameter is found to weigh 77 lb. Find to three decimal places the weight of 1 cu. in. of this brass.

MISCELLANEOUS PROBLEMS IN MEASUREMENT

1. How many cubic yards of stone and cement are required to build a dam 300 ft. long, 15 ft. high, 10 ft. wide at the bottom, and 4 ft. wide at the top?

2. A freight car 36 ft. long and 8 ft. 6 in. wide, inside measurements, is filled with wheat to a depth of 5 ft. Allowing .8 bu. to a cubic foot, and 60 lb. to a bushel, find the weight of the load. If the capacity of the car is 80,000 lb., how much does it lack of being loaded to its full capacity?

3. The largest possible cone is turned in a lathe out of a cylinder 6 in. long and 3 in. in diameter. What part of the cylinder goes into shavings? How many cubic inches are there in the cone?

4. Allowing $7\frac{1}{2}$ gal. to a cubic foot, find how many gallons of water a cistern will hold that is 9 ft. in diameter and 10 ft. deep. Since $31\frac{1}{2}$ gal. = 1 bbl., how many barrels will it hold?

5. The cylindrical tank of a large sprinkling car used by a city street car company in sprinkling the tracks is 6 ft. in diameter and 26 ft. long. How many gallons will it hold?

6. Find the weight of a brass tube 20 in. long whose inner diameter is 1 in. and whose outer diameter is $1\frac{1}{2}$ in., if the weight of brass is taken as .3 lb. per cu. in.

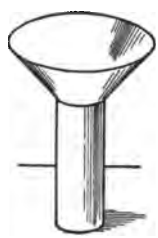
7. The piston of a steam pump is 8 in. in diameter, and makes a stroke of 18 in. How many gallons of water will it deliver in an hour, if it makes 30 strokes a minute in each direction, and delivers water with each stroke forward and each stroke back. (231 cu. in. = 1 gal.)

8. An engine's boiler 4 ft. in diameter and 16 ft. long is traversed by 60 pipes, each 3 in. in diameter, which convey the heat through the water. How many gallons of water will the boiler hold?

9. A fruit raiser has a round pile of apples that is 12 ft. across at the bottom, and tapers to a point that is 3 ft. high at the middle. How many bushels are there in the pile? (Count 4 bu. to 5 cu. ft.)

10. A wash boiler 12 in. deep, 10 in. wide, and 20 in. long has round ends; *i.e.* each end is a half cylinder. How many gallons does it hold?

11. The figure represents a view of a rain gauge, an instrument used for measuring the amount of rainfall. The opening at the top is 12 in. in diameter, and the cylindrical stem is 4 in. in diameter. Suppose that in a rain storm the stem is filled to a depth of 4 in. What is the precipitation? (That is, what is the depth of the rainfall on level ground?)



12. A farmer has a pile of ear corn approximately in the form of a cone whose height is 8 ft. and the width at the bottom is 16 ft. How many bushels of shelled corn will it make? (Count 2 bu. of shelled corn to 5 cu. ft. of ear corn.)

13. A farmer wishes to know how many tons of hay there are in a stack in the form of a cylinder 16 ft. in diameter and 7 ft. high, surmounted by a cone 8 ft. high. Allowing 512 cu. ft. to the ton, find the amount of hay in the stack.

14. In a steam boiler 94 flues, or cylindrical pipes, each 2 in. in outside diameter and 12 ft. long, convey the heat from the fire box through the water. How much heating surface do they apply to the water?

15. A horizontal oil tank is 24 ft. long and 8 ft. in diameter. Allowing $7\frac{1}{2}$ gal. to a cubic foot, find how many gallons it will hold.

16. Which will hold more and how much, two tanks 16 ft. long and 4 ft. in diameter, or one tank 16 ft. long and 8 ft. in diameter?

17. A corn crib is 16 ft. long and 10 ft. wide. It is filled the entire length with ear corn to a depth of 10 ft. along the back side and to a depth of 7 ft. along the front side. Allowing $2\frac{1}{2}$ cu. ft. to a bushel, find how many bushels the crib contains.

18. A haymow in a barn is 30 ft. wide, 40 ft. long, and 12 ft. high to the eaves of the roof. The gable is 30 ft. wide, and the highest point of the roof is 15 ft. above the level of the eaves. If the mow is filled to the roof with hay, how many tons does it contain?

19. A man feeds his hogs in a V-shaped trough that is 12 ft. long, 11 in. wide at the top, and $5\frac{1}{2}$ in. deep. Counting $7\frac{1}{2}$ gal. to a cubic foot, find how many gallons it will hold.

20. A railroad fill for double tracks is 300 ft. long, 28 ft. wide at the top, 40 ft. wide at the bottom, and 4 ft. high. How many cubic yards of earth did it take to make the fill?

21. A cement wall is 60 ft. long, 2 ft. wide at the top, 4 ft. wide at the bottom, and 6 ft. high. How many cubic feet does it contain?

22. If the driving pulley on a shaft is 6 ft. in diameter and makes 90 revolutions per minute, how many revolutions per minute will the driven pulley which is 42 in. in diameter make?

23. If a point on the belt connecting two equal pulleys, each 20 in. in diameter, is traveling at the rate of 1000 ft. per minute, how many revolutions per minute are the pulleys making?

24. When an automobile with 34-inch tires is traveling 30 miles per hour, how many revolutions per minute are the wheels making?

25. How high must a milk can 14 in. in diameter be to hold 10 gal., not including nor filling the tapered top?

26. If a milk can 24 in. high holds 10 gal., what is its diameter?

27. A large watering tank 8 ft. long, 30 in. wide, and 24 in. deep has parallel sides and semicircular ends. How many gallons will it hold?

28. A cylinder 4 in. in diameter and 10 in. long is turned on a lathe from a block $4\frac{1}{2}$ in. square and 10 in. long. What fractional part of the block is cut away?

29. If a bowling ball 8 in. in diameter weighs 12 lb., what was the weight of the cube from which it was turned, allowing $\frac{1}{2}$ in. extra on each dimension of the cube for waste in turning?

30. If a ball of lead 4 in. in diameter is melted and run into a cylinder $2\frac{1}{2}$ in. in diameter, how long will the cylinder be?

31. If a bar of lead 1 in. by $\frac{3}{4}$ in. by 8 in. is melted and run into a triangular prism the sides of whose base are each 1 in., how long will the prism be?

32. A farmer has a pile of grain heaped into one corner of his granary forming $\frac{1}{4}$ of a cone 6 ft. high with a radius of 12 ft. If he puts it into a bin 4 ft. by 6 ft., to what depth will it fill the bin?

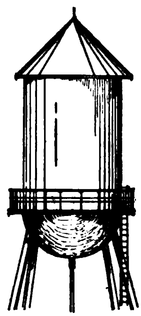
33. If a bag of grain holding 2 bu. is emptied on a floor forming a cone 12 in. high, what will be the diameter of the base?

34. A dealer delivers 12 tons of coal, putting it into a bin 9 ft. by 12 ft. Allowing 63 lb. to the cubic foot, to what depth will it fill the bin?

35. What must be the depth of a cylindrical vessel, 18 in. in diameter, to hold a bushel?

36. How many ash cans 18 in. in diameter and 30 in. high can be emptied into a wagon with a capacity of 3 cu. yd.?

37. Water runs through a sewer pipe at the rate of 60 ft. per minute. If the pipe is 18 in. in diameter, how many gallons per day are discharged?



38. The drawing represents a water tank, with a conical roof. The tank is in the form of a cylinder with a hemisphere at the bottom. The height of the cylindrical part is 16 ft., and its diameter is 10 ft. Find the capacity of the tank in gallons.

39. In the tank of Problem 38, the diameter of the roof is 12 ft. and its height 6 ft. How many squares (a square = 100 sq. ft.) of metal are required to cover the roof?

CHAPTER VII

LITERAL AND GRAPHIC REPRESENTATION OF NUMBERS

156. The formula. — Many of the principles which the student has learned in the preceding chapters, as well as practical rules which are used in the trades and sciences, are expressed by using letters to represent the numbers involved. Such an expression is called a **formula.** Thus the area of a square may be expressed by $A = s^2$, where A represents the number of square units in the area and s represents the number of linear units in one side. In the following exercises, each letter represents the number of units in the quantity represented.

157. Signs of operation. — In *literal notation*, or notation involving the use of letters, all of the signs of operation used in the preceding work are used. In addition to these, the absence of a sign between two letters denotes multiplication. Thus ab means $a \times b$.

EXERCISES

1. If l represents the length, w the width, and A the area of a rectangle, write the formula, expressing the relation between l, w, and A.

2. In the formula $A = lw$, find A when $l = 16$ and $w = 8$.

3. If h represents the height, b the base, and A the area of a triangle, write the formula.

4. In the formula $A = \frac{1}{2}hb$, find A when $b = 16$ and $h = 12$.

5. If c represents the circumference of a circle, and d the diameter, express the formula for the circumference in terms of the diameter.

6. If r represents the radius, and c the circumference, represent the circumference in terms of the radius.

7. In the formula $c = \pi d$, find c when $d = 16$.

8. In the formula $c = 2\pi r$, find c when $r = 28$.

9. If A represents the area of a circle, and r the radius, write the formula for A in terms of r.

10. In the formula $A = \pi r^2$, find A when $r = 16$.

11. Write a formula for the area of a circle in terms of its diameter.

12. In the formula $A = \dfrac{\pi d^2}{4}$, find A when $d = 16$.

13. If V represents the volume of a rectangular prism and l, w, and h represent its three dimensions, write the formula for the volume.

14. In the formula $V = hlw$, find V when $h = 5$, $l = 7$, and $w = 10$.

15. If b represents the area of the base of a prism and h its height, write the formula for its volume.

16. In the formula $V = bh$, find V when $b = 16$ and $h = 6$.

17. If $V =$ volume, $h =$ altitude, and $b =$ base of a pyramid, write the formula for the volume.

18. In the formula $V = \frac{1}{3} bh$, find V when $b = 25$ and $h = 9$.

19. Interpret $V = \pi r^2 h$ when the letters refer to lines in a cylinder.

20. Interpret $V = \frac{1}{3} \pi r^2 h$. Find V when $r = 6$ and $h = 12$.

21. Interpret $S = 4\pi r^2$ when referring to a sphere.

22. Interpret $V = \frac{4}{3} \pi r^3$ when referring to a sphere.

23. If $V = \frac{4}{3}\pi r^3$, find V when $r = 16$.

24. If $S = 4\pi r^2$, find S when $r = 12$.

25. The area of a trapezoid is expressed by the formula $A = \frac{1}{2}h(a + b)$. Explain what it means.

26. If $A = \frac{1}{2}h(a+b)$, find A when $h=6$, $a=6$, and $b=10$.

27. Find the value of $d(x + y)$ when $d = 5$, $x = 6$, and $y = 8$.

28. Find the value of $m(n - d)$ when $m = 10$, $n = 14$, and $d = 8$.

NOTE. — The following formulæ are met in the various trades. They may be omitted by those who have no interest in them.

29. The power to overcome the resistance of the wind in driving an automobile is given by the formula:

$$\text{H. P.} = .96\,PAM.$$

In this formula,

H. P. = horsepower required to overcome wind resistance,
P = pressure of wind in pounds per square foot,
A = front area of body of car in square feet,
M = speed of car in miles per hour.

Find the resistance when $P = .13$, $A = 10$, $M = 30$.

30. Perhaps the most used formula for finding the horsepower of an automobile engine is the following:

$$\text{H. P.} = \frac{D \times D \times S \times R \times N}{15{,}000}.$$

In this formula,

D = bore in inches (inside diameter of cylinder),
S = length of stroke of piston in inches,
R = number of revolutions per min. of driving shaft,
N = number of cylinders.

Find H. P. when bore = 3 in., stroke = 5 in., $R = 2000$, number of cylinders = 8.

31. The indicated horsepower of an engine is represented by the formula H.P. $=\frac{p \times l \times a \times n}{33,000}$, where $p =$ the mean effective pressure on the end of piston in pounds per square inch, $l =$ the length of stroke in feet, $a =$ area of piston in square inches, $n =$ twice the number of revolutions per minute. Compute the H. P. of an engine in which a test showed p to be 35 lb. per square inch, and the number of revolutions 60, while the area of the piston was 706.8 sq. in., and the length of stroke 48 in.

32. The horsepower that can be transmitted by a shaft with a working stress of 5000 lb. per square inch is H. P. $=\frac{n \times d^3}{64}$, where $n =$ number of revolutions per minute, and $d =$ diameter of shafting in inches. How many horsepower can be transmitted by such a shafting 4 in. in diameter, making 76 revolutions per minute? One 5 in. in diameter, making 100 revolutions per minute?

33. The stress P, in pounds per square inch, due to centrifugal force in the rim of a fly-wheel, is found by the formula $P = \frac{W \times V^2}{32.2\,R}$, where $W =$ weight of 1 ft. length of rim 1 sq. in. in cross section, $V =$ velocity of rim in feet per second, $R =$ radius of rim in feet. In a cast iron wheel, $W = 3.1$ lb. Find the stress in the rim of a cast iron fly-wheel 8 ft. in diameter, running at 160 revolutions per minute.

34. The formula for tie-rods for beams supporting brick arches is: Horizontal thrust in pounds per linear foot of arch $=\frac{1.5 \times W \times S^2}{R}$, where $W =$ weight on arch in pounds per square foot, $S =$ span of arch in feet, $R =$ rise of arch in

inches. Find the stress on the tie-rod in an arch on which the weight is 360 lb. per square foot, the span 4 ft., and the rise of the arch 18 in.

35. If D inches is the deflection in the middle of a beam supported at both ends and loaded in the middle, then $D = \dfrac{W \times L}{48 \times E \times I}$, where W = weight of load in pounds, L = length of beam in inches, and E and I are numbers depending upon the form and material of the beam. In a wrought-iron bar 2 in. deep and 1 in. wide $I = \frac{2}{3}$ and $E = 29{,}000$. If the bar is 6 ft. long, find the deflection due to a load of 3200 lb.

36. If a beam L in. in length is supported at both ends and loaded uniformly throughout its length with W lb. per foot, the maximum deflection D, in inches, is obtained by the formula $D = \dfrac{5 \times W \times L}{384 \times E \times I}$. If a beam in which $E = 30{,}000{,}000$ and $I = \frac{8}{3}$ is 12 ft. long, and the load 2000 lb. per foot, find the maximum deflection.

37. In an electric current, R, the combined resistance of the resistances r_1, r_2, r_3, in parallel, is found by the formula $R = \dfrac{r_1 \times r_2 \times r_3}{r_1 \times r_2 + r_1 \times r_3 + r_2 \times r_3}$. What is the combined resistance of resistances of 5.2 ohms, 6.4 ohms, and 7.6 ohms, put in parallel?

38. The difference in the elevations of two points on the earth's surface is found by use of the barometer from the formula $D = \dfrac{55000\,(L - U)}{L + U}$, where D = difference in elevation in feet, L = reading of barometer at lower point, and U = reading at upper point.

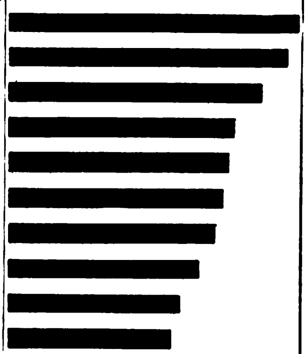
At the foot of a mountain the barometer reads 26.54, and at the top 20.94. Find its height.

39. The discharge of a pump in gallons per minute is obtained from the formula $G = .03264\ Td^2$, where G = number of gallons, T = travel (total distance traveled) of piston in feet per minute, d = diameter of cylinder in inches. Suppose that the diameter of the cylinder of a pump is 18 inches, that the stroke of the piston is 24 inches, and that it makes 40 revolutions per minute. Find the discharge.

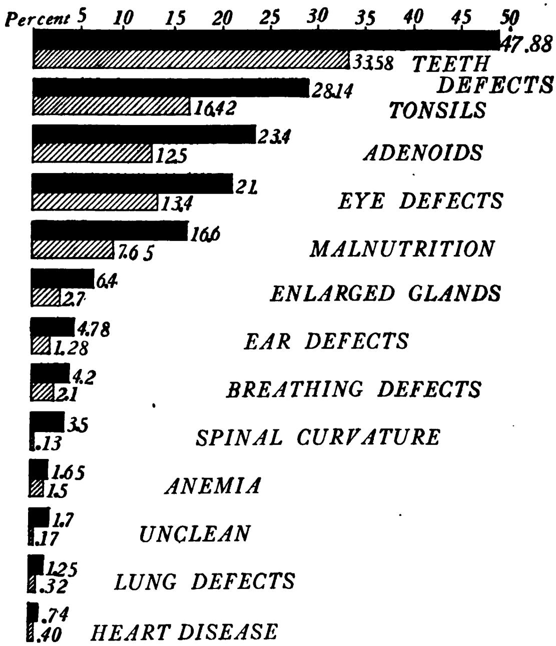
158. Quantities represented by lines. — For purposes of showing the relations between quantities that may be seen without computation, such quantities are represented by lines accurately drawn to a scale. When the quantities are independent of each other, a series of straight lines is used. Thus, the following from *The World's Work*, December, 1915, shows a diagram of this kind.

FOREIGN TRADE TONNAGE OF THE WORLD'S TEN GREATEST SEAPORTS

	PORT	YEAR	TONNAGE TOTAL ENTERED PLUS CLEARED
1	New York	6/30/11 to 6/30/12	27,223,000
2	Antwerp	1911	26,656,000
3	Hamburg	1911	23,776,000
4	Rotterdam	1911	21,853,000
5	Liverpool	1911	21,834,000
6	London	1911	20,978,000
7	Hongkong	1911	20,491,000
8	Shanghai	1911	18,600,000
9	Marseilles	1910	16,348,000
10	Singapore	1911	15,456,000

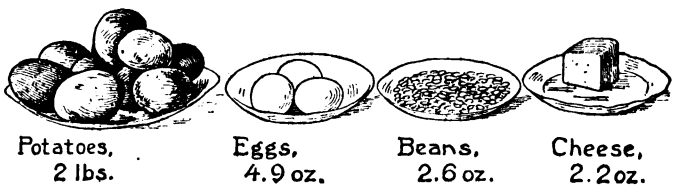


A recent study gives the comparative percentage, shown graphically, of country (heavy line) and city (shaded line) children suffering from various diseases.

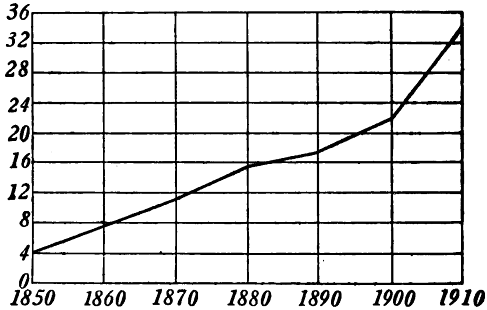


159. Quantity represented by pictures. — Sometimes newspapers, popular magazines, and trade journals use pictures to bring out relations. The following is an example.

PROTEIN THE SAME IN EACH

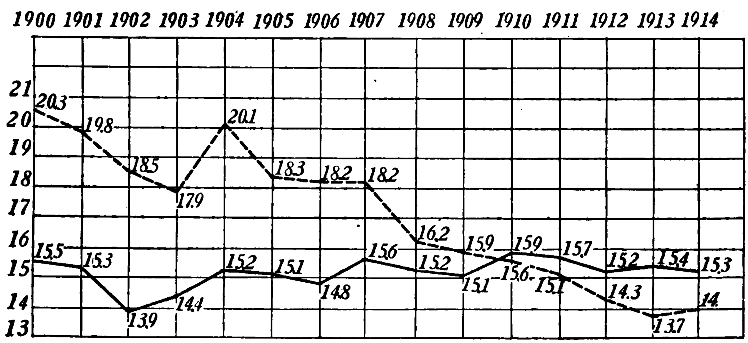


160. Graphs or lines showing relations. — Where two sets of quantities are so related that a variation of one causes a variation of the other, each is said to be a **function** of the other. In all such relationships as time and rainfall, time and productions, earnings and expenses, etc., the relationship may be pictured by a continuous line called a **graph.**



Thus we find the growth of the foreign trade from New York pictured in a recent magazine article as in the margin. That is, in 1850, the amount was $4,000,000,000; in 1860, about $7,500,000,000; in 1910, $33,000,000,000. If the growth continues to increase from 1910 to 1920 as it did from 1900 to 1910, show by continuing the graph in the same direction what the amount will be.

As another example, the graph below was given in the



New York Times, Sunday, April 2, 1916, showing the result of an investigation by Dr. Thomas D. Wood, professor of physical education, Columbia University, of the "Death rate of New York City (broken line) compared with the death rate in rural New York (solid line)."

EXERCISES

1. Keep a record of the temperature at a given hour for a fixed number of days and make a graph showing the variations, as in § 160.

2. The areas of the continents, in millions of square miles, are: North America 8, South America 6.85, Europe 3.8, Asia 17, Africa 11.5, and Australia 3. By lines as in § 158 show these relations.

3. The sugar production of the world in tons is given in the table in the margin. On the same squared paper show graphs of the beet and the cane productions. Use the nearest hundredth of a million. Thus call 5,590,992 5.59 million, etc.

Year	Cane	Beet
1900	3,056,294	5,590,992
1902	4,079,742	6,913,504
1904	4,234,203	6,089,468
1906	6,731,165	7,216,060
1908	6,917,663	7,002,474
1910	8,327,069	6,597,506
1912	9,066,030	6,820,266
1914	9,865,016	8,908,470

4. The table in the margin shows the growth of the automobile business from June, 1904, to June, 1915. Make a graph showing the growth.

Year	Number	Year	Number
1904	21,700	1910	187,000
1905	25,000	1911	210,000
1906	34,000	1912	378,000
1907	44,000	1913	485,000
1908	85,000	1914	515,000
1909	126,500	1915	703,527

Suggestion. Represent the years in the horizontal and the number of automobiles in the perpendicular, using 25,000 as each unit space.

5. On a single chart, make a graph of the following:

YEARLY MARKETINGS OF LIVE STOCK

(From "The Agricultural Outlook" of the Department of Agriculture)

The combined receipts of hogs, cattle and sheep at Chicago, Kansas City, Omaha, St. Louis, Sioux City, St. Joseph, and St. Paul, yearly, since 1901, were as follows:

YEAR	CATTLE	HOGS	SHEEP	YEAR	CATTLE	HOGS	SHEEP
1901	7,708,839	20,339,864	7,798,359	1908	8,827,360	22,863,701	9,833,640
1902	8,375,408	17,289,427	9,177,050	1909	9,189,312	18,834,641	10,284,905
1903	8,878,789	16,780,250	9,680,692	1910	9,265,412	15,685,435	12,406,767
1904	8,690,699	17,778,827	9,604,812	1911	8,768,456	20,453,530	13,556,107
1905	9,202,083	18,988,933	10,572,259	1912	8,159,888	20,265,667	13,755,579
1906	9,373,825	19,223,792	10,684,437	1913	7,904,552	19,924,331	14,037,830
1907	9,590,710	19,544,617	9,857,877	1914	7,182,239	18,272,091	13,272,491

6. The table gives the average weights of a man 5 ft. 10 in. in height at different ages, based upon life insurance records. Draw a curve representing the growth in weight.

Age. .	15–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64
Wt. .	154	159	164	167	170	171	172	173	174

7. The Journal of the American Medical Association gives the following record of deaths caused by Fourth of July celebrations. Show the relation graphically.

YEAR	DEATHS	YEAR	DEATHS	YEAR	DEATHS
1904	183	1908	163	1912	41
1905	182	1909	215	1913	32
1906	158	1910	131	1914	40
1907	164	1911	57	1915	30

8. The following is a table of the foreign-born people in the United States in 1910. Show the relations by lines.

England	876,455	Switzerland	124,834
Scotland	261,034	Portugal	57,623
Wales	82,479	Spain	21,977
Ireland	1,352,155	Italy	1,343,070
Germany	2,501,181	Russia	1,602,752
Norway	403,858	Finland	129,669
Sweden	665,183	Austria	1,174,924
Denmark	181,621	Hungary	1,495,600
Netherlands	120,053	Roumania	65,920
Belgium	49,397	Bulgaria and Serbia	21,451
France	117,236	Greece	101,264

9. Show by a chart the increase in the production of crude petroleum in the United States since 1906.

YEAR	BBL. OF 42 GAL.	YEAR	BBL. OF 42 GAL.
1906	134,717,580	1911	209,557,248
1907	126,493,936	1912	220,449,391
1908	166,095,335	1913	222,935,044
1909	178,527,355	1914	248,446,230
1910	183,170,874	1915	290,312,535

10. Hastings measured (in kilograms) the strength of the right fore-arm of 5,476 children, of ages from 5 years to 16 years. He found the average strength to be as follows:

Age	5	6	7	8	9	10	11	12	13	14	15	16
Strength	4.9	7.0	9.2	10.6	13.1	14.7	18.0	19.7	22.6	25.4	28.9	33.3

Draw a curve to show the relation of the growth of strength to the increase of age.

11. Make a graph showing the world's production of rubber since 1905.

Year	Tons	Year	Tons	Year	Tons
1905	61,000	1909	69,000	1913	115,000
1906	66,000	1910	70,000	1914	124,000
1907	69,000	1911	76,000	1915	142,000
1908	65,000	1912	99,000		

If the rate of increase continues to grow as it has since 1912, show by continuing the curve what may be expected in 1916; in 1920; in 1930.

12. Bring to class and interpret graphs found in magazines, books, and daily papers.

CHAPTER VIII

PERCENTAGE AND ITS APPLICATIONS

161. The meaning of per cent. — **Per cent** is only another name and notation for *hundredths*.

Thus, $6\% = \frac{6}{100} = .06$,

and $.17 = \frac{17}{100} = 17\%$.

It is seen, then, that per cent, like a fraction expressed in any other way, is a *relation number*. That is, it is an expression of the relation of one thing to another. Hence, the use of the term per cent is meaningless unless there is coupled with it a distinct specification of the thing used as a basis for comparison. Thus, to say that a man made 20% is meaningless unless we add "of the cost", or "of the sales", or of whatever the basis for comparison happens to be.

EXERCISES

Change to decimals:

1. 35%.	**5.** 12%.	**9.** 200%.	**13.** .3%.
2. 8%.	**6.** $6\frac{1}{4}\%$.	**10.** 150%.	**14.** 2.6%.
3. 145%.	**7.** $\frac{2}{5}\%$.	**11.** 1750%.	**15.** 13.46%.
4. 90%.	**8.** $\frac{5}{8}\%$.	**12.** 900%.	**16.** 1.25%.

Change to per cent:

17. .26.	**21.** .725.	**25.** .015.	**29.** 1.5.
18. .38.	**22.** .0725.	**26.** .004.	**30.** 3.04.
19. .07.	**23.** 1.25.	**27.** .0075.	**31.** 1.645.
20. .065.	**24.** 3.5.	**28.** .265.	**32.** 2.01.

162. The problems of percentage. — The three problems that arise in percentage are:

1. *To find a part of a number when the part to be found is expressed as a per cent of the number.*

2. *To compare one number with another and express the relation in terms of per cent.*

3. *To find a number when a part of it, expressed as a per cent of it, is known.*

NOTE. — The first two of these problems are by far the most common in business practice, yet the third has a definite use.

The following are illustrations of these three problems:

1. An agent sold a house and lot for \$8750 and received a fee of $2\frac{1}{2}\%$ of the selling price. Find his fee.

SOLUTION

$$\begin{array}{r} \$8750 \\ .025 \\ \hline 43\,750 \\ 175\,00 \\ \hline \$218.75 \end{array}$$

EXPLANATION. — $2\frac{1}{2}\%$ of \$8750 means .025 of it, or $.025 \times \$8750$. Hence, to solve such a problem, the per cent is expressed as a decimal fraction and the work is performed as in multiplication of decimals.

2. A manufacturer received \$1243.50 for goods that cost him \$875 to manufacture and sell. His profit equaled what per cent of the cost to manufacture and sell them?

SOLUTION

$$\begin{array}{r} \$1243.50 \\ 875. \\ \hline \$368.50 \end{array}$$

$$.42\tfrac{4}{35} = 42\tfrac{4}{35}\%$$

$$\begin{array}{r} 875)368.50 \\ 350\,0 \\ \hline 18\,50 \\ 17\,50 \\ \hline 1\,00 \end{array}$$

EXPLANATION. — The gain is first found. Then its relation to the cost is seen to be $\frac{368.50}{875}$. This relation is changed to the complex decimal $.42\frac{4}{35}$, and this is expressed in the notation of per cent.

Had the relation to the selling price been asked, 1243.50 would have been the divisor.

It must be kept in mind that the quotient, when dividend and divisor are like numbers, shows the relation of the dividend to the divisor.

3. A manufacturer finds that it costs him \$102.60 to manufacture and sell a certain article. At what price shall he list it in order to make a profit of 24 % of the list price?

SOLUTION

```
      $1 35.
.76)$102.60^00
     76
     26 6
     22 8
      3 80
      3 80
```

EXPLANATION. — Since 100 % of anything is all of it, if 24 % of the list price is profit, 76 % of it, or the remainder after deducting 24 %, is cost. Hence, $.76 \times$ the list price = cost. Then, cost $\div .76$ = list price. That is, $.76 \times$ list price = \$102.60, and hence, $\$102.60 \div .76 = \$135$, list price.

SECOND SOLUTION

$$76\% \text{ of the list price} = \$102.60.$$
$$1\% \text{ of the list price} = \tfrac{1}{76} \text{ of } \$102.60 = \$1.35.$$
$$100\% \text{ of the list price} = 100 \times \$1.35 = \$135.$$

EXPLANATION. — This is called the **unitary analysis** method. Here one reasons from the known per cent to 1 %, then to 100 %, or *all.*

NOTE. — The second method is a very common "schoolroom method", but no business man would use it. The first method is the one used in practical life.

EXERCISES

At sight give:

1.	2.	3.	4.
1 % of 400.	1 % of 600.	1 % of 150.	1 % of 175.
2 % of 400.	2 % of 800.	3 % of 780.	5 % of 180.
5 % of 400.	7 % of 500.	6 % of 430.	9 % of 150.

5.	6.	7.	8.
100 % of 400.	100 % of 75.	200 % of 120.	150 % of 300.
200 % of 400.	300 % of 85.	500 % of 850.	125 % of 600.
500 % of 400.	600 % of 35.	400 % of 620.	250 % of 800.

9.	10.	11.	12.
$\frac{1}{2}\%$ of 800.	$\frac{1}{5}\%$ of 750.	$\frac{2}{5}\%$ of 80.	$.4\%$ of 600.
$\frac{1}{4}\%$ of 800.	$\frac{1}{6}\%$ of 180.	$\frac{3}{4}\%$ of 120.	$.3\%$ of 800.
$\frac{1}{3}\%$ of 900.	$\frac{1}{2}\%$ of 840.	$\frac{2}{3}\%$ of 150.	$.2\%$ of 750.

Change to hundredths, then to per cent:

13. $\frac{1}{2}$.	**18.** $\frac{3}{25}$.	**23.** $\frac{1}{8}$.	**28.** $\frac{63}{100}$.
14. $\frac{1}{4}$.	**19.** $\frac{17}{25}$.	**24.** $\frac{3}{8}$.	**29.** $\frac{86}{200}$.
15. $\frac{1}{5}$.	**20.** $\frac{17}{50}$.	**25.** $\frac{1}{3}$.	**30.** $\frac{94}{100}$.
16. $\frac{3}{4}$.	**21.** $\frac{27}{50}$.	**26.** $\frac{2}{3}$.	**31.** $\frac{120}{1000}$.
17. $\frac{4}{5}$.	**22.** $\frac{19}{25}$.	**27.** $\frac{1}{6}$.	**32.** $\frac{680}{2000}$.

Change to common fractions in lowest terms:

33. 25%.	**37.** 70%.	**41.** $12\frac{1}{2}\%$.	**45.** $62\frac{1}{2}\%$.
34. 50%.	**38.** 60%.	**42.** $37\frac{1}{2}\%$.	**46.** $33\frac{1}{3}\%$.
35. 75%.	**39.** 80%.	**43.** $82\frac{1}{2}\%$.	**47.** $66\frac{2}{3}\%$.
36. 40%.	**40.** 30%.	**44.** $6\frac{1}{4}\%$.	**48.** $8\frac{1}{3}\%$.

A TABLE OF EQUIVALENTS

(*To be memorized*)

$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{1}{5} = 20\%$	$\frac{4}{5} = 80\%$
$\frac{1}{4} = 25\%$	$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{2}{5} = 40\%$	$\frac{3}{4} = 75\%$
$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{3}{5} = 60\%$	$\frac{3}{8} = 37\frac{1}{2}\%$

Other Equivalents Less Important

$\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$
$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$	$\frac{1}{16} = 6\frac{1}{4}\%$

PROBLEMS

1. A man's yearly income is \$2750. He spends 20% of it for rent, 45% of it for living expenses, and 8% of it for

recreation. How much does he spend for each? What per cent is left for other things?

NOTE. — Sometimes solutions may be *checked* by solving the problem in two or more ways. Thus in Problem 1, by subtracting the amounts spent for rent, living, and recreation, \$742.50 is left. \$742.50 ÷ \$2750 = .27 = 27 %. Also, 100 % − (20 % + 45 % + 8 %) = 27 %, a second solution.

2. A man delivered 3650 pounds of milk to a creamery. It tested 3.8 % butter fat. He was paid at the rate of 24 ¢ per pound for the butter fat. Find for how much the milk was sold.

3. A crate of live chickens weighs 146 pounds. The empty crate weighs 12 pounds. If the chickens shrink $27\frac{1}{2}$ % on being dressed, what will be the dressed weight of the lot?

4. A merchant's sales for the year amounted to \$175,500. His average gross gain was 28.2 % of the sales, but his overhead charges, as light, heat, management, etc., were 13.7 % of the sales. Find his net profit. (Solve in two ways.)

5. A real estate agent sold a farm of 145 acres at \$95 per acre, and the stock and utensils for \$2890. His fee was $2\frac{1}{2}$ % of the price of the farm and 3 % of the price of stock and utensils. How much did he receive for his services?

6. A merchant's sales for the year were \$156,250. His gross gain was \$22,122. The overhead charges were \$12,356.38. His net profit was what per cent of his sales?

7. If the cost of productive labor in a certain department is \$1728 per week, and the cost of operating the department for the same time is \$576, what per cent must be added to the cost of labor on each article manufactured to meet the entire cost of labor and operating?

8. In 1914 the value of the butter made on the farms in the United States was \$222,861,400. The value of that

made in factories was $179,510,600. Compare each with the other. Also compare each with the total.

9. The export trade in the automobile industry for 1914 and 1915 is given below. Find the per cent of increase or decrease over 1914 in each kind.

KINDS	1914	1915
Commercial automobiles	$ 1,181,611	$ 39,140,682
Passenger automobiles	25,392,963	21,113,953
Automobile tires	3,505,267	4,963,270
Automobile engines	1,391,893	1,405,334
Automobile parts	6,624,232	7,853,183

10. In 1800 the population of the United States was 5,308,480. In 1915 the estimated population was 100,264,480. Find the per cent of increase.

11. The total expenses of the war and navy departments of the United States in 1850 were $17,591,750. In 1915 they were $315,942,491. Find the per cent of increase.

12. The total expenses of the government for the year ending June 30, 1915, to the nearest thousand dollars was $731,400,000. Of this, $172,973,000 went to the war department, $141,836,000 to the navy, and $164,388,000 to pensions. Find what per cent of our national expense went to each of these three departments.

13. The total salaries paid to teachers in public schools in the United States during the year of 1914–1915 were $303,538,000. Compare this with the total expenses of the three departments given in Problem 12. Also compare the amount with that of each department separately.

14. The production of corn in 1900 was 2,105,102,516 bu. In 1914 it was only 1,702,599,000 bu. Find the per cent of decrease.

15. At the end of the year a merchant found that his net profits were \$15,360.50 after paying out \$1250 rent, \$6580 clerk hire, and \$1364.50 for sundry expenses. He also found that his gross profits were $16\frac{2}{3}\%$ of his sales. Find the amount of his sales. His net profits were what per cent of his sales? (For definitions of terms used, see page 196.)

16. After paying rent \$600, clerk hire \$2100, heating and lighting \$320, and sundries \$340, a merchant found that he had a net profit of \$3900 from his business. If his sales for the year amounted to \$64,450, his gross gain was what per cent of his sales? Of the cost?

17. A merchant's sales for the year amounted to \$175,460. His gross gain was 22% of his sales. If clerk hire, rent, and sundries amounted to \$17,546, his net gain was what per cent of his sales? Of the cost of the goods?

18. A merchant's sales for the year were \$246,540. His gross gain was \$63,460. The clerk hire and overhead charges amounted to 18.8% of the sales. The net profit was what per cent of the sales? Of the cost?

19. If the material and labor in a manufactured article amount to \$110 and the overhead charges amount to \$20.40, at what price must the manufacturer list it to make a net profit of 20% of the list price? Of the total cost?

20. A dealer sold goods costing him \$3880 at an advance of 25% of the cost. If the overhead charges amounted to $16\frac{2}{3}\%$ of the sales, the net profit was what per cent of the selling price?

21. A man owing \$196,500 failed in business and paid his creditors only 38¢ on the dollar. What per cent of his

indebtedness did he pay? What per cent of their money did the creditors lose? How much did the creditors lose?

22. A merchant failing in business finds his resources to be only \$140,950 while his liabilities are \$165,750. What per cent of his indebtedness can he pay? How much will a creditor lose to whom he owes \$28,640?

23. If the retailer, wholesaler, and manufacturer make 25%, 10%, and $12\frac{1}{2}$%, respectively, of what they received for an article, find how much each made upon an article for which the retailer received \$48.

24. In Problem 23, change the word "received" to "paid" and solve.

25. Coffee costing \$.$16\frac{1}{2}$ per pound lost 15% in weight in roasting. For how much per pound must the roasted coffee sell to give a profit of $16\frac{2}{3}$% of the selling price? To make a profit equal to 20% of the cost?

26. A house costing \$12,500 rents for \$90 per month. The taxes are \$2.12 per \$100 on an assessment of 80% of the cost. The upkeep amounts to \$192.25 per year. The net income per year is what per cent of the cost?

NOTE.—Net income means the income from the house after the year's bills have been paid.

27. If the house described in Problem 26 is idle 15% of the time, find the yearly rate of income upon the cost. (The taxes and upkeep do not change.)

28. The iceman pays \$5 per ton for ice and retails it at 50¢ per 100 lb. The cost of delivery is \$2 for every ton bought. He loses 20% by melting. What per cent of his sales does he make?

29. A merchant buys sugar at $4\frac{1}{2}$¢ per pound and sells it at 6¢ per pound. If the loss in downweights, drying out,

etc., is 10% of the amount bought, what per cent of the cost is he making? What per cent of the sales?

30. A dealer bought 2000 barrels of apples at $1.50 per barrel. He sold $\frac{3}{4}$ of them at $2.25 per barrel, $\frac{1}{8}$ of them at $1.75 per barrel, and the remainder at $1.25 per barrel. Find the net rate of gain on the cost of all. On the total sales. Can you solve without a pencil?

31. A dealer bought 75 boxes of oranges at $3.50 per box of 156 each. If there is a loss of 240 from decay, at what price per dozen shall the others be sold to realize a profit of 20% of the cost? Of 20% of the selling price?

32. Goods costing $1500 are sold at a profit of 25% of the cost. Find the profit.

33. Goods costing $1500 are sold at a profit of 25% of the selling price. Find the profit.

34. Goods selling for $1500 give a profit of 25% of the cost. Find the profit.

35. Goods costing me $24,500 were sold by my agent for $28,650. His fee was $2\frac{1}{2}$% of the sales. Other expenses amounted to $47.50. My net profit was what per cent of the cost?

36. An importer sold some goods to a wholesale dealer at an advance of 20% on the import cost to him. The wholesaler sold them to a retailer at an advance of 15% on the price that he paid the importer. The retailer sold them to his customers at an advance of 25% above what he paid the wholesaler. If the customers paid $3885 for the goods, how much did they cost the importer?

37. Suppose the goods in Problem 36 cost the importer $1658. How much will they finally cost the customers of the retailer?

38. I bought an article for \$24.50, and marked it so as to make 20% on the cost after giving a discount of \$5.60. The discount was what per cent of the marked price?

39. After increasing in value $12\frac{1}{2}$% over the preceding year for two consecutive years, a farm was worth \$9112.50. What was its value before the increase?

40. A grocer bought a lot of peaches at 90¢ per basket. He sold 90% of them at \$1.20 per basket. The remainder spoiled. What per cent on the whole cost did he make?

41. If I sell $\frac{5}{8}$ of a bill of goods for $\frac{3}{4}$ of the cost of it, what per cent on the cost am I making? What per cent on the selling price?

42. After deducting 5% of the sales for his commission and \$17.25 for transportation charges, an agent remitted the consignor \$1792.50. Find the amount of the sales.

43. At what price must a manufacturer list an article which cost him \$40.625 to manufacture so as to make 25% of the list price after deducting 10% of the list price?

Make other problems from these data, and solve.

44. From what price can I deduct $33\frac{1}{3}$% and still make 20% of the cost on a suit costing \$9.60? How can you check your solution?

45. A merchant sold goods at an advance of 25% on what they cost him, and deducted 5% of the selling price for cash payment. If the net gain was \$127.50, find the cost.

Make and solve other problems from these data.

46. A merchant did a credit business one year of \$78,568. The loss in bad debts amounted to $2\frac{1}{2}$% of the sales. If the goods were sold at an average advance of $33\frac{1}{3}$% of the cost, find his net gain, allowing overhead charges of $17\frac{1}{2}$% of the sales.

47. If goods are retailed at an average advance of 40% of the cost, and bad debts are 5% of the sales, and the cost of selling is 20% of the sales, find what per cent of the cost is made. Find what per cent of the sales is made.

48. If ham, in boiling and slicing, loses 45% of its weight, what is the cost of sliced boiled ham if the raw ham costs 24¢ per pound?

49. Boiled smoked tongue can be bought at a delicatessen shop for 65¢ per pound. The raw tongue can be bought for 26¢ per pound. If the loss in trimming and boiling is 55%, and fuel for boiling costs 2¢ per pound (raw tongue), how much per pound (boiled tongue) is saved by a housewife who buys and boils the raw tongue?

50. A man bought a house and lot for $9500. The insurance averages $18, the taxes $168, and repairs $60 per year. At what price per month must he rent it in order to make a net profit of 6% each year on his original investment?

TRADE OR COMMERCIAL DISCOUNT

163. Commercial discount. — It is a custom among certain wholesalers, manufacturers, and publishers to have a fixed **price list** for their merchandise. These price lists are printed in their catalogues. The list prices are usually higher than the actual market prices, for a deduction is allowed to "the trade"; that is, to retail dealers handling their kinds of goods. This deduction is called **trade** or **commercial discount.**

164. Successive discounts. — Usually the list prices remain the same for long periods, but when the market changes new discounts are made. If the market price increases, a smaller discount is allowed; but if the price decreases, the discount is increased. The increase in discount is usually made by stating a new per cent to be applied to the previous

net price. When two or more discounts to be deducted in this way are allowed, they are called **successive discounts.**

165. Gross amount. — The **gross amount** of a bill is the amount at the regular list price before any discount has been deducted.

166. Net amount. — The net amount of a bill is the amount to be paid after all discounts have been deducted.

PROBLEMS

Oral

1. If a bill of goods listed at $90 is discounted 20 %, how much will they cost?

2. Goods listed at $150 cost $120. What was the rate of discount?

3. A bill of goods listed at $125 was discounted 20 % and 10 %. How much did they cost?

4. After getting a discount of 25 %, goods cost me $120. At what were they listed?

5. The gross amount of a bill is $800. The discounts are 25 % and 10 %. What is the net amount?

6. The gross amount of a bill is $750. The discounts are $33\frac{1}{3}$ % and 20 %. What is the net amount?

7. If a merchant gets a 25 % discount from the list price, what discount can he give from the same list price to make 20 % of the cost to him?

Solution. — The goods cost the merchant 75 % of the list price. He is to sell them to make 20 % of 75 % of the list price. Hence he must make 15 % of the list price. Then he must sell them for 90 % of the list price, or 10 % *less* than the list price. That is, he can give a discount of 10 %.

DRILL TABLE

Give at sight:

	List Price	Discount	Net Cost		List Price	Discount	Net Cost
1.	$20	15 %		16.	$400	20 %	
2.	25	10 %		17.	800	15 %	
3.	48	25 %		18.	700	5 %	
4.	64	$12\frac{1}{2}$ %		19.	900	12 %	
5.	36	25 %		20.	2000	40 %	
6.	35	20 %		21.	3000	30 %	
7.	60	5 %		22.	3600	$33\frac{1}{3}$ %	
8.	42	$16\frac{2}{3}$ %		23.	7200	20 %	
9.	80	10 %		24.	9600	$33\frac{1}{3}$ %	
10.	75	$33\frac{1}{3}$ %		25.	6400	$12\frac{1}{2}$ %	
11.	120	10 %		26.	8400	$16\frac{2}{3}$ %	
12.	150	5 %		27.	7500	$33\frac{1}{3}$ %	
13.	250	2 %		28.	3800	5 %	
14.	300	8 %		29.	4200	2 %	
15.	350	2 %		30.	5400	3 %	

8. What discount from the list price can a merchant give on goods which he bought at a discount of 40 % in order to make 25 % of what they cost him?

9. A merchant made 40 % of the cost on goods bought at a discount of 40 %. What discount did he give from the list price?

10. If a merchant gets a discount of 20 %, and sells at the list price, what per cent of the cost does he make?

11. If goods are bought at a discount of 40 % and sold at a discount of 20 %, what per cent of the cost is made?

SOLUTION. — The cost is 60 % of the list price. The selling price is 80 % of the list price. The gain is 20 % of the list price. The rate of gain on the cost $= 20 \div 60 = 33\frac{1}{3}$ %.

12. In Problem 11, what is the rate of gain on the selling price?

13. When goods are bought at 10% below the list price and sold at 20% above the list price, the gain is what per cent of the cost? Of the selling price?

14. A dealer sold a piano listed at \$850 at a discount of 30%. He got a discount of 50%. What per cent of the cost did he make? What per cent of the selling price? (Answer without using \$850.)

15. A bill of goods quoted at \$400 is sold at 25% and 10% off. What is the net cost?

16. After receiving discounts of 20% and 10% on goods listed at \$1000, how much will they cost me?

By inspection give the net price:

	List Price	Discounts	Net Price		List Price	Discounts	Net Price
17.	\$ 600	20%, 10%		**22.**	\$2400	25%, 10%	
18.	900	$33\frac{1}{3}$%, 10%		**23.**	3600	$33\frac{1}{3}$%, $12\frac{1}{2}$%	
19.	1000	40%, 15%		**24.**	1600	25%, 5%	
20.	1200	$16\frac{2}{3}$%, 20%		**25.**	3500	$14\frac{2}{7}$%, 2%	
21.	1800	$33\frac{1}{3}$%, 20%		**26.**	6000	10%, 10%	

PROBLEMS IN DISCOUNT

1. Find the net amount of a bill of \$1280 less $33\frac{1}{3}$% and 25%.

Solution

$$\begin{array}{r} 4)\$1280 \\ \underline{320} \\ 3)960 \\ \underline{320} \\ \$640 \end{array}$$

Explanation. — In any problem in which two or more discounts are allowed, the order in which they are taken does not affect the result. It is much shorter in this problem to take $\frac{1}{4}$ (25%) and then $\frac{1}{3}$ ($33\frac{1}{3}$%), than to take $\frac{1}{3}$ and then $\frac{1}{4}$.

A still shorter method is to reason that the goods cost $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{1}{2}$ of the list price, hence, $\frac{1}{2}$ of \$1280.

2. The gross amount of a bill of hardware was \$1680. The discounts were $33\frac{1}{3}\%$, 25%, 10%. Find the net price. Can you solve without a pencil?

3. A bill of hardware was listed at \$96, 40% and 10% off. Find the net price.

4. Find the net cost of a bill of \$85, 40% and 5% off.

5. Find the net cost of 8 doz. drip pans listed at \$4.45 per dozen, and 15 coal hods listed at \$2.10 per dozen; discounts 60% and 10%.

6. Find the net cost of 24 doz. basting spoons at \$3 per dozen, and $\frac{1}{4}$ gross galvanized buckets at \$58 per gross; discounts 75% and 10%.

7. A dealer received a bill of window glass listed at \$730, but the discounts were 90% and 15%. Find the net cost.

8. Find the net cost of a bill of \$850, 50% and 15% off.

9. A bill of chinaware listed at \$736 had discounts of $66\frac{2}{3}\%$ and 10%. Find the net price, including \$8.36 for boxing, freight, and drayage.

10. One third of the gross amount of a bill of silverware amounting in all to \$846 was discounted at 40%, 10%, and 10%, and the remainder at 40% and 15%. Find the net amount of the bill. If the dealer retails the entire bill at an average of 90% of the list price, what does he make? What per cent of the net cost is this? What per cent of the selling price?

Find short methods of finding the net price when the successive discounts are as follows:

11. $33\frac{1}{3}\%$ and 25%.

12. 60% and 25%.

13. $66\frac{2}{3}\%$ and 10%.

14. $33\frac{1}{3}\%$ and 10%.

15. 40% and $16\frac{2}{3}\%$.

16. 64% and $16\frac{2}{3}\%$.

17. $37\frac{1}{2}\%$ and 20%.

18. 20% and $12\frac{1}{2}\%$.

At sight give the net price:

19. \$1650 less $33\frac{1}{3}$ % and 25 %.

20. \$3500 less 60 % and 25 %.

21. \$4200 less $66\frac{2}{3}$ % and 10 %.

22. \$1500 less $33\frac{1}{3}$ % and 10 %.

23. \$1940 less 25 % and $33\frac{1}{3}$ %.

24. \$2400 less 40 % and $16\frac{2}{3}$ %.

25. \$6000 less 64 % and $16\frac{2}{3}$ %.

26. \$1365 less $37\frac{1}{2}$ % and 20 %.

27. \$1790 less 20 % and $37\frac{1}{2}$ %.

28. \$5000 less 20 % and $12\frac{1}{2}$ %.

29. A dealer receives a bill the gross amount of which is \$334. The discounts are 40 % and 10 %. The freight, drayage, and sundry expenses amount to \$12.50. If he sells the goods at an average of 85 % of the list price, what per cent of the selling price does he make?

30. If a dealer gets discounts of 40 % and 15 %, what discount can he give on the list price to make 30 % of the cost? Of the sales?

31. A jobber sold a plate glass for \$150, less discounts 50 %, 30 %, and 10 %, and still made a profit of 25 % of what it cost him. What did it cost the jobber?

32. A dealer received the following invoice of wagons: 3 listed at \$79 each; 2 listed at \$81 each; 4 listed at \$103 each; and one listed at \$85. The discounts are 40 % and 5 %. Find the net invoice. If a further discount of 5 % of the net bill is given for cash, what will he save by paying cash?

33. A dealer bought disk harrows listed at \$18.50, at 35 % and 10 % off, and sold them through an agent at 10 % off from the list price. The agent got 25 % of the amount he received for the harrows. Find the net profit of both the dealer and the agent on each harrow.

34. Three salesmen, A, B, and C, offer me the same kind of goods at the same list price. A offers to discount 25 % and 15 %; B, 20 % and 20 %; and C, 15 %, 15 %, and 10 %.

With which will it be best for me to deal, and how much should I save from a list price of $200?

167. Comparing single with successive discounts. — A merchant may wish to know the single discount equivalent to two or more successive discounts in order to compare net costs on different propositions. Three methods are shown by examples.

EXAMPLE. — Which is better, 45%, or 40% and 10%?

FIRST SOLUTION

$$\begin{array}{rr} & 100\% \\ & \underline{40\%} \\ & 60\% \\ & \underline{6\%} \\ & 54\% \\ \text{Dis.} = & 46\% \end{array}$$

EXPLANATION. — A single discount of 40% leaves 60% of the list price. 10% of 60% = 6%. Deducting 6% leaves a *net cost* of 54% of the list price. This is 46% *less* than list price. Hence, successive discounts 40% and 10% = a single discount of 46%; hence, this is 1% of the list price better than 45%.

SECOND SOLUTION

$$\begin{array}{r} 100\% \\ \underline{40\%} \\ 60\% \\ \underline{.10} \\ 6\% \\ \underline{40\%} \\ 46\% \end{array}$$

EXPLANATION. — The first discount is 40%, leaving 60% of the list price. The second discount is 10% of 60%, or 6%. The sum of the two discounts, 40% + 6%, gives the single discount equivalent to the two.

THIRD SOLUTION

$$\begin{array}{r} 40\% \\ \underline{10\%} \\ 50\% \\ \underline{4\%} \\ 46\% \end{array}$$

EXPLANATION. — In this method, the two discounts are added and their product subtracted from their sum. The reason for this is clear to those having had algebra. For the net price = 90% of 60% of the list price. That is, to $(100\% - 10\%) \times (100\% - 40\%)$ of the list price. And, in general, the net price $= (100\% - r\%) \times (100\% - r'\%) = 100\% - (r\% + r'\%) + r\% \times r'\%$. Grouping, this $= 100\% - [(r\% + r'\%) - r\% \times r'\%]$. Hence the total discount is $r\% + r'\% - r\% \times r'\%$.

EXERCISES

By inspection, give the single discount equivalent to:

1. 10% and 10%.
2. 20% and 10%.
3. $33\frac{1}{3}\%$ and 10%.
4. 40% and 25%.
5. 40% and $33\frac{1}{3}\%$.
6. 50% and 10%.
7. $66\frac{2}{3}\%$ and 10%.
8. 20% and 25%.
9. 40% and 20%.
10. 60% and 20%.
11. 60% and 25%.
12. 30% and 10%.
13. 40% and $16\frac{2}{3}\%$.
14. 20% and $12\frac{1}{2}\%$.
15. 40% and 15%.
16. 50% and 5%.
17. 50% and 25%.
18. 50% and 40%.
19. 60% and $12\frac{1}{2}\%$.
20. 60% and 25%.
21. 55% and $33\frac{1}{3}\%$.
22. 40% and 30%.
23. 25% and $33\frac{1}{3}\%$.
24. 36% and $12\frac{1}{2}\%$.
25. 52% and $8\frac{1}{3}\%$.
26. 28% and $16\frac{2}{3}\%$.
27. $37\frac{1}{2}\%$ and 20%.
28. 30% and $14\frac{2}{7}\%$.
29. 30% and 20%.
30. 30% and 30%.
31. 40% and 10%.
32. 20% and 20%.
33. 50% and 30%.
34. 50% and 20%.
35. 40% and 40%.
36. 40% and 5%.
37. $37\frac{1}{2}\%$ and 10%.
38. $37\frac{1}{2}\%$ and 5%.
39. 35% and 20%.
40. 35% and 10%.
41. 25% and 25%.
42. 25% and 10%.
43. $12\frac{1}{2}\%$ and $14\frac{2}{7}\%$.
44. 15% and 10%.
45. 10% and 5%.
46. 20% and 5%.
47. 60% and $12\frac{1}{2}\%$.
48. 60% and 10%.
49. 60% and 30%.
50. 60% and 40%.
51. 60% and 60%.
52. 50% and $33\frac{1}{3}\%$.

Solve the following by inspection, and check by computation:

53. 20%, 20%, and 10%.
54. 25%, 20%, and 5%.
55. $33\frac{1}{3}$%, 20%, and 10%.
56. 30%, 20% and $12\frac{1}{2}$%.
57. 25%, 25% and $14\frac{2}{7}$%.
58. 40%, 25%, and $16\frac{2}{3}$%.
59. 50%, 25%, and 10%.
60. 40%, 20%, and $12\frac{1}{2}$%.
61. $37\frac{1}{2}$%, 20%, and 5%.
62. 50%, 20%, and 20%.

168. Billing goods to "the trade".—The following forms of bills sent by wholesalers to retailers show the discounts and terms of sale. Sometimes the words "sold to", as in Problem 1, are used, and sometimes "bought of" as in Problem 2. The first form is most common.

PROBLEMS

1. Check the following bill:

CHICAGO, ILL., May 1, 1916

A. G. SPAULDING & BROS.

ATHLETIC GOODS, 147 WABASH AVE., CHICAGO

SOLD TO W. D. Williams & Co.,
Urbana, Ill.

Terms: Net Cash.

3	Doz. Tennis Rackets	$18.00	54	00		
6	Doz. Tennis Balls	3.25	19	50		
$\frac{1}{2}$	Doz. Tennis Nets	17.60	8	80		
			82	30		
		Less 10%	8	23		
					74	07

2. Check the following:

INDIANAPOLIS, July 6, 1916

Mr. R. H. Brabb,
Springfield, Mo.

Bought of HOLLWEG & REESE

IMPORTERS OF CHINA, GLASS, AND QUEENSWARE

Terms: 60 days or 2% off 10 days.

6	Doz.	5274	Plates	$2.25	13	50		
6	"	"	Tea Cup	2.50	15	00		
$\frac{7}{12}$	"	"	Coffee Cup	2.40	1	40		
6	"	"	Fruit	1.50	9	00		
2	"	"	Deep Soup	2.00	4	00		
1	Only	"	Covd. Dish	1.65	1	65		
3	"	"	Casse-role	1.65	4	95		
					49	50		
			Less 40%		19	80	29	70
			Less 10%				2	97
							26	73

3. If Mr. Brabb pays the bill within 10 days, what is the net amount of the bill?

4. Bill from same firm to A. H. Harris, Decatur, Ill., the following: 12 doz. plates at $3.20 per doz.; 7 covered dishes at $1.35 each; 12 doz. tea cups at $2.10 per doz.; 5 casseroles at $2.15 each. Discounts 30% and 30%. Terms: net 60 days, 2% off in 10 days.

5. Complete the following bill:

CHICAGO-KENOSHA HOSIERY CO.

MANUFACTURERS OF

SEAMLESS AND FULL-FASHIONED HOSIERY

SOLD TO Messrs. J. R. Doe & Co.,
Decatur, Ill.

Terms: 2 per cent 10 days. Net 30 days.

JULY 15, 1916

NO. OF DOZ. PR.	DESCRIPTION	PRICE			TOTALS		
$2\frac{1}{2}$	Boys' Stockings	1.90					
$1\frac{1}{2}$	" "	2.00					
$7\frac{1}{2}$	" "	2.10					
4	" "	2.25					

If Doe & Co. remit before July 25, what should be the amount of the remittance? What must they remit any time between July 25 and Aug. 15?

6. Bill from the same firm to A. L. Dickson & Son, Jan. 4, 1916, the following: $3\frac{1}{2}$ doz. pr. hose at \$3.25; $7\frac{1}{2}$ doz. pr. children's stockings at \$1.80; $12\frac{1}{2}$ doz. pr. men's half hose at \$2.50; $8\frac{1}{2}$ doz. pr. boys' stockings at \$2.25. What will settle the bill within 10 days? What within 30 days?

7. Sidney Shepherd & Co., Buffalo, N. Y., sold to H. M. Murphy & Son, March 13, 1916, the following:

$\frac{1}{4}$ gro. pails at \$39.60; $\frac{1}{12}$ gro. steamers at \$28.80; $\frac{1}{12}$ gro. steamers at \$32.40; all less 10%. Also $\frac{1}{2}$ gro. pails at

\$31 net; $1\frac{1}{2}$ doz. drip pans at \$4.45 less 60%, 10%, and 10%; $\frac{1}{4}$ doz. bond boxes at \$5.10 less 10%. Terms: 2% off 10 days; net 60 days.

What will settle the bill before March 23?

8. Check the following:

Description		Price		Gross Amount		Net Amount	
$\frac{1}{4}$ doz. alum. trays and scrapers	net	4	00			1	00
$\frac{1}{4}$ doz. alum. trays and scrapers	net	4	00			1	00
1 doz. favorite cake spoons	net		75				75
1 doz. galvanized tubs				5	70		
1 doz. galvanized tubs				5	30		
$\frac{1}{2}$ doz. galvanized tubs		7	20	3	60		
$\frac{1}{2}$ doz. galvanized tubs		7	20	3	60		
1 doz. skimmers			70		70		
1 doz. rings			25		25		
$\frac{1}{12}$ gro. dippers		7	80		65		
$\frac{1}{6}$ doz. bread raisers		4	80		80		
$\frac{1}{6}$ doz. bread raisers		5	70		95		
$\frac{1}{6}$ doz. bread raisers		6	90	1	15		
1 doz. cup dippers			65		65		
				23	35		
	less 10%					21	02
$\frac{1}{4}$ gro. slop jars		36	00	9	00		
	less 10%					8	10
1 doz. dippers	net		55				55
2 doz. basting spoons, per gr.		22	00	3	67		
2 doz. basting spoons, per gr.		29	00	4	83		
2 doz. basting spoons, per gr.		36	00	6	00		
				14	50		
	less 75% and 10%					3	27
1 doz. soup strainers	net	1	00			1	00
1 doz. pie tins	net		45				45
$\frac{1}{12}$ doz. dippers	net	11	40				95
						38	09

PROFIT AND LOSS

169. Profit and loss. — The problems discussed here do not differ from many of those already discussed. The object of taking up the topic here is to give a more thorough discussion of the practices and customs of reckoning profits, losses, costs, selling prices, etc., that arise in business transactions.

170. Net and gross costs. — The amount actually paid for an article is called its **prime** or **net cost**. When transportation charges, commission for buying, insurance, etc., are added, the result is the **gross cost**.

171. Net and gross profits. — The difference between the gross cost and the actual selling price is the **gross profit**. The gross profit less the selling costs such as salaries, commissions, traveling expenses, etc., gives the **net profit**. In case the gross cost is greater than the net selling price, evidently there is a **loss**.

172. Reckoning the rate of profit or loss. — There is no uniform agreement among business men as to what should be used as the basis in reckoning the rate of profit. Some reckon the profits upon the *net cost*, some upon the *gross cost*, and others upon the *selling price*. No confusion arises, however, if the basis is stated. But to say that a man made 25% has no meaning unless the basis upon which it is reckoned is stated.

Thus, if the sales of a firm amounted to \$180,000 and the net cost of the goods was \$120,000, and the operating expenses were \$40,000, one may say that the net gain equaled $11\frac{1}{9}\%$ of *the sales*, $12\frac{1}{2}\%$ of *the gross cost*, or $16\frac{2}{3}\%$ of *the net cost;* or, that the gross gain equaled $33\frac{1}{3}\%$ of *the sales*, $37\frac{1}{2}\%$ of *the gross cost*, or 50% of *the net cost.*

It is now almost the universal custom to reckon the cost

of operating upon the sales. Thus, from the data given above, the operating cost is $22\frac{2}{9}\%$ of *the sales*. It will be observed, then, that $22\frac{2}{9}\%$ of the sales subtracted from the gross gain of $33\frac{1}{3}\%$ of the sales leaves a net gain of $11\frac{1}{9}\%$ of the sales as it should; but $22\frac{2}{9}\%$ of the sales could not be subtracted from the gross gain of $37\frac{1}{2}\%$ of the gross cost, or from 50 % of the net cost. For this reason, it is a growing custom among the retail trade to reckon both the net and gross gains upon the sales.

NOTE. — Formerly textbooks have taught that loss or gain is *always* reckoned "upon the cost," but this is not true in modern business practice. It is becoming a very common practice to reckon loss or gain upon the net sales. This may be verified by asking almost any business authority.

PROBLEMS

1. A manufacturing concern shows total sales, $1,465,850; cost of labor and material, $1,063,400; and operating expenses, $125,650. Find the rate of net profit upon the sales; upon the cost of labor and material; upon the gross cost. Also find what per cent each item is of the sales.

2. Complete the following "Profit and Loss Statement". Find the rate of net gain on the net sales in each department, and also in the "totals". Find the same upon the cost of the goods.

	Dry Goods Dept.		Shoe Dept.		Men's Wear Dept.		Totals	
Sales	$165,780	00	$96,370	00	$142,960	00		
Goods ret'd . .	630	00	250	00	610	00		
Net sales . . .								
Cost of goods .	100,740	00	70,015	00	96,370	00		
Gross gain .								
Overhead charges . .	16,480	00	7,216	00	10,860	00		
Net profit . .								

3. In the "Profit and Loss Statement" in Problem 2, find what per cent of the gross sales were returned in each department and in the totals.

4. In the same "Statement", to what per cent of the net sales did the overhead charges amount in each department and in the totals?

5. In the same "Statement", compare the overhead charges in each department with the total overhead charges. Can you check your results by adding the three?

6. A fruit dealer bought a carload of apples which cost him \$1280. He paid a commission for buying equal to 5% of the cost. The transportation charges were \$22.50. Since some of the apples were damaged, he had to sell the entire carload for \$1125. The total loss was what per cent of the purchase price? Of the gross cost? Of the selling price?

7. An invoice of hardware amounted to \$1760 less $33\frac{1}{3}$% and 10%. If it was then marked at an advance of 25% of the net cost, find for how much it was marked. At what must it be marked to make 25% of the selling price?

8. If the whole invoice in Problem 7 is sold at a gain of 25% of the net cost, what per cent of the selling price is made? If sold at a gain of 25% of the selling price, what per cent of the cost is made?

9. A dealer buys shoes at \$45 per dozen pairs. The expense of purchasing them amounts to 6¢ per pair. It is estimated that the cost of selling will amount to 34¢ per pair. At what price must he mark them to make 40% of the gross cost? To make $33\frac{1}{3}$% of the selling price?

10. A merchant's sales for the year amounted to \$75,000. The gross gain was 22% of the sales. The clerk hire was \$2400. The general expenses amounted to \$1250. The

clerk hire and the general expenses were each what per cent of the sales? The net gain was what per cent of the sales?

11. One year a merchant's sales amounted to \$192,000. The returned goods amounted to \$1020. The cost of selling the goods was \$18,960. His net profit was \$14,800. The gross profit was what per cent of the gross sales? To what per cent of the gross sales did the returned goods amount?

12. From the data in Problem 11, reckon the cost of the goods, and the rate of net gain on the cost.

13. During the year a merchant's sales amounted to \$246,500. The average gross gain was 20% of the sales. Rent, insurance, clerk hire, and sundries amounted to \$28,200. The net gain was what per cent of the sales? Of the purchase price?

14. In Problem 13, find what per cent of the sales were required to pay the expenses?

15. An importer bought green coffee at $14\frac{1}{4}$¢ per pound. After roasting it he sold it for 24¢ per pound. In roasting it lost 15% in weight. The cost of selling amounted to $2\frac{1}{4}$¢ per pound. What per cent of the selling price did he make?

16. A grocer bought 4000 bushels of potatoes at 45¢ per bushel. He sold $\frac{3}{4}$ of them at 65¢ per bushel and the remainder at 50¢ per bushel. The cost of selling amounted to 9¢ per bushel. The net gain was what per cent of the gross amount of the sales?

17. The "Profit and Loss Statement" of the three departments of a business one year showed the following: Clothing department, sales \$94,500, gross gain 32% of the sales; shoe department, sales \$26,400, gross gain 24% of the sales; men's furnishings department, sales \$19,680, gross gain 35% of the sales. The total cost of selling averaged 18% of the

sales throughout the store. That left a total net profit of how much in the three departments?

18. One year the "Profit and Loss Statement" of a manufacturing company showed that the net sales amounted to \$1,601,204. The cost to manufacture the goods that were sold amounted to \$924,860. It cost \$87,400 to sell the goods. The administration expenses were \$35,200. Find the gross profit. The net profit. The net profit is what per cent of the sales?

19. In Problem 18, the manufacturing cost, the selling cost, and the cost of administration are each what per cent of the net sales?

20. A manufacturing company had one year the following account of cost: Raw material, \$78,650; factory pay roll, \$98,345; management expense, \$26,500; sundry expenses, \$15,680. Analyze the account as to the various items; that is, show the relation (per cent) of each item to the whole cost of production (material, labor, management, etc.).

21. If the net sales of the company described in Problem 20 amounted to \$298,850, the net profit was what per cent of the whole cost of production?

22. If the net sales had amounted to \$196,780, the loss would have been what per cent of the cost of production?

23. *Rule a form like that of page 201 and fill blanks showing:*

a. The gross profits by departments.

b. The per cent of gross profits reckoned on the sales.

c. The net profits by departments.

d. The per cent of net profits reckoned on the sales.

e. The total gross profits of the store.

f. The per cent of gross profits reckoned on the total sales.

g. The total net profits of the store.

h. The per cent of net profits reckoned on the total sales.

	Dept. Sales		Cost of Goods Sold		Gross Profits		Per Cent of Gross Profits	Expenses		Net Profits		Per Cent of Net Profits
1.	$57,628	40	$42,090	68				$8765	35			
2.	98,206	38	71,732	42				12,062	40			
3.	76,842	57	54,328	65				9840	60			
4.	86,398	30	61,149	28				10,090	42			
5.	91,063	48	64,396	28				10,980	25			
6.	50,280	20	38,964	22				7965	20			
Total												

173. Marking goods. — Some manufacturers list their goods at a price that will give a profit of a certain per cent *of the cost of production*, and others list them so as to make a certain per cent *of the list price*. In general, however, the list or selling price is not *exactly* computed in order to give a certain per cent of either cost or sales, but the price is regulated by competition.

PROBLEMS

1. If the cost of production of a certain article is $84.50, find a list price that will give a profit of 15 % of the cost. A profit of 15 % of the list price.

SOLUTION OF FIRST

```
 $84.50
   1.15
 422 50
 845 0
8450
$97.17 50
```

Hence, list price = $97.18.

SOLUTION OF SECOND

```
       $99.41
.85)$84.50 00
     76 5
      8 00
      7 65
        35 0
        34 0
         1 00
```

Hence, list price = $99.41.

EXPLANATION. — The first solution is evident. In the second, since 15 % of the list price is profit, the rest must be cost. That is, 85 % of the list price is cost. Hence the cost could be found by multiplying the list price, were it known, by .85. Therefore, the cost ÷ .85 = the list price. That is, \$84.50 ÷ .85 = \$99.41, the list price.

2. If the total cost to produce an article is \$345.75, find the list price that gives a profit of 20 % of the cost. A profit of 20 % of the list price.

3. By what is the cost divided to give a profit of 35 % of the list price? 50 % of the list price? 60 % of the list price? 40 % of the list price? 25 % of the list price?

Make a list price for goods that will meet the following conditions:

NOTE. — There are two answers to each problem.

	Cost of Production	Gain on Cost	Gain on List Price		Cost of Production	Gain on Cost	Gain on List Price
4.	\$26.30	15 %	15 %	12.	\$ 39.80	35 %	35 %
5.	35.80	20 %	20 %	13.	72.85	20 %	20 %
6.	42.60	12 %	12 %	14.	348	25 %	25 %
7.	65.80	22 %	22 %	15.	652	18 %	18 %
8.	38.40	18 %	18 %	16.	765	40 %	40 %
9.	27.50	15 %	15 %	17.	896	$33\frac{1}{3}$ %	$33\frac{1}{3}$ %
10.	36.50	25 %	25 %	18.	930	$16\frac{2}{3}$ %	$16\frac{2}{3}$ %
11.	84.20	30 %	30 %	19.	1050	25 %	25 %

20. Show that a profit of 20 % of the list price is the same as 25 % of the cost.

21. Show that if goods sell at a profit of 50 % of the cost, the gain is but $33\frac{1}{3}$ % of the selling price.

22. A manufacturer sells goods at a profit of 40 % of the cost to produce. The total overhead charges amount to

25 % of the sales. Show that he is making a net profit of but 5 % of the cost.

23. In Problem 22, what per cent of the sales are the net profits?

24. If the cost of selling goods equals 20 % of the sales, and the gross profit is 40 % of the net cost, the net profit is what per cent of the sales?

SOLUTION

$$\begin{array}{r} 5\overline{)140\,\%} \\ \underline{28\,\%} \\ 112\,\% \end{array}$$

$$\begin{array}{r} .08\tfrac{4}{7} = 8\tfrac{4}{7}\,\% \\ 140\overline{)12.00} \\ \underline{11\ 20} \\ 80 \end{array}$$

EXPLANATION. — The selling price = 140 % of the cost. The cost of selling = 20 % of 140 % of the cost = 28 % of the cost. The net gain = 12 % of the cost. 12 % of the cost ÷ 140 % of the cost = 12 ÷ 140 = $.08\frac{4}{7}$. Hence the net gain = $8\frac{4}{7}$ % of the selling price.

DRILL TABLE

	Gain on Cost on Sales	Cost of Selling	Net Gain on Cost	Net Gain on Sales		Gain on Cost on Sales	Cost of Selling	Net Gain on Cost	Net Gain on Sales
25.	25 %	10 %			**35.**	50 %	18 %		
26.	25 %	15 %			**36.**	50 %	20 %		
27.	25 %	18 %			**37.**	50 %	25 %		
28.	$33\frac{1}{3}$ %	10 %			**38.**	60 %	10 %		
29.	$33\frac{1}{3}$ %	15 %			**39.**	60 %	15 %		
30.	$33\frac{1}{3}$ %	20 %			**40.**	60 %	20 %		
31.	40 %	8 %			**41.**	60 %	25 %		
32.	40 %	16 %			**42.**	75 %	15 %		
33.	40 %	25 %			**43.**	75 %	40 %		
34.	40 %	12 %			**44.**	80 %	40 %		

45. The cost to manufacture a certain article is $42.50; the estimated cost to sell it is $18.20. At what must it be

listed to give a profit of $22\frac{1}{2}\%$ of the cost to manufacture and sell?

46. Using the data in Problem 45, for how much must it be listed to give a profit of $22\frac{1}{2}\%$ of the list price?

47. Using the same data, at what must it be listed to give a profit of $22\frac{1}{2}\%$ of the total cost after discounting the list price 40%?

48. Using the same data, at what must it be listed to give a profit of $22\frac{1}{2}\%$ of the list price after discounting the list price 40%?

49. Using the same data, at what must it be listed to give a profit of $22\frac{1}{2}\%$ of the selling price after discounting the list price 40%?

50. An article costing \$76.40 to manufacture and sell was listed to give a profit of 15% of the cost after discounting the list price 25%. The profit was what per cent of the list price? Of the net selling price?

51. An article is listed at \$74.40. This gives a profit of $16\frac{2}{3}\%$ of the list price. The profit is what per cent of the cost?

52. An article is listed at \$74.40. This gives a profit of $16\frac{2}{3}\%$ of the cost. The profit is what per cent of the list price?

53. After discounting the list price $33\frac{1}{3}\%$ and 10%, a wholesaler is making a profit of 16% of the list price. What per cent of the cost is he making? What per cent of the net selling price is he making?

54. After discounting the list price $33\frac{1}{3}\%$ and 10%, a wholesaler is making a profit of 16% of the net selling price. What per cent of the cost is he making? What per cent of the list price is he making?

55. In Problem 54, if the profit of 16% had been on the cost, it would have been what per cent of the list price? Of the net selling price?

56. If goods are listed to give a profit of 25% of the list price after allowing discounts of $33\frac{1}{3}$% and 10%, the gross profit is what per cent of the net selling price? If the cost of selling is $17\frac{1}{2}$% of the net selling price, the net gain is what per cent of the selling price? Of the cost? Of the list price?

COMMISSION AND BROKERAGE

174. Commission and brokerage. — Some kinds of merchandise are bought and sold through agents called **commission merchants** or **brokers.** The person for whom the business is transacted is called the **principal.** The fee for agents' services is sometimes reckoned as a per cent of the amount of the sales or of the amount bought; and sometimes it is a certain price for each unit bought or sold, as so much per ton, per barrel, per bushel, etc. The fee for buying or selling is called the **commission** or **brokerage.**

175. Business terms used. — The **principal,** or **consignor,** refers to the goods as a **shipment** and to his agent as the **consignee.** The commission merchant calls the goods a **consignment.** The **gross proceeds** from a sale is the entire amount received for them by the consignee. The **net proceeds** is the amount left after the commission, freight, storage, and any other charges have been deducted. In the commission and brokerage business the **prime** or **net cost** of a purchase is the amount the agent actually pays for the goods. The **gross cost** is the prime cost plus all charges incurred in buying, as commission, traveling expenses, insurance, storage, etc.

DRILL TABLE

	Sales	Rate	Commission		Sales	Rate	Commission
1.	$ 85,000	4%		11.	$115,000	8%	
2.	175,000	2%		12.	260,000	$2\frac{1}{2}$%	
3.	250,000	5%		13.	270,000	6%	
4.	160,000	$2\frac{1}{2}$%		14.	187,000	10%	
5.	240,000	5%		15.	275,000	8%	
6.	280,000	4%		16.	364,000	5%	
7.	190,000	3%		17.	106,000	9%	
8.	165,000	2%		18.	76,500	4%	
9.	285,000	4%		19.	82,300	4%	
10.	110,000	3%		20.	163,500	5%	

PROBLEMS

1. An agent bought 500 barrels of apples for me at $1.20 per barrel. Find his commission at 3%. How much per barrel did it add to the cost?

2. My agent bought a carload (720 bu.) of potatoes for me at 40¢ a bushel, commission $2\frac{1}{2}$%. What was his commission? How much per bushel did it add to the cost? If freight, drayage, and other expenses are 9¢ a bushel, for how much a bushel must I sell them to make 30% of the total cost? To make 30% of the sales?

3. One month an agent bought 20,000 bu. of peaches at an average price of 50¢ per bu. What did he earn at $2\frac{1}{2}$% commission?

4. A speculator bought 50,000 bu. of wheat, through a broker, for $86\frac{1}{8}$¢ per bu., and sold it at $93\frac{1}{4}$¢ per bu. What did he make, brokerage being $\frac{1}{8}$¢ per bushel for buying and the same for selling? What was the broker's fee?

5. If a speculator bought 1500 bbl. of pork at $14.05 per bbl. and sold it at $$12.87\frac{1}{2}$ per bbl., what was his total

loss, not including interest, brokerage being $2\frac{1}{2}$¢ per barrel for buying and the same for selling? What was the broker's fee?

6. What will 3500 bales (500 lb. each) of cotton cost at 9.6¢ per pound, brokerage 5¢ per bale? What will the brokerage be?

7. A speculator bought 2800 bales of cotton (500 lb. each) at 10.2¢ per lb. and sold at 9.8¢ per lb. What was the total loss, brokerage 5¢ per bale for each transaction?

8. A commission merchant sent me \$829 from the sale of 500 barrels of apples at \$1.75, after paying \$20 freight and \$8.50 storage. What commission did he charge me? How much per barrel was this?

9. I shipped produce to a commission merchant which he sold as follows: 68 crates of strawberries at \$2.56 per crate; 2340 bunches of radishes at $2\frac{1}{4}$¢ a bunch; and 15 baskets of lettuce (18 pounds each) at 12¢ a pound. How much did I receive if I paid 5% commission?

10. A commission merchant sold a carload of melons (1450) at 48¢ each, on a 5% commission. How much commission did he get from the sale?

NOTE. — A commission merchant who deals in hay, grain, and beans usually charges a commission per car, ton, bushel, etc.

11. A commission merchant sold 198 bales of clover hay averaging 135 lb. per bale at \$16 per ton; and 186 bales of timothy hay averaging 138 lb. per bale at \$18 per ton. After retaining his commission of \$1 per ton for selling and \$28.50 for freight and drayage, how much should he remit?

12. A commission merchant sold 430 bags of beans (165 lb. per bag) at \$2.85 per bu. (60 lb.). After retaining his commission of 5¢ per bag, and \$42.60 for freight, how much should he remit?

13. Complete the following *account sales:*

NEW YORK HAY EXCHANGE ASSOCIATION

June 1, 1916

Sold for the account of **M. Dawson**

		Description	No. Bales	Weight		
1916						
May	1	No. 1 timothy @ $20 per ton	164	22,445		
"	7	" 3 " @ 16 " "	180	24,625		
"	7	" 1 clover @ 16 " "	170	23,440		
"	9	" 2 " @ 14 " "	160	21,435		
		Freight $90.40. Drayage $16.50.				
		Storage $45.				
		Commission $1 per ton				
		Amount of draft				

NOTE. — The number of bales in Problems 13 and 14 has nothing to do with the computation. The price is quoted on the ton.

14. Mr. Mooreman paid the following prices for 3 carloads of hay: No. 1 baled timothy $14; No. 2 baled timothy $12; No. 1 clover $12. He shipped it to the Brooklyn Hay and Grain Co., who sold it as follows: 300 bales No. 1 timothy, 42,225 pounds at $20; 120 bales No. 2 timothy, 16,345 pounds at $18; 90 bales No. 1 clover, 13,440 pounds at $17. Commission $1 per ton. Freight and drayage $96.75. Storage $36. Find the net gain and the gain per cent on what Mr. Mooreman paid for the hay. Make out the account sales.

15. A dealer paid $1.80 per bushel (60 lb.) for beans, put them in bags of 165 pounds each, and sold them per carload

f. o. b. (free on board cars) at $2.25 per bushel, through a commission merchant. The bags cost $165 per 1000. The commission merchant's fee was 5 cents per bag. Find the net profit on a carload of 320 bags.

16. A dealer bought 3 carloads of wheat (3460 bu. in all), paying an average price of 82 ¢ per bu. It was sold by a commission merchant for 86 ¢ per bu. (f. o. b.). Find the net gain, commission being $5 per car.

NOTE. — **Dairy and poultry products are usually sold on a 5% commission.**

17. A commission merchant sold 1580 lb. of chickens at $15\frac{1}{4}$ ¢ a pound, and 56 cases of eggs (30 doz. each) at $18\frac{3}{4}$ ¢ a dozen. He charged 5 % commission and deducted $7.25 for freight and other charges. How much did he remit?

18. Find the net proceeds and the net gain from the following sale: 120 cases of eggs (30 doz. each) at 26 cents, 730 eggs broken; 60 crates of spring chickens, averaging 90 lb. per crate at 22 cents; and 40 tubs dairy butter, 80 lb. each, at 28 cents. Freight $86. Drayage $32. Storage $28. Commission for selling 5 %. The average price paid was eggs, 18 ¢; chickens, 16 ¢; butter, 26 ¢.

BORROWING AND LOANING MONEY

176. Interest.—**Interest** is money paid for the use of money or the amount paid for an accommodation on an unpaid debt. Interest is reckoned as a certain per cent of the debt, which is called the **principal**, for a year's use of it, even though the interest may be collected semiannually or quarterly.

177. A promissory note. — A **promissory note** is a signed promise by one party to pay a certain sum of money to another party at a specified time. A note is given to cover a loan or to settle any other kind of indebtedness. If the note

bears interest, the words "with interest" are used and the rate of interest is usually specified. If not specified, the legal rate of the state is understood. If a note does not bear interest until after the date at which it falls due, as often occurs when a note is given in payment of a debt or a purchase, the words "with interest" are omitted. Such a note is called a non-interest bearing note.

USUAL MANNER OF DRAWING A NOTE

$500.00 NEW YORK Apr. 3, 1916.

Four MONTHS AFTER DATE I PROMISE TO PAY TO
Henry J. Walker OR ORDER
Five hundred and no/100 DOLLARS
AT National City Bank of New York.
FOR VALUE RECEIVED, with interest at 6%.
DUE Aug. 3. 1916. John B. Anderson.

178. Security. — One who loans money wishes, of course, to become reasonably secure from loss; that is, reasonably sure that the money will be repaid. If the financial standing of the borrower is good, it may be that his own signature is sufficient security. Sometimes two or more sign the same note. Any one of those who sign the note thus becomes responsible for its payment. This method is known as **personal security**. Sometimes the repayment is secured by the borrower making over certain real estate or personal property to the one of whom the money is borrowed. This is called a **mortgage**, and becomes void when the money is repaid.

179. Date of maturity. — To find the date upon which a note is due, the usual custom is to count forward the number

of months or days according as months or days are stated in the note. Thus, the note shown in § 177 fell due on Aug. 3, 1916.

In computing the interest due, when the time is less than one year, 30 days are considered an interest month, and 360 days, a year.

180. Interest problems. — The only problem with which the business man is concerned is to find the interest (or *amount*, *i.e.* interest plus the principal) when the time and rate are given. From the meaning of interest it is seen that,

To obtain the interest for any period of time, multiply the interest for one year by the number of years.

NOTE. — In actual business practice, interest is usually collected annually, semiannually, or quarterly. Necessity for finding interest, then, for more than a year rarely arises. A few problems, however, are given here in which the time exceeds 1 year.

EXERCISES IN INTEREST

1. At 5 %, what is the interest of $ 400 for 1 year? For 6 months? For 2 years? For 2 years 6 months?

2. At 6 %, what is a year's interest of $ 500? 8 months'?

3. At 5 %, what is a year's interest of $ 800? 9 months'?

At 6 %, find the interest of:

4. $ 500 for 6 mo.
5. $ 300 for 8 mo.
6. $250 for 1 yr.
7. $150 for 10 mo.

At 5 %, find the interest of:

8. $800 for 6 mo.
9. $ 200 for 4 mo.
10. $ 300 for 8 mo.
11. $400 for 3 mo.
12. $900 for 9 mo.
13. $700 for 9 mo.

14. At 5 % find the interest of $1950 for 1 yr. 4 mo.

SOLUTION

$$\frac{4}{3} \times \frac{5}{100} \times \$1950 = \frac{\$390.00}{3} = \$130.$$

EXPLANATION. — $\frac{5}{100} \times$ \$1950 gives the interest for 1 yr. But 1 yr. 4 mo. = $1\frac{1}{3}$ or $\frac{4}{3}$ yr. Hence $\frac{4}{3} \times \frac{5}{100} \times$ \$1950 gives the total interest.

NOTE. — This is called the **General Method** of finding interest.

Find the interest of:

15. \$1560 at 5 % for $1\frac{1}{2}$ yr.
16. \$2830 at 6 % for 8 mo.
17. \$1960 at 5 % for 9 mo.
18. \$3850 at 6 % for 10 mo.
19. \$3260 at 5 % for 11 mo.
20. \$1640 at $5\frac{1}{2}$ % for 7 mo.
21. \$1784 at 6 % for 8 mo.
22. \$3620 at 4 % for 11 mo.
23. \$4200 at 4 % for 1 yr. 2 mo.
24. \$1950 at 5 % for $\frac{3}{4}$ yr.
25. \$3840 at 6 % for 5 mo.
26. \$2680 at 5 % for 3 mo.
27. \$4900 at 4 % for 5 mo.
28. \$2650 at 5 % for 1 yr. 3 mo.
29. \$4820 at 5 % for 1 yr. 7 mo.
30. \$5460 at 6 % for 1 yr. 5 mo.
31. \$6300 at 5 % for 11 mo.
32. \$7200 at 6 % for 6 mo.
33. Find the interest of \$8460 at 6 % for 115 days.

SOLUTION

$$\frac{115}{360} \times \frac{6}{100} \times \$8460 = \$162.15.$$

EXPLANATION. — For periods less than 1 yr., 360 days are considered a year. Hence 115 da. = $\frac{115}{360}$ yr.

Find the interest of:

34. \$500 at 5 % for 117 da.
35. \$650 at 6 % for 201 da.
36. \$720 at 6 % for 98 da.
37. \$640 at 5 % for 109 da.
38. \$960 at 5 % for 180 da.
39. \$730 at 6 % for 95 da.

40. \$1780 at 5 % for 41 da.
41. \$9680 at 4 % for 86 da.
42. \$1780 at 5 % for 72 da.
43. \$6800 at 4 % for 26 da.
44. \$1960 at 6 % for 36 da.
45. \$2800 at 6 % for 108 da.

Find the interest of:

46. \$4500 at 6 % for 2 mo. 20 da.

SOLUTION

$$\frac{80}{360} \times \frac{6}{100} \times \$4500 = \$60.$$

EXPLANATION. — 2 mo. 20 da. = 80 da. 80 da. = $\frac{80}{360}$ yr.

47. \$9460 at 6 % for 6 mo. 12 da.
48. \$1350 at 5 % for 8 mo. 20 da.
49. \$3460 at 4 % for 9 mo. 10 da.
50. \$4860 at 4 % for 7 mo. 15 da.
51. \$6500 at 5 % for 11 mo. 10 da.
52. \$1400 at 6 % for 6 mo. 2 da.
53. \$3050 at 6 % for 4 mo. 20 da.
54. \$5100 at 5 % for 2 mo. 5 da.
55. \$1950 at 4 % for 2 mo. 12 da.
56. \$2500 at 6 % for 2 mo. 20 da.
57. \$4675 at 4 % for 3 mo. 15 da.
58. \$3700 at 5 % for 8 mo. 8 da.
59. \$1280 at 6 % for 4 mo. 17 da.
60. \$6400 at 5 % for 1 mo. 21 da.

61. Find the interest on a note of \$1250, dated June 7, 1916, at 6 %, due Jan. 15, 1917.

WORK

1917	1	15
1916	6	7
	7	8

$$\frac{\overset{109}{\cancel{218}}}{\underset{30}{\cancel{\underset{\cancel{60}}{\cancel{360}}}}} \times \frac{\cancel{6}}{100} \times \$1250 = \frac{\$136.250}{3} = \$45.42.$$

7 mo. 8 da. = $\frac{218}{360}$ yr.

62. What is the value, on the 5th of January, 1916, of a 6% note for \$1475, dated June 18, 1915?

63. What must I pay on the 8th of December, 1917, to redeem my note for \$960, dated February 19, 1917, with interest at 6%?

64. How much is due July 20, 1917, on a note for \$1725, given October 6, 1916, with interest at 5%?

65. Find the interest of \$7685, from April 25 to November 11, 1917, at 5%.

66. What is the amount of the principal and interest of \$2850 loaned at 5% from July 2, 1916, until December 25, 1916?

67. I took a 6% mortgage of \$6000 for a loan made on February 28, 1916. How much interest will have accrued by November 30, 1916?

68. If you loan a man \$1540 at 5%, on March 12, 1916, and he pays it back to you on January 20, 1917, how much must he pay you in settlement?

69. A man bought a city lot for \$2500, paying \$1000 cash and giving his note, secured by a mortgage on the property, at 6%, payable in one year, for the balance. How much must he pay at the end of the year to cancel the note and mortgage?

70. A man bought a house for \$9500, paying \$3500 cash, and giving his note (and mortgage) for the balance at 5%. How much a year is his interest?

71. Instead of paying rent of \$40 a month, a man buys a house for \$6500. He pays \$2500 cash and gives his note at 6% for the balance. If his taxes are \$70 a year, how much less does he pay out annually in interest and taxes than he paid in rent?

72. A man who is paying \$50 per month rent has \$3000 in the savings bank earning 4% interest. Can he save money by buying a \$7000 house with the \$3000 and a 6% note, if taxes are \$65 a year, and other expenses are \$80? How much per year will he gain or lose by buying the house?

SUGGESTION. — How much a year is he now paying out in excess of the interest from the bank? How much will he pay out yearly in interest, taxes, and repairs? Which is more, and how much?

73. A man who has \$4000 invested in shares of a building and loan association that pays him 5% interest annually, rents a flat for \$45 a month. How much will he gain or lose yearly by buying a house for \$6500 with the \$4000 and a 6% note, if all expenses on the house are \$175 a year?

181. Short methods of finding interest. — The person who has to compute interest constantly uses a *book of tables*. There is but little need, then, of learning short methods, since the *general method* is sufficient for the average person, who rarely computes interest. Two methods in very common use, easily derived from the general method, are given below.

BANKERS' METHOD

1. Find the interest of \$1750 at 6% for 113 days.

SOLUTION BY GENERAL METHOD

$$\frac{113}{\underset{60}{\cancel{360}}} \times \frac{\cancel{6}}{100} \times \$1750 = \frac{\$197.750}{6} = \$32.96.$$

From the above solution it is evident that:

To find the interest on any principal at 6% *for any number of days, multiply the principal by the number of days, point off three more decimal places, and divide by* 6.

This is called the **Bankers' Method.**

Find the interest at 6 % of:

2. $1850 for 95 days.
3. $2680 for 107 days.
4. $5400 for 113 days.
5. $9680 for 34 days.
6. $1160 for 46 days.
7. $1560 for 19 days.
8. $2340 for 31 days.
9. $6300 for 35 days.
10. $7340 for 16 days.
11. $9630 for 58 days.
12. Find the interest of $3450 at 6 % for 48 days.

SOLUTION

$$\begin{array}{r} \$3450 \\ 8 \\ \hline \$27.600 \end{array}$$

EXPLANATION. — Instead of multiplying by 48 and dividing by 6, work is saved by dividing 48 by 6 and multiplying by the result.

Find the interest at 6 % of:

13. $3460 for 18 days.
14. $5650 for 24 days.
15. $3760 for 12 days.
16. $9340 for 36 days.
17. $7360 for 42 days.
18. $8600 for 30 days.
19. $7520 for 54 days.
20. $1650 for 72 days.
21. $1780 for 48 days.
22. $1950 for 24 days.
23. $3200 for 78 days.
24. $5400 for 18 days.
25. $6300 for 42 days.
26. $3500 for 84 days.
27. Find the interest of $960 at 5 % for 42 days.

SOLUTION

$$\begin{array}{r} \$960 \\ 7 \\ \hline 6)\overline{6.720} \\ 1.120 \\ \hline \$5.60 \end{array}$$

EXPLANATION. — $6.72 is the interest at 6 %. $1.12 is the interest at 1 %. $5.60 is the interest at 5 %.

28. Study the solution in Exercise 27 and show a similar way of finding interest at 7 %.

29. Find the interest of $1780 at 7 % for 56 days.

Find the interest at 5% *of:*

30. \$8400 for 36 days.

31. \$7800 for 48 days.

32. \$7500 for 18 days.

33. \$5400 for 24 days.

34. \$6300 for 54 days.

35. \$7800 for 66 days.

36. \$1750 for 114 days.

37. \$2680 for 135 days.

38. \$9300 for 126 days.

39. \$3750 for 96 days.

40. \$8430 for 110 days.

41. \$3870 for 117 days.

ALIQUOT PART METHOD

1. Find the interest of \$1960 at 6% for 60 days.

SOLUTION BY GENERAL METHOD

$$\frac{\cancel{6}\cancel{0}}{\cancel{3}\cancel{6}\cancel{0}} \times \frac{\cancel{6}}{100} \times \$1960 = \$19.60.$$

From the above solution it is evident that:

To find the interest of any principal at 6% *for* 60 *days, move the decimal point two places to the left.*

2. Find the interest of \$1760 at 6% for 96 days.

SOLUTION

$$\begin{array}{r} \$17.60 \\ 8.80 \\ 1.76 \\ \hline \$28.16 \end{array}$$

EXPLANATION. — By pointing off two decimal places, \$17.60 is the interest for 60 days. $\frac{1}{2}$ of \$17.60 is the interest for 30 days. $\frac{1}{10}$ of \$17.60 is the interest for 6 days.

This is called the **Aliquot part method**.

Find the interest at 6% *of:*

3. \$960 for 66 days.

4. \$985 for 80 days.

5. \$840 for 90 days.

6. \$960 for 96 days.

7. \$750 for 72 days.

8. \$640 for 75 days.

9. \$940 for 86 days.

10. \$820 for 126 days.

11. \$750 for 132 days.

12. \$870 for 93 days.

13. \$650 for 70 days.

14. \$1250 for 30 days.

15. \$1050 for 36 days.

16. \$1500 for 45 days.

At 5%, find the interest of:

17. \$840 for 75 days.

SOLUTION	EXPLANATION
\$8.40	int. at 6% for 60 da.
2.10	int. at 6% for 15 da.
6)10.50	int. at 6% for 75 da.
1.75	int. at 1% for 75 da.
\$8.75	int. at 5% for 75 da.

18. \$820 for 70 days.

19. \$950 for 63 days.

20. \$720 for 93 days.

21. \$875 for 72 days.

22. \$970 for 85 days.

23. \$450 for 20 days.

24. \$120 for 50 days.

25. \$830 for 45 days.

26. \$650 for 45 days.

27. \$810 for 75 days.

28. \$960 for 70 days.

29. \$875 for 110 days.

30. \$760 for 96 days.

31. \$940 for 57 days.

182. Partial payments on a note. — In general, modern business custom will not allow an advance payment upon a note unless it is so stipulated in the note. Such stipulations are usually to the effect that such payments are to be made at the end of interest-paying periods. There was a time, however, when borrowing among individuals was more common than now, and when the payments of interest and any other payments were more irregular. To govern the final settlements in such transactions, the United States Supreme Court decreed that:

Partial payments of notes must first be used to cancel the interest due. Any balance remaining may be used to lessen the principal. If, however, the payment is too small to pay

the interest due, the unpaid interest must not be used to increase the principal, which must never represent more than the money actually and previously due.

PROBLEMS

1. What is due Jan. 16, 1918, on a note of $1600 drawing 6% interest, given May 3, 1914, the following payments having been made: May 3, 1915, $200; Dec. 18, 1915, $80; June 25, 1916, $300; April 16, 1917, $450?

2. What is due July 23, 1918, on a note of $2400 drawing 5% interest, given Jan. 4, 1913, the following payments having been made: Sept. 4, 1913, $300; July 19, 1914, $50; Jan. 4, 1915, $300; April 16, 1916, $250; Dec. 26, 1917, $500?

NOTE. — Since the payment on July 19, 1914, was insufficient to pay the interest then due, the principal left on Sept. 4, 1913, will have to be used again. Work will be saved by finding the amount from Sept. 4, 1913, to Jan. 4, 1915, and then deducting $350, or both payments.

3. A note of $3500, dated Aug. 15, 1915, interest 6%, has the following indorsements: Aug. 15, 1916, $500; Feb. 15, 1917, $100; July 10, 1917, $400; Dec. 15, 1917, $1500. What is due Aug. 15, 1918?

4. A note of $6500, dated April 3, 1915, interest 6%, has the following indorsements: April 3, 1916, $1500; Aug. 10, 1916, $300; Feb. 7, 1917, $450; July 10, 1917, $1640. What is due Nov. 10, 1917?

5. A note of $1750, dated Nov. 6, 1912, interest 5%, has the following indorsements: Nov. 6, 1913, $500; April 10, 1914, $300; Aug. 20, 1914, $100; Dec. 20, 1914, $150; April 15, 1915, $200. What is due Feb. 20, 1916?

6. Find what is due on Nov. 10, 1917, on a note of $5000, at 6%, dated May 15, 1914, with the following indorsements:

May 15, 1915, $800; Sept. 1, 1915, $200; Feb. 15, 1916, $100; June 20, 1916, $200; May 6, 1917, $300.

7. The following indorsements were made on a note of $4500, at 5 %, dated June 10, 1914: June 10, 1915, $200; Aug. 10, 1915, $200; Dec. 1, 1915, $300. What is due Oct. 15, 1916?

8. A note of $6500 dated June 20, 1913, interest 6 %, has the following indorsements: June 20, 1914, $300; July 10, 1914, $200; Dec. 15, 1914, $600; May 20, 1916, $1000. What is due Dec. 20, 1916?

BANKING

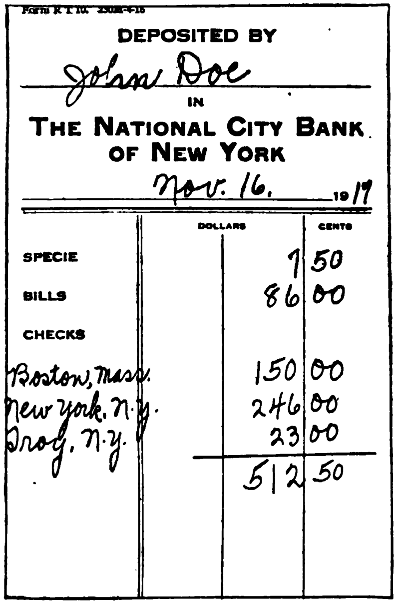
183. A bank. — A **bank** is an institution where money is deposited for safe-keeping, and from which it may be withdrawn when wanted; and it is a place where money is loaned on personal or other security.

Banks, in the commercial sense, are classified under three heads: (1) banks of deposit; (2) banks of discount; (3) banks of circulation. All or any two of these functions may be exercised by the same association. As to their general kinds, banks are national, state, and savings banks, also private banks and loan and trust companies.

184. The capital of a bank. — Before a bank can organize to keep deposits and to loan money, it must have money of its own. This is called the **capital of a bank.** This capital is usually furnished by a large number of persons called the **stockholders,** and cannot be withdrawn by them but is held as a guarantee that money deposited will be paid when called for. The large part of the money in any bank is that of the depositors. It is from the loans of these deposits that the bank earns money.

185. Opening an account. — When making your first deposit with a bank, you will be given a **pass book** in which

your deposit is entered to your credit, and a **check book** for writing out orders on the bank to pay out money from your deposit. You will also be asked to write your name in a **signature book**, or on a **signature card**, in order that the signature to future checks or other indorsements may be identified. Hence your signature should be written as you intend to write it ever after in signing or indorsing checks, etc.



DEPOSITED BY

John Doe

IN

THE NATIONAL CITY BANK OF NEW YORK

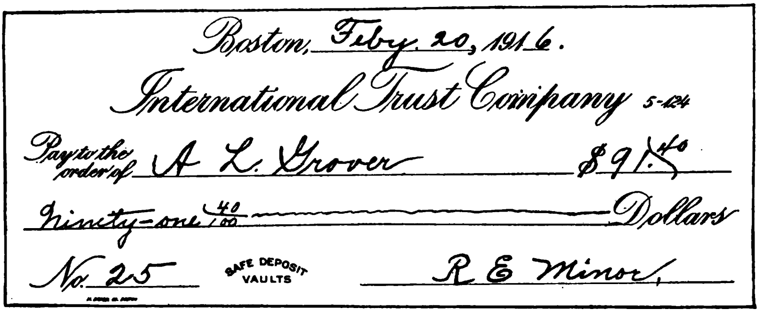
Nov. 16, 1917

	DOLLARS	CENTS
SPECIE	1	50
BILLS	86	00
CHECKS		
Boston, Mass.	150	00
New York, N. Y.	246	00
Troy, N. Y.	23	00
	512	50

186. Ticket of deposit. — The deposit ticket is merely a memorandum of your deposits. It should be filled out by the **depositor** and checked by the **teller**. It serves as a sort of receipt for the transaction, for, in the future, if any question arises as to the deposit, the ticket in your own handwriting and checked by the teller is an unquestionable proof of the transaction.

187. Making out a check. — In making out a check one should seek to protect himself from any dishonest future holder. Hence a check should be written in ink. Begin to write the amount as far to the left as possible and fill the remainder of the space intended for the amount with a heavy line in order that nothing else may be added.

USUAL MANNER OF DRAWING A CHECK


Boston, Feby. 20, 1916.
International Trust Company 5-124
Pay to the order of A. L. Grover $91 40/100
Ninety-one 40/100 Dollars
No. 25 SAFE DEPOSIT VAULTS R. E. Minor.

The words "the order of" make the check **negotiable.** That is, by indorsement it may be transferred by Mr. Grover to another person for collection. All forms of checks, notes, drafts, and certificates of deposit that are transferable by indorsement are called **negotiable paper.**

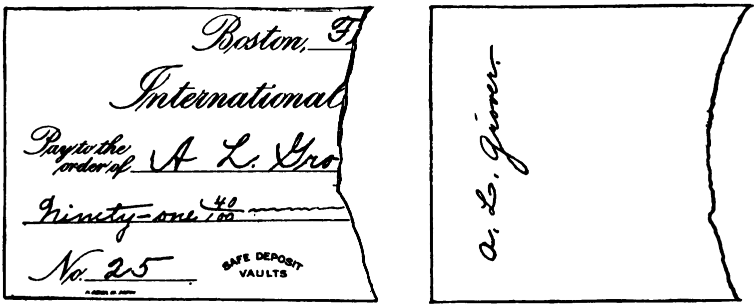
In case the maker of a check is unknown to the party to whom it is payable, the maker may have it **certified** by his bank to insure its acceptance. Thus, in the check shown above, if R. E. Minor is unknown to A. L. Grover, it is well for Mr. Minor to have his bank certify or accept it. This is done by stamping across the face of it with the stamp shown in the margin. This is then called a **certified check** and means that the bank acknowledges that an amount sufficient to pay the check has been set aside and charged to Mr. Minor's account. That is, the bank becomes responsible for the payment.

No. 468
CERTIFIED
AT THE REQUEST OF THE MAKER
FEB. 20, 1916
INTERNATIONAL TRUST CO.
BOSTON, MASS.
J. A. Smith CASHIER.

188. Indorsing a check. — Checks may be made payable to "order" or to "bearer." If made to "order", the **payee** (the one to whom the money is payable) must be known to

be the proper person to receive the money, and must **indorse** it by writing his name, as it appears in the check, across the back. As the teller turns the check over toward himself the indorsement should be across the left end.

USUAL FORM OF INDORSEMENT



In case the payee is depositing the check through some other party, he should indorse it by writing "pay to the order of" the party, before signing his name; then if lost or stolen, it cannot be cashed by the parties having thus obtained it. This is called a **restrictive indorsement.**

Always present checks for payment as soon as possible. The **drawer** of a check prefers to have it paid without delay so as not to have to keep track of outstanding checks. Loss may result from keeping a check an undue length of time, for the drawer may have closed his account with the bank and forgotten that the check was yet unpaid.

189. Certificate of deposit. — If one does not wish to open an account with a bank, but wishes merely to make a deposit for safe-keeping, the amount is not entered in a pass book but a **certificate of deposit** is given for the amount. This amount is not subject to check, but is withdrawn by presenting the certificate. A low rate of interest is usually paid upon such deposits when left for a definite period.

CERTIFICATE OF DEPOSIT

Certificate of Deposit

The Market Exchange Bank Co.

Columbus, O. Feby. 9, 1916. No. 525

Richard Roe has deposited in this Bank $1,000 00/100

One thousand and no/100 Dollars in current funds

payable to the order of Himself

in current funds on the return of this Certificate properly endorsed.

NOT SUBJECT TO CHECK. John Doe Cashier

190. Loans and discounts. — When banks loan money for short periods, the interest is usually collected in advance. This interest paid in advance is called **bank discount.**

Banks discount negotiable notes for their customers, charging interest upon the *maturity value* of the note (to the nearest dollar) for the time the note then has to run.

USUAL MANNER OF DRAWING A NOTE

$600.00/100 Boston, Mass. Feby. 10, 1917

Three months after date I promise to pay

to the order of International Trust Company of Boston,

Six hundred 00/100 Dollars

Payable at International Trust Co., Boston, Mass.

Value received

No. 34 Due May 10, 1917 J. L. Turner

Mr. Turner receives, on Feb. 10, 1916, $600, immediately pays back the discount (interest) of $9.00, and carries away or deposits on his

account, the balance, $591.00, called the proceeds. On April 10, 1916, he must pay back to the bank $600.

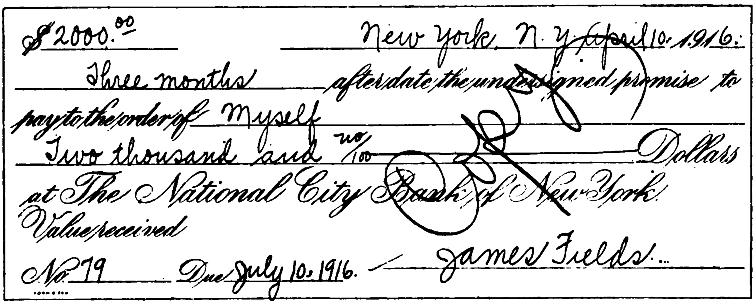
The **term of discount** is taken by days instead of by months. Thus the term of discount from Feb. 10 to March 10 (except leap years) is 28 days; from Aug. 10 to Sept. 10 (any year) it is 31 days; etc.

In some cases the interest on a bank loan is not collected in advance, but the borrower pays the interest with the face of the note at maturity. In that case the rate of interest is named in the note.

Loans are sometimes made subject to **call** or **demand** instead of maturing at a fixed definite date. These notes are drawn for the exact sum loaned and the interest is collected monthly or quarterly.

191. A form of note payable to self. — There are seasons when depositors at a bank may not have sufficient funds to meet certain needs. At such times a note of the following form is sometimes given.

A COMMON FORM OF NOTE TO SELF



$2000.00 New York, N. Y. April 10, 1916.
Three months after date the undersigned promise to
pay to the order of Myself
Two thousand and no/100 Dollars
at The National City Bank of New York
Value received
No. 79 Due July 10, 1916. James Fields.

This note being payable to himself, Mr. Fields indorses it and usually has it further indorsed by a second party.

If this note is accepted, the bank computes the interest (discount) for 90 days at the regular rate, subtracts it from $2000, and places the balance on Mr. Fields' checking account, *e.g.* if the rate is 6 %, the bank places to Mr. Fields' credit $1970. On the day the note is due, the bank pays itself $2000 from the funds Mr. Fields has on deposit, just as it would pay any check written by him, and returns the canceled note together with his "paid" checks, on the regular day for giving customers a statement of their accounts. (Statements are usually given on the first of the month.)

PROBLEMS

1. Find the discount at 6 % on a note of $2450 to run 90 days.

2. Find the discount at 6 % on a note of $1850 to run from June 15, 1917, to Sept. 15, 1917.

3. Find the discount at 6 % on a note dated June 20, 1917, to run 3 months.

4. Find the discount on a note for the same amount at the same rate, and for 3 months, if dated Jan. 20, 1917.

5. What are the proceeds on a note of $1600, dated Jan. 10, 1917, to run 4 months, rate of discount 6 %?

6. If a note of $1200 to run one year at 5 % interest is discounted at 6 % 45 days before it is due, what are the proceeds?

192. Protests. — If a note is not paid at maturity, or if a check comes in and there are not enough funds to the maker's credit to pay it, the note or check is **protested**. The *protest* is a legal document drawn up by a notary public, giving notice of non-payment.

Thus, to illustrate, a bank cannot honor a check that comes in from a maker who has not the funds to his credit to pay it. But the check is turned over to a notary public who again presents the check to the bank at the close of

the business day for payment. If payment is refused him, he protests it. That is, he notifies all parties whose names appear on the check and attaches a certificate to the dishonored paper stating what he has done. This protest makes all indorsers as well as the maker reponsible for payment.

BANK PROTEST FORM

UNITED STATES OF AMERICA }
State of New Jersey. } *s.s.*

On the *tenth* day of *June* in the year of our Lord, One Thousand Nine Hundred *sixteen* at the request of the FIRST NATIONAL BANK OF TRENTON, the holder of the original check, I, *A. L. Jones,* Notary Public duly commissioned and sworn, residing at Trenton, did present the same check to *R. E. Smith* Cashier of said Bank, at their Banking House, and demanded of him payment of the same, to which he replied that he could not *for want of funds of the Maker;* wherefore I duly give notice to the endorsers of the non-payment thereof. Wherefore I, the said notary, at the request aforesaid, did protest, and by these presents do publicly and solemnly protest, as well against said Drawer and Endorser of said check as against all others whom it doth or may concern, for exchange, re-exchange, and all costs, charges, and damages already incurred, and to be hereafter incurred, for want of payment of said check.

This done and protested at Trenton, in the County of Mercer, State of New Jersey.

In witness whereof, I have hereunto set my hand and affixed my Notatorial Seal.

A. L. Jones,

Notary Public.

193. Finding the term of discount. — Since banks receive interest for the exact number of days a note has to run, the time can be much more quickly found by a table than by adding the days of each month.

THIS TABLE SHOWS THE NUMBER OF DAYS FROM ANY DAY OF ANY MONTH TO THE SAME DAY OF ANY MONTH NOT MORE THAN ONE YEAR LATER.

FROM	To Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

PROBLEMS

By the table find how many days:

1. From Jan. 10, 1917, to June 5, 1917.
2. From May 15, 1918, to Aug. 21, 1918.
3. From Feb. 3, 1916, to Apr. 18, 1916.
4. From June 1, 1917, to Dec. 15, 1917.
5. From Dec. 10, 1916, to April 13, 1917.

6. From Oct. 7, 1916, to May 30, 1917.

7. From Nov. 21, 1917, to Feb. 27, 1918.

By the Bankers' Method, page 215, find the discount on:

8. \$1350 for 42 da. at 6%.
9. \$2680 for 57 da. at 6%.
10. \$2850 for 83 da. at 6%.
11. \$2930 for 50 da. at 6%.
12. \$2930 for 60 da. at 5%.
13. \$5400 for 13 da. at 6%.
14. \$3640 for 84 da. at 5%.
15. \$9680 for 34 da. at 5%.
16. \$1260 for 112 da. at 6%.
17. \$1780 for 107 da. at 5%.
18. \$2240 for 103 da. at $5\frac{1}{2}$%.
19. \$3680 for 72 da. at $5\frac{1}{2}$%.

At sight give the discount on:

20. \$1000 for 60 da. at 6%.
21. \$1550 for 60 da. at 6%.
22. \$1800 for 90 da. at 6%.
23. \$2400 for 30 da. at 6%.
24. \$3200 for 90 da. at 6%.
25. \$2800 for 75 da. at 6%.
26. \$2400 for 80 da. at 6%.
27. \$3600 for 60 da. at 5%.
28. \$4200 for 60 da. at 5%.
29. \$5400 for 60 da. at 5%.
30. \$3000 for 90 da. at 5%.
31. \$1800 for 90 da. at 5%.
32. \$2400 for 80 da. at 5%.
33. \$3600 for 80 da. at 5%.

34. If you borrow \$875 at a bank for 112 da. at 6%, what will the discount be? What will the proceeds be, *i.e.* how much will you actually receive? How much do you pay the bank at the end of 112 days?

35. Write a note promising to pay yourself \$1500 in 90 days. Date it Aug. 12, 1917.

36. When is the note which you have written due?

37. Have the note indorsed by some one besides yourself.

38. At 6% discount, with how much will your bank credit you for the note?

39. When and for how much will your account be charged for this note?

40. Write a note promising to pay the First National Bank of Burlington, Vt., for a loan of $1250 for 75 days, dated Nov. 10, 1917.

41. Find the date of maturity and the discount at 6%.

42. Compare Bank Discount and Simple Interest, by supposing $1500 borrowed on each plan for 123 days at 6%. That is, show how much you would pay in each case and how much money you would get to use in each case for the 123 days.

43. A note of $1800, dated March 6, 1916, to run 8 months at $5\frac{1}{2}$%, was discounted Sept. 16, 1916, at 6%. Find the proceeds.

EXPLANATION.—The holder of the note was to receive $1800 plus $66 interest, on Nov. 6, 1916. As he wished to borrow money before the note was due to him, he presented the note at the bank on September 16, 1916, as security for a loan. Int. for 8 mo. at $5\frac{1}{2}$% was added to the face to get the maturity value. The note is due Nov. 6. Time from Sept. 16 to Nov. 6 is 51 days. Int. of $1866 for 51 da. at 6% gives the discount. The discount subtracted from the maturity value gives the proceeds.

SOLUTION

Face	= $1800
Int. 8 mo. at $5\frac{1}{2}$%	= 66
Maturity value	= $1866
Discount for 51 da.	= 15.86
Proceeds	= $1850.14

The holder of the note, then, received from the bank $1850.14 on Sept. 16, 1916. On Nov. 6, 1916, the maker of the note must pay *the bank* $1866. If he fails to do so, this sum must be paid to the bank by the man who placed the note in the bank, and he himself collects from the maker of the note if he can.

44. A note of $1750, given Jan. 15, 1917, to run one year at $5\frac{1}{2}$%, was discounted at 6% on Nov. 28, 1917. Find the proceeds.

45. A note of $1800, given Aug. 28, 1917, to run 8 months at 6 %, was discounted on Dec. 15, 1917, at 6 %. Find the proceeds.

46. A note of $2750, given Nov. 30, 1917, to run 6 months at 5 %, was discounted on April 2, 1918, at 6 %. Find the proceeds.

47. A note of $3640, given May 6, 1917, to run 10 months at 5 %, was discounted Feb. 10, 1918, at 6 %. Find the proceeds.

Find proceeds of notes under these conditions:

	Face	Int. Rate	Date of Note	Time to Run	Discounted at 6% on
48.	$1650	5%	Jan. 10, 1917	8 months	Aug. 15, 1917
49.	2400	5%	June 17, 1917	6 months	Oct. 6, 1917
50.	3750	5½%	July 10, 1918	1 year	April 7, 1919
51.	4200	5½%	Dec. 20, 1918	1 year	Nov. 13, 1919
52.	1350	6%	May 16, 1918	6 months	Oct. 20, 1918
53.	1650	6%	April 7, 1918	6 months	Sept. 1, 1918
54.	3260	5%	Feb. 6, 1918	10 months	Oct. 13, 1918
55.	4500	5%	May 11, 1918	1 year	March 21, 1919

56. A man sold his farm for $12,500, getting 40 % of it in cash and taking a note due in 120 days without interest for the balance. If he discounted the note at 6 % on the day it was made, how much did he get for the farm?

57. If you have a note for $2500, bearing 5½ % interest, dated Dec. 20, 1916, to rur 1 year, what will a bank give you for it on Sept. 10, 1917, if the rate of discount is 6 %?

58. In order to receive $1000 from a bank charging 6 % discount, for a note dated May 10, 1917, to run 4 months, what must be the face of the note you discount?

194. Finding interest by tables. — When one has interest or discount to compute constantly, time is saved by using **interest tables.**

EXERCISES

1. By the tables, find the interest of $2400 for 135 days at 6%.

SOLUTION

135 da. = 4 mo. 15 da.		
Int. of $2000 for 15 da.	=	$5.00
Int. of $400 for 15 da.	=	1.00
Int. of $2000 for 4 mo.	=	40.00
Int. of $400 for 4 mo.	=	8.00
		$54.00

By the tables, find the interest at 6 % *of:*

2. $850 for 18 da.
3. $970 for 33 da.
4. $680 for 42 da.
5. $910 for 47 da.
6. $780 for 68 da.
7. $3400 for 71 da.
8. $5600 for 83 da.
9. $7200 for 91 da.
10. $8600 for 113 da.
11. $7840 for 69 da.
12. $645 for 27 da.
13. $837 for 39 da.
14. $972 for 41 da.
15. $895 for 111 da.
16. $908 for 117 da.
17. $675 for 175 da.
18. $861 for 134 da.
19. $276 for 142 da.

Find the bank discount and proceeds of:

20. $450 from Aug. 6, 1917, to Nov. 15, 1917, at 6%.
21. $960 from Jan. 3, 1918, to June 18, 1918, at 6%.
22. $4300 from March 18, 1918, to June 3, 1918, at 6%.
23. $1850 from Oct. 1, 1917, to Jan. 19, 1918 at 6%.
24. $3800 from Aug. 30, 1918, to Nov. 3, 1918, at 6%.
25. $4250 from Nov. 24, 1917, to Feb. 6, 1918, at 6%.

INTEREST TABLE. INTEREST AT 6%

DA.	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000	DA.
1	0.017	0.033	0.050	0.067	0.083	0.100	0.117	0.133	0.150	0 167	1
2	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	0.333	2
3	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	3
4	0.067	0.133	0.200	0.267	0.333	0.400	0.467	0.533	0.600	0.667	4
5	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	5
6	0.100	0.200	0.300	0.400	0.500	0 600	0.700	0.800	0.900	1.000	6
7	0.117	0.233	0.350	0.467	0.583	0.700	0.817	0.933	1.050	1.167	7
8	0.133	0.267	0.400	0.533	0.667	0.800	0.933	1.067	1.200	1.333	8
9	0 150	0.300	0.450	0.600	0.750	0.900	1.050	1.200	1.350	1.500	9
10	0.167	0.333	0.500	0.667	0.833	1.000	1.167	1.333	1.500	1.667	10
11	0.183	0.367	0.550	0.733	0.917	1.100	1.283	1.467	1.650	1.833	11
12	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000	12
13	0.217	0.433	0.650	0.867	1.083	1.300	1.517	1.733	1.950	2.167	13
14	0.233	0.467	0.700	0.933	1.167	1.400	1.633	1.867	2.100	2.333	14
15	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	15
16	0.267	0.533	0.800	1.067	1.333	1.600	1.867	2.133	2.400	2.667	16
17	0.283	0.567	0.850	1.133	1.417	1.700	1.983	2.267	2.550	2.833	17
18	0.300	0.600	0.900	1.200	1.500	1.800	2.100	2.400	2.700	3.000	18
19	0.317	0.633	0.950	1.267	1.583	1.900	2.217	2.533	2.850	3.167	19
20	0.333	0.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	20
21	0.350	0.700	1.050	1.400	1.750	2.100	2.450	2.800	3.150	3.500	21
22	0.367	0.733	1.100	1.467	1.833	2.200	2.567	2.933	3.300	3.667	22
23	0.383	0.767	1.150	1.533	1.917	2.300	2.683	3.067	3.450	3.833	23
24	0.400	0.800	1.200	1.600	2,000	2.400	2.800	3.200	3.600	4.000	24
25	0.417	0.833	1.250	1.667	2.083	2.500	2.917	3.333	3.750	4.167	25
26	0.433	0.867	1.300	1.733	2.167	2.600	3.033	3.467	3.900	4.333	26
27	0.450	0.900	1.350	1.800	2.250	2.700	3.150	3.600	4.050	4.500	27
28	0.467	0.933	1.400	1.867	2.333	2.800	3.267	3.733	4.200	4.667	28
29	0.483	0.967	1.450	1.933	2.417	2.900	3.384	3.867	4.350	4.833	29

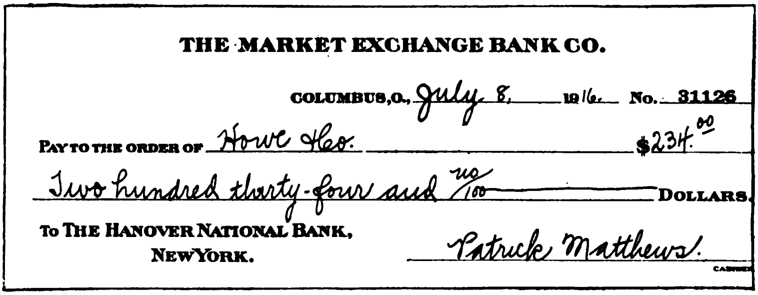
MO.	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000	MO.
1	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	1
2	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	2
3	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.000	13.500	15.000	3
4	2.000	4.000	6.000	8.000	10.000	12.000	14.000	16.000	18.000	20.000	4
5	2.500	5.000	7.500	10.000	12.500	15.000	17.500	20.000	22.500	25.000	5
6	3.000	6.000	9.000	12.000	15.000	18.000	21.000	24.000	27.000	30.000	6
7	3.500	7.000	10.500	14.000	17.500	21.000	24.500	28.000	31.500	35.000	7
8	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000	8
9	4.500	9.000	13.500	18.000	22.500	27.000	31.500	36.000	40.500	45.000	9
10	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	10
11	5.500	11.000	16.500	22.000	27.500	33.000	38.500	44.000	49.500	55.000	11

YR.	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000	YR.
1	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	60.00	1
2	12 00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	120.00	2
3	18.00	36.00	54.00	72.00	90.00	108.00	126.00	144.00	162.00	180.00	3
4	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	240.00	4
5	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	300.00	5

195. Bank drafts: exchange. — A **draft** is a written order from one bank to another bank to pay a specified sum to a third party. Drafts are usually known in the commercial world as **exchange.**

A draft is bought at a bank by any person wishing to make a remittance to some distant place to save the transmission of the actual money. The sender pays the face of the draft plus a small fee charged by the bank for the accommodation.*

USUAL FORM OF BANK DRAFT



THE MARKET EXCHANGE BANK CO.

COLUMBUS, O., July 8, 1916. No. 31126

PAY TO THE ORDER OF Howe & Co. $234.00

Two hundred thirty-four and no/100 DOLLARS.

TO THE HANOVER NATIONAL BANK,
NEW YORK.

Patrick Matthews.
CASHIER

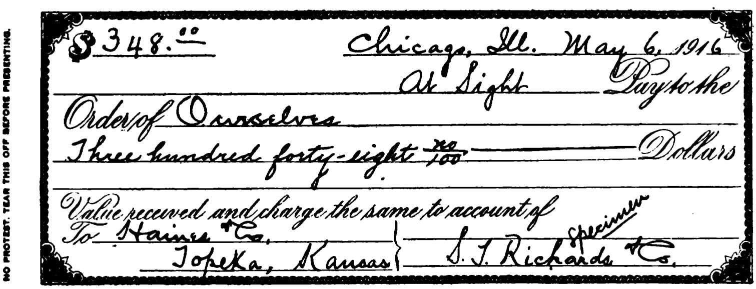
* When selling drafts, banks are ordering money paid out which they have on deposit in the banks of large commercial centers. This would soon exhaust their balances at these banks and thus require the large expense of frequent transfers of large amounts of actual money, were it not for the fact that they are also constantly receiving drafts on other banks from their customers which are mailed to the banks upon which they issue drafts. In this way they keep up their deposits without often having to send money when the accounts between these banks are regularly balanced. As there must always be a balance in favor of the bank upon which the drafts are drawn, the loss of interest and the necessary office expenses make it necessary for banks to charge a small fee above the actual face of the draft, say 10 cents for sums less than $100, 25 cents for sums from $100 to $500, etc. This is called a **premium.** The premium upon large drafts is usually some small per cent of the face of the draft. The rate of exchange is not at all uniform among banks.

196. Indorsements. — In buying a draft, one usually has it made out to himself, then, across the back of it, he writes an order for it to be paid to the party that he wishes to have collect the money.

Thus, if D. E. Cole wishes to pay a sum to Robert Smith, he will buy a draft payable to D. E. Cole, then across the back write "Pay to Robert Smith, or order, D. E. Cole". This is called an "**indorsement** in full" or a "restrictive indorsement". If Mr. Cole merely signs his name, any one having the draft in his possession may collect it. If Mr. Cole wishes, he may have the draft made out directly to Robert Smith, in which case it will not need Mr. Cole's indorsement. The restrictive indorsement, however, is the usual one, for in effect it is a receipt for the payment, for it shows of whom Robert Smith received the payment.

197. Commercial drafts. — Instead of the debtor sending the creditor a draft, as in the last article, the creditor may draw upon the debtor directly by a draft. This form of draft is called a **commercial draft.**

Thus, suppose Haines & Co. of Topeka, Kans., have bought goods of S. T. Richards & Co. of Chicago, and have not paid the bill when due. Richards & Co. may make out a draft as follows, indorse it, and deposit it with their bank in Chicago for collection:


NO PROTEST. TEAR THIS OFF BEFORE PRESENTING.

$348.00 Chicago, Ill. May 6, 1916

At Sight Pay to the

Order of Ourselves

Three hundred forty-eight 00/100 Dollars

Value received and charge the same to account of

To Haines & Co. }

Topeka, Kansas } S. T. Richards & Co.

Specimen

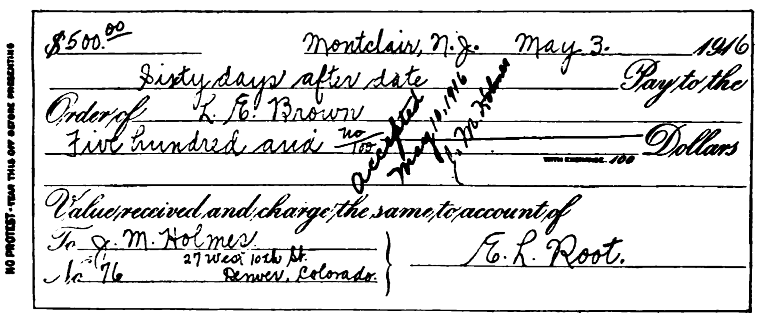
The Chicago bank now sends this draft to some bank in Topeka. The Topeka bank sends a messenger to Haines & Co. and presents the draft for payment. If collected, the Topeka bank sends the money, or its equivalent, to the Chicago bank. This bank notifies S. T. Richards & Co. and places the amount, less a small fee for collection, to their credit.

In case Haines & Co. refuse to pay it, the draft is returned to the Chicago bank and Richards & Co. are notified. They must now take other means of collecting. It is seen, then, that a draft in no way forces a payment.

198. Sight and time drafts. — The form shown in § 197 is that of a **sight draft**. If "Thirty days after date", or some other time, is written in place of "At sight", the draft becomes a **time draft**. Some houses sell goods to be paid for in 30, 60, or 90 days and at once deposit with their bank a time draft for the payment. This does not imply that this is a means of forcing collection, but merely a convenience.

Had a time draft been presented to Haines & Co., either they would have written across the face, "Accepted", with the date, and the firm's signature, or they would have refused to pay it. When a time draft is accepted, it really becomes a promissory note and may be discounted at a bank.

USUAL FORM OF AN ACCEPTED TIME DRAFT



NO PROTEST - TEAR THIS OFF BEFORE PRESENTING

$500.00 Montclair, N.J. May 3. 1916

Sixty days after date Pay to the

Order of K. E. Brown

Five hundred and no/100 Dollars

Value received and charge the same to account of

To J. M. Holmes
No. 76 27 West 10th St. Denver, Colorado

E. L. Root.

The form shown on the preceding page is sometimes called a **three-party draft.** In this draft J. M. Holmes owes E. L. Root $500 in sixty days. Mr. Root either owes L. E. Brown or is selling him (discounting) the draft in order to cancel the debt or to get cash before the amount is due from Mr. Holmes, and hence he is asking Mr. Holmes to pay the amount due to Mr. Brown.

In making out a time draft to meet the terms of payment on an invoice of goods, the firm usually orders payment made to the order of "ourselves", then indorses it and deposits it for collection with their bank. Sometimes the order is to pay to the bank with which they are doing business.

EXERCISES

1. Mr. Jones deposits a 90-day time draft for $2000 with his bank. If the bank discounts it at 6%, for how much will they credit Mr. Jones's account?

2. Instead of sending a bill payable in 30 days, Mr. Haines draws up 30-day drafts upon his customers. What will he get in cash for an order of $1250?

3. A 90-day draft for $1565 was dated April 10, 1916, and accepted April 17. Find the discount from the time it was accepted until due. (Use 6% when rate is not given.)

4. A 90-day draft for $1860 was accepted 12 days after date. Find the cash value on the day of acceptance.

5. F. Alton & Co., New York, on Aug. 3, 1915, drew a time draft on F. Fay & Bros., Decatur, Ill., for a bill of $2800 due Nov. 1, 1915. Write out the proper form of draft.

6. Accept this draft on Aug. 18, 1915. That is, sign it for the proper firm named above.

7. Find the proceeds of this draft discounted at 6%. Who gets the proceeds?

8. Write a draft of which you are the maker, your teacher the payee, and a neighboring bank the drawee.

9. Suppose that you bought of E. L. Holman & Co. a bill of hardware as follows: 26 doz. axes at $4.80; 32 doz. files at $1.20; 36 doz. saws at $3.40. Discount 40% and 10%. Terms: 2% discount for cash in 10 days; net 60 days.

Make out a bill for this, showing the net cash price. Send a draft in payment. What is the face of the draft? Where will you get it? To whom will you have it payable? If made payable to yourself, how will you indorse it before sending it?

10. F. R. Smith & Sons bought of J. L. Morris & Co. the following: 2 safes at $12.50; 3 armchairs at $9.00; 2 rockers at $9. Terms: net 60 days.

Make out the proper bill, finding the price in 60 days. Draw a 60-day draft for the amount. Suppose the draft is accepted 5 days after date. What are the proceeds?

11. Suppose you bought of M. E. Dawson & Co. the following: 4 kitchen cabinets at $13.25; 3 kitchen cabinets at $15.25; and 1 kitchen cabinet at $17.75. Terms: 2% off for cash in 10 days; net 30 days.

Make out the proper bill. Find the net cash price. Send them your certified check for the payment. What will Dawson & Co. do with this check when they receive it?

12. A. L. Morgan of Salem, Ill., bought of Reed, Murdock, & Co., Chicago, the following: 86 lb. tea at 43¢; 280 lb. coffee at 28¢; 250 lb. prunes at $11\frac{1}{2}$¢; 116 lb. raisins at 14¢; 144 cans salmon at 15¢; 86 lb. apricots at 18¢. Terms: 60 days net; 2% off for cash in 10 days.

Make out the bill for settlement in 60 days. At the end of 60 days the firm draws a sight draft on Morgan.

Make out the proper form. What would Reed, Murdock, & Co. do with this draft? What then becomes of it?

13. R. L. Stevens & Sons sold to J. H. Boyce the following on 90 days' time, drawing a 90-day draft on Boyce when the goods were billed: 2 rockers at $13.50; 1 rocker at $8.25; 2 Morris chairs at $15.00; 3 chairs at $3.75; 1 hall tree at $11.50; 1 hall tree at $13.75. Terms: net 90 days.

Make out the proper bill; the proper draft. The draft is "accepted" and discounted at 6% in 5 days from date. How much do Stevens & Sons get in cash for the goods?

14. I. C. Carpenter bought of the Van Cleve Glass Co. the following: 3 boxes 7 × 9, single, at $26.75; 5 boxes 10 × 14, single, at $28.25; 2 boxes 16 × 20, single, at $30.00; 5 boxes 12 × 20, single, at $28.00; 6 boxes 18 × 24, single, at $31.75. Discounts 85% and 10%. Terms: 60 days net or 2% off in 10 days.

Write out a check for the cash payment.

15. Suppose that on Dec. 10, 1916, you bought goods of A. C. Black & Co. of Chicago amounting to $786 net 30 days. On Jan. 10, 1917, Black & Co., not having received payment, draw upon you by draft. Make out the proper form of draft payable to "ourselves" to deposit with the State Bank of Chicago. Show their indorsement and suppose that they sent it to the First National Bank of your home town for collection. Whom will you pay? How much?

16. Suppose that you bought an invoice of goods on Jan. 6, 1917, from D. C. Smith & Co. of St. Louis, Mo., amounting to $1648; terms, 60-day time draft from date. On Jan. 8, you receive the draft from them direct and immediately indorse it and return it. Smith & Co. deposit it on Jan. 10 with their bank for collection (*i.e.*, discount it). For how much will they receive credit on Jan. 10?

199. An arrival draft. — Some merchants ship their goods subject to their own order. The shipper receives a **bill of lading** (that is, an acknowledgment of the receipt of goods for transportation) from the freight agent of the transportation company that is to carry the goods. This, properly indorsed, becomes an order upon the agent at their destination to deliver the goods to the party named. This bill of lading is attached to an **arrival draft** and deposited by the shipper at his bank. This bank, through its *correspondent bank*, sends the draft to a bank in the city to which the goods are sent. Upon the arrival of the goods, the one who has bought them is notified and pays the draft and receives the bill of lading, which must be presented before the transportation company will deliver the goods; that is, unseal the car.

The following forms show an invoice sent to the buyer and the draft which is sent with the bill of lading.

INVOICE

DETROIT, MICH., March 24, 1916

E. B. Hodges & Co.,
Norfolk, Va.

Bought of MARTIN DAWSON

Via M. C. & N. & W.
Car M. C. No. 42829

HAY, GRAIN, AND SEEDS

164 bales, 22445 lb. No. 1 Timothy at $11.75 f. o. b. Detroit	131	86		
Arrival Draft			131	86

ARRIVAL DRAFT

$131.86	*Detroit, Mich., March 24, 1916*
On arrival M. C. & N. & W. car No. 42829 pay to the order of Detroit Savings Bank	
One hundred thirty-one and $\frac{86}{100}$ *Dollars*	
Value received, and charge the same to account of	
To E. B. Hodges & Co.,	
Norfolk, Va.	*Martin Dawson*

200. Savings banks. — A **savings bank** is an institution for receiving and investing savings. It pays interest at stated intervals, usually semiannually, upon the deposits. As the bank's income varies with market conditions the rate of interest paid is usually decided at the end of each interest period. It is usually 3%, $3\frac{1}{2}$%, or 4%.

201. Compound interest. — When interest due at the end of any interest period is added to the principal and thus draws interest for the next interest period, and so on, we have **compound interest.** Thus, in a savings bank, the interest due at any interest-paying date is credited to one's account and thus draws interest, giving *compound interest.*

NOTE. — It is interesting and surprising to know that one cent at 10% simple interest from the beginning of the Christian Era to the present time (1916) would amount to but $1.93, while compounded annually it would amount to more than $10^{75} \times \$6$, that is, $6 with seventy-five zeros annexed.

202. Use of compound interest. — In most states the collection of compound interest is illegal. In modern practice, then, the subject is merely of use to large investors, as building and loan associations, life insurance companies, banking

corporations, etc., who wish to compute the final incomes from reinvesting all interest as it falls due. Such computations are made by the use of compound interest tables.

COMPOUND INTEREST TABLE

(The amount of one dollar principal)

YEARS	2%	$2\frac{1}{2}$%	3%	$3\frac{1}{2}$%	4%	5%	6%	YEARS
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0500	1.0600	1
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.1025	1.1236	2
3	1.0612	1.0769	1.0927	1.1087	1.1248	1.1576	1.1910	3
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.2155	1.2625	4
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2763	1.3382	5
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3401	1.4185	6
7	1.1487	1.1887	1.2299	1 2723	1.3159	1.4071	1.5036	7
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4775	1.5938	8
9	1.1951	1.2489	1.3048	1.3629	1.4233	8.5513	1.6895	9
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.6289	1.7908	10
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.7103	1.8983	11
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.7969	2.0122	12
13	1.2936	1.3785	1.4685	1.5639	1.6651	1.8857	2.1329	13
14	1.3195	1.4130	1.5126	1.6187	1.7319	1.9800	2.2609	14
15	1.3459	1.4483	1.5580	1.6754	1.8009	2.0789	2.3966	15
16	1.3727	1.4845	1.6047	1.7340	1.8729	2.1829	2.5404	16
17	1.4002	1.5216	1.6529	1.7949	1.9479	2.2920	2.6928	17
18	1.4283	1.5597	1.7024	1.8575	2.0258	2.4066	2.8543	18
19	1.4568	1.5987	1.7535	1.9225	2.1069	2.5269	3.0256	19
20	1.4860	1.6386	1.8061	1.9898	2.1911	2.6533	3.2071	20
21	1.5156	1.6796	1.8603	2.0594	2.2788	2.7860	3.3996	21
22	1.5461	1.7216	1.9161	2.1315	2.3700	2.9253	3.6035	22
23	1.5770	1.7646	1 9736	2.2055	2.4647	3.0715	3.8198	23
24	1.6076	1.8087	2.0328	2.2835	2.5633	3.2251	4.0489	24
25	1.6405	1.8539	2.0938	2.3628	2.6658	3.3864	4.2919	25

PROBLEMS

1. If you deposit $400 in a savings bank which pays 4% interest, payable semiannually, how much will you have at the end of two years?

WORK, NOT USING TABLES

$400
8 int. for ½ yr.
$408 amt. at end of ½ yr.
8.16 int. for ½ yr.
$416.16 amt. at end of 1 yr.
8.32 int. for ½ yr.
$424.48 amt. at end of 1½ yr.
8.49 int. for ½ yr.
$432.97 amt. at end of 2 yr.

EXPLANATION.—Interest of $400 for ½ yr. at 4 % = $8. This is added to $400 to give the amount due. Next, interest on $408 for ½ yr. at 4 % = $8.16. This is added to $408, giving $416.16, etc.

WORK, USING TABLES

$1.0824
400
$432.9600

EXPLANATION. — Since 4 %, payable yearly, is 2 % for ½ yr., or each period, we find the amt. of $1 for 4 periods at 2 %. Since amt. of $1 = $1.0824, amt. of $400 = 400 × $1.0824. The difference of 1 cent in results comes from using a 4-place table.

2. Find in both ways the amount of $600 for 4 years at 4 %, payable semiannually.

3. If, on the day of your birth, your father had deposited $800 for you in a savings bank paying 4 %, compounded semiannually, how much would you have on your 21st birthday?

EXPLANATION AND SOLUTION. — The time is 21 years at 4 % semiannually, hence 42 periods at 2 % each. Since the table gives the *amount* of $1.00 for no more than 25 periods, we shall have to find what $1.00 amounts to for say 21 periods at 2 %, which is $1.5156; then we find that this sum will in the remaining 21 periods amount to 1.5156 times what $1.00 would amount to in the same time, *i.e.* to 1.5156 × $1.5156. As $800 was to be deposited, we get for the final amount 800 × 1.5156 × $1.5156, or $1837.64.

4. $1200 deposited at 4 %, compounded annually, will amount to how much in 25 years?

5. Find the amount of $5000 at 5 %, compounded annually, for 20 years.

6. If a man at 21 inherits $12,000 which he deposits in a bank at 6%, compounded annually, how much will he have to his credit at 65?

7. If you could deposit $500 annually in a savings bank paying 4%, compounded annually, how much would you have at the end of 20 years?

EXPLANATION AND SOLUTION. — The first $500 is in for 20 yr., the next for 19 yr., the next for 18 yr., etc. down to one yr. Time may be saved by adding the first twenty amounts under the 4% column before multiplying by 500. To the fourth decimal place, the sum is $30.9692. 500 × $30.9692 = $15,484.60.

8. How much can one accumulate in 10 years by depositing $800 annually, interest 4%, compounded annually?

9. To what will an annual payment of $564 for 10 years amount, interest $3\frac{1}{2}$%, compounded annually?

10. What may one draw at the end of 10 years if he regularly deposits $400 semiannually in a savings bank paying 4%, compounded semiannnally?

11. If one can invest $600 yearly from his salary where it will yield him 6% yearly, and can keep all the interest reinvested at the same rate as fast as it accrues, how much can he accumulate in 20 years?

12. A man gave to his son on his sixth birthday and on each succeeding birthday including his 21st, $100, which, together with the interest, he kept in a savings bank paying 4% interest, annually. How much had the young man to his credit from this source on his 21st birthday?

13. A man 40 years old can take out a $1000 15-year endowment insurance policy for $63.23, payable annually. This same payment invested at 6%, compounded annually, would amount to what in the same time?

14. If a man can begin investing \$500 annually at 6 % when 25 years of age and continue it until he is 60, keeping all interest reinvested, how much will he have accumulated?

15. Suppose that one invests \$200 yearly at 6 % from ages 20 to 30; \$500 annually from 30 to 40; and \$800 annually from 40 to 50. Keeping all interest reinvested, what will he have accumulated at 50?

16. A man 21 years of age can take out a 20-year endowment life insurance policy for \$5000 by paying \$238.75 annually for the 20 years. If he lives until the end of that time, he will receive the \$5000. How much more would he have had at that time if he had made the same deposits in a savings bank paying 4 % interest annually? (Use the table on page 246 to save adding the amounts.)

17. At the age of 50 a man can take out a 25-year endowment policy for \$10,000 by paying \$510.50 annually. If he lives the 25 years, compare, as in Problem 16, with the same amount invested in a savings bank paying $3\frac{1}{2}$ % interest, payable annually.

18. Compare the amount of \$2500 at the end of 25 years at 6 % simple interest with the amount of the same sum at 4 % compound interest payable annually.

203. Sinking fund. — A sum set aside at certain periods to meet a future obligation is called a **sinking fund**. Thus, a corporation may borrow money for some purpose and give notes coming due in 10, 15, and 20 years. To meet these demands, a sum will be set aside each year at compound interest. The problem arising is not one that comes logically under "banking", but it depends upon compound interest and hence it is treated here.

TABLE SHOWING AMOUNT ACCUMULATED AT END OF A PERIOD OF YEARS BY PAYING $1 AT BEGINNING OF EACH YEAR IN THE PERIOD

Year	2 Per Cent	$2\frac{1}{2}$ Per Cent	3 Per Cent	$3\frac{1}{2}$ Per Cent	4 Per Cent	5 Per Cent	6 Per Cent	Year
1	1.020	1.025	1.030	1.035	1.040	1.050	1.060	1
2	2.060	2.076	2.091	2.106	2.122	2.152	2.184	2
3	3.122	3.153	3.184	3.215	3.246	3.310	3.375	3
4	4.204	4.256	4.309	4.362	4.416	4.526	4.637	4
5	5.308	5.388	5.468	5.550	5.633	5.802	5.975	5
6	6.434	6.547	6.662	6.779	6.898	7.142	7.394	6
7	7.583	7.736	7.892	8.052	8.214	8.549	8.897	7
8	8.755	8.955	9.159	9.368	9.583	10.027	10.491	8
9	9.950	10.203	10.464	10.731	11.006	11.578	12.181	9
10	11.169	11.483	11.808	12.142	12.486	13.207	13.972	10
11	12.412	12.796	13.192	13.602	14.026	14.917	15.870	11
12	13.680	14.140	14.618	15.113	15.627	16.713	17.882	12
13	14.974	15.519	16.086	16.677	17.292	18.599	20.015	13
14	16.293	16.932	17.599	18.296	19.024	20.579	22.276	14
15	17.639	18.380	19.157	19.971	20.825	22.657	24.673	15
16	19.012	19.865	20.762	21.705	22.698	24.840	27.213	16
17	20.412	21.386	22.414	23.500	24.645	27.132	29.906	17
18	21.841	22.946	24.117	25.357	26.671	29.539	32.760	18
19	23.297	24.545	25.870	27.280	28.778	32.066	35.786	19
20	24.783	26.183	27.676	29.269	30.969	34.719	38.993	20
21	26.299	27.863	29.537	31.329	33.248	37.505	42.392	21
22	27.845	29.584	31.453	33.460	35.618	40.430	45.996	22
23	29.422	31.349	33.426	35.667	38.083	43.502	49 816	23
24	31.030	33.158	35.459	37.950	40.646	46.727	53.865	24
25	32.671	35.012	37.553	40.313	43.312	50.113	58.156	25
26	34.344	36.912	39.710	42.759	46.084	53.669	62.706	26
27	36.051	38.860	41.931	45.291	48.968	57.403	67.528	27
28	37.792	40.856	44.219	47.911	51.966	61.323	72.640	28
29	39.568	42.903	46.575	50.623	55.085	65.439	78.058	29
30	41.379	45.000	49.003	53.429	58.328	69.761	83.802	30
31	43.227	47.150	51.503	56.334	61.701	74.299	89.890	31
32	45.112	49.354	54.078	59.341	65.210	79.064	96.343	32
33	47.034	51.613	56.730	62.453	68.858	84.067	103.184	33
34	48.994	53.928	59.462	65.674	72.652	89.320	110.435	34
35	50.994	56.301	62.276	69.008	76.598	94.836	118.121	35
36	53 034	58.734	65.174	72.458	80.702	100.628	126.268	36
37	55.115	61.227	68.159	76.029	84.970	106.710	134.904	37
38	57.237	63.783	71.234	79.725	89.409	113.095	144.058	38
39	59.402	66.403	74.401	83.550	94.026	119.800	153.762	39
40	61.610	69.088	77.663	87.510	98.827	126.840	164.048	40

PROBLEMS

1. A city has \$25,000 in bonds coming due in 15 years. How much must be set aside each year at 4%, compounded annually, to meet this debt?

SOLUTION

\$25,000 ÷ \$20.825 = 1200.48.
Hence, \$1200.48.

EXPLANATION.—From the table, \$1 each year for 15 yr. at 4% will amount to \$20.825. But the amount needed is \$25,000, which is 1200.48 times as large. Hence, the amount to set aside is \$1200.48.

NOTE.—This assumes that immediately upon borrowing the \$25,000 the city sets aside \$1200.48, and that during the last (15th) year, the city does not save anything to apply to payment of these bonds.

2. If a city issues \$100,000 in bonds due in 10 years, and \$100,000 more due in 15 years, to build a new school building, how much must be set aside yearly at 4% compound interest to meet the entire obligation?

3. To meet a bond issue of \$500,000 due in 10 years, a sinking fund earning $3\frac{1}{2}$% compound interest is provided. How much must be set aside annually?

4. On May 10, 1915, a corporation issued \$3,000,000 in bonds to mature May 10, 1935. How much must be set aside annually to create a sinking fund to meet this amount at maturity if the fund earns 4%, compounded annually?

5. How large an annual investment paying 6%, all interest being reinvested as it accrues, must a man at 25 years of age make to have \$50,000 when he is 60 years old?

STOCK INVESTMENTS

204. A corporation. — A **corporation** consists of a number of individuals united by the consent of the state into one body or company, and empowered by the state to act in a

certain capacity, or to transact business of some form. The list of powers, rights, and duties of the corporation are stated in writing in an instrument called the **charter.**

205. Stock. — **Stock** is the name given to the capital of a corporation. The **capital stock** of a corporation is divided into **shares.** They are usually $100 each but may be of any value.

206. A stockholder. — Any person owning one or more shares in a corporation is a **stockholder** in that corporation. As evidence of ownership, each stockholder receives a **stock certificate** showing the number of shares he owns and the par value of each.

A STOCK CERTIFICATE


INCORPORATED UNDER THE LAWS OF THE STATE OF MISSISSIPPI

347

$100

Hebron-Dean Company

Authorized Capital Stock $20,000

This Certifies that B. N. Mason is the owner of Fifty Shares of the Capital Stock of Hebron-Dean Company transferable only on the books of this Corporation in person or by Attorney upon surrender of this Certificate properly endorsed.

In Witness Whereof the said Corporation has caused this Certificate to be signed by its duly authorized officers and its Corporate Seal to be hereunto affixed the eighth day of March A.D. 1916

J. M. Dixon, PRESIDENT

C. K. Snyder, SECRETARY

SPECIMEN

SHARES $100 EACH

207. Par value. — The **par value** of stock is the value named on the stock certificate. The value is determined by the corporation, and depends upon the number of shares into which, at the time of organization, it seemed desirable to divide the capital. The par value is no indication of the real or market value of the stock. Thus the par value of each share represented in the certificate on page 248 is $100, but the market value may be more or less than $100.

NOTE. — In the problems of this book, consider the par value $100.

208. Market value. — The **market value** is the price at which stock can be bought or sold in open market. A number of factors affect the market price of stock, chief among which are: (1) the real or prospective earning power of the corporation; and (2) the confidence of the buying public, or the lack of it, in the general stability of the enterprise. When the real or anticipated earnings are small, the price is low. When large, the price is high.

NOTE. — The price of stock often changes several points (*i.e.* several dollars per share) during a single day. Thus the chance that a note sent by President Wilson in December, 1916, to the warring nations of Europe might bring an early peace caused many stocks to drop several points on the day the note was published.

209. Assessments. — An **assessment** is a certain per cent of the par value that is sometimes levied against the stockholders to pay certain losses.

210. The dividends. — The earnings of a corporation that are divided among its stockholders are called the **dividends.** They are distributed as a per cent of the par value of the stock. Thus a 12 % dividend is a dividend of $12 per share on stock whose par value is $100.

211. Two kinds of stock. — Two kinds of stock are issued: **common stock** and **preferred stock.** The rate of dividend on

the *preferred stock* is fixed by the corporation before the stock is sold. The rate usually ranges from 5% to 7% of the par value. These dividends thus become an obligation of the corporation, and are paid from the earnings before making a distribution of dividends among the holders of the *common stock*. The dividends paid the holders of common stock are reckoned as a per cent of the par value of the common stock. Such dividends vary greatly, depending upon the net earnings of the company.

212. Buyers of stock. — There are two classes of buyers of stock; viz., the **investor**, who buys stock and holds it for the dividends it will pay, just as he loans money for the interest, or buys and rents a house for the rent he will receive; and the **speculator**, who buys expecting the price to advance so that he may sell at a profit.

When buying for an investment, one must bear in mind that there is a great uncertainty as to the dividends; and also that the price at which he can sell is subject to great fluctuations as shown in the following tables.

TABLE SHOWING CHANGES IN PRICES DURING THE YEAR 1915 IN THREE SELECTED COMMON STOCKS

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Bethlehem Steel	High	$54\frac{3}{8}$	$57\frac{3}{4}$	$91\frac{1}{2}$	155	159	$172\frac{3}{4}$	275	311	369	600	500	485
	Low	$46\frac{1}{4}$	$47\frac{3}{8}$	$54\frac{1}{4}$	$83\frac{3}{4}$	125	135	$164\frac{1}{4}$	250	$283\frac{1}{2}$	362	400	450
General Motors	High	$94\frac{1}{2}$	$94\frac{5}{8}$	127	150	$146\frac{1}{2}$	159	$189\frac{1}{2}$	224	375	395	476	558
	Low	82	82	$91\frac{7}{8}$	122	125	$135\frac{1}{4}$	$153\frac{7}{8}$	181	$219\frac{1}{4}$	320	365	450
Cuban-Am. Sugar	High	47	$45\frac{3}{4}$	$52\frac{1}{4}$	$63\frac{1}{2}$	$70\frac{1}{2}$	91	125	125	$123\frac{7}{8}$	122	$174\frac{7}{8}$	177
	Low	38	40	40	51	51	68	$86\frac{3}{4}$	103	113	112	111	140

NOTE. — On Dec. 2, 1916, Bethlehem Steel sold for 625; General Motors for 810; and Cuban-American Sugar for 220. Compare these with the prices at the time you are studying this.

TABLE SHOWING THE RANGE IN PRICES OF SOME COMMON STOCKS OVER A PERIOD OF 5 YEARS

	1911		1912		1913		1914		1915	
	High	Low	High	Low	High	Low	High	Low	High	Low
Union Pacific R.R. . .	$192\frac{3}{8}$	$153\frac{1}{2}$	$176\frac{3}{4}$	$150\frac{3}{4}$	$162\frac{7}{8}$	$137\frac{3}{4}$	$164\frac{3}{8}$	112	$141\frac{1}{2}$	$115\frac{1}{4}$
Baltimore and Ohio .	$109\frac{3}{4}$	$93\frac{1}{2}$	$111\frac{7}{8}$	$101\frac{1}{4}$	$106\frac{3}{8}$	$90\frac{5}{8}$	$98\frac{1}{2}$	67	96	$63\frac{3}{4}$
Brooklyn Rapid Transit	$84\frac{3}{4}$	72	$94\frac{1}{2}$	$76\frac{3}{4}$	$92\frac{3}{4}$	$83\frac{3}{4}$	$94\frac{1}{4}$	79	93	$83\frac{1}{4}$
U. S. Steel	$82\frac{1}{8}$	50	$80\frac{3}{4}$	$58\frac{1}{4}$	$69\frac{1}{8}$	$49\frac{7}{8}$	$67\frac{1}{4}$	48	$89\frac{1}{2}$	38
Bethlehem Steel . . .	$38\frac{1}{8}$	$18\frac{3}{4}$	$51\frac{5}{8}$	$27\frac{3}{4}$	$41\frac{1}{2}$	25	$46\frac{5}{8}$	$29\frac{1}{2}$	600	$46\frac{1}{4}$
General Motors . . .	$51\frac{3}{4}$	35	$42\frac{7}{8}$	30	40	25	99	$37\frac{3}{8}$	558	82
Sears, Roebuck and Co.	$192\frac{3}{4}$	$123\frac{3}{4}$	$221\frac{1}{2}$	140	$214\frac{3}{4}$	155	$197\frac{1}{2}$	$168\frac{1}{2}$	215	$131\frac{1}{2}$
Western Union . . .	$84\frac{1}{2}$	$71\frac{5}{8}$	$86\frac{1}{4}$	72	$75\frac{1}{8}$	$54\frac{1}{8}$	$66\frac{7}{8}$	$53\frac{3}{8}$	90	57

213. A stock broker. — In general, one wanting to buy stock in a certain corporation does not know who has it for sale; or if he wishes to sell, he does not know a buyer. Hence he has to buy or sell through an agent called a **stock broker.** The broker usually belongs to an association called a **stock exchange.** The broker's usual fee is $\frac{1}{8}\%$ of the par value for buying, and the same for selling.

214. Problems in stocks. — The problems involved in buying or selling stock are very simple when one understands the terms involved. Hence a careful study of the preceding sections should be made. The two most important problems are:

1. *Stock bought at one price and sold at another gives a profit or loss of how much?*

2. *Stock selling at a certain price and paying a certain dividend yields an income of what rate upon the market value?*

PROBLEMS

1. If the capital stock of a corporation is $500,000, and there are 5000 shares, what is the par value of each share?

2. If this company is able to distribute $35,000 of its earnings among its stockholders, what rate of dividend can it declare?

3. If a 7% dividend is declared, how much is paid the holder of each $100 share? The holder of 150 such shares?

4. If $100 at interest will earn $6 per year, and a $100 share of stock earns an 8% dividend, which would you prefer to own, $100 or a share of stock?

5. About how much money loaned at 6% will produce $8 interest? Then about how much could you pay for a share of such stock to get 6% yearly on your money?

6. Get a daily paper and read the stock quotations. Tell what the quotations mean and give a probable reason for the market price of each.

7. Select certain stocks from the quotation and find how much 100 shares of each would cost. Follow the quotations of the same stocks for several weeks and see how much you would have made or lost if you could have bought 100 shares. (Allow $\frac{1}{8}$% of the par value for brokerage both in buying and in selling.)

8. Chicago, Milwaukee, and St. Paul (C. M. & St. P.) sold for 199$\frac{5}{8}$ in 1906. Get a daily paper and find the price at the time you are studying this. What has been the per cent of increase or decrease?

9. Baltimore and Ohio stock sold for 78$\frac{3}{8}$ on Nov. 23, 1907. Compare that with the present price and see how much a man would have gained or lost had he bought 250

shares then and sold now. (Allow brokerage for both buying and selling.)

10. In 1906 the highest price of Union Pacific was $195\frac{1}{8}$. In 1907 the highest price was 183. If bought in 1906 and sold in 1907, find the total loss (including brokerage) on 350 shares.

11. Suppose you bought Union Pacific as quoted above for 1906 and sold to-day. How much would you have made or lost on 150 shares?

12. On Jan. 11, 1911, U. S. Steel sold for 74. Compare the present price with that of 1911 and find the per cent of increase or decrease.

13. On Jan. 11, 1911, American Express stock sold for 225. Compare, as in Problem 12, with the price to-day.

14. If you own 150 shares of Chi. & N. W. and a 7% dividend is declared, how much will you receive?

15. At the time you study this a 7% dividend from Chi. & N. W. gives an income of what per cent of the market value?

16. What can you afford to pay for stock paying a 10% dividend to get an income of 6% upon your investment?

17. How much could you pay for 5% stock to get 6% of your investment?

18. As in Problems 16 and 17, find prices at which stock paying the following dividends will yield 6%: 3%, 8%, 12%, 15%, 20%, 30%.

19. The capital stock of a company is $8,500,000. The gross earnings one year were $860,000. The operating expenses were $250,000. $100,000 was set aside for a sinking fund. What rate of dividend was declared?

20. A corporation with a capital of \$36,500,000 had a gross income one year of \$22,645,328. The total expenses for the year were \$20,634,916. Find the largest whole per cent of dividend that could be declared and the amount of profits left undivided.

21. The capital stock of a certain corporation is \$2,700,000, \$200,000 of this is 7 % preferred stock. One year the gross earnings were \$296,465. The total expenses were \$132,465. The net profits were distributed as dividends. How much went to the holders of preferred stock, and what per cent of dividend did the holders of common stock get?

22. A certain corporation with a capital of \$2,000,000 (common stock) reported for the year: Gross earnings, \$596,528.31; operating expenses, \$357,934.85; interest paid, \$120,206.44; preferred stock dividends, \$25,000. What is the largest whole per cent of dividend on the common stock the directors can declare? How much of the surplus earnings will not be distributed?

23. If you should buy 100 shares of N. Y. Central at $104\frac{1}{2}$, brokerage $\frac{1}{8}$ %, and hold for the dividends, and if the dividends are 6 % each year, how much more or less would you receive yearly than you would from loaning your money at $5\frac{1}{2}$ %?

24. Suppose you can buy Willys-Overland at $207\frac{1}{4}$ or Studebaker Co. at 146. If the Willys-Overland pays 11 % and the Studebaker Co. pays 7 %, which is the better investment and how much?

25. If you buy 100 shares of stock at $125\frac{1}{4}$, receive a 12 % dividend, and sell in one year at $131\frac{1}{8}$, both transactions through a broker, how much do you make?

26. In Problem 25, how much more do you make than you would have made by loaning your money at 6 %?

27. If you should buy 100 shares of stock at $80\frac{3}{4}$, receive two 5 % dividends, and sell in 2 years at 65, both transactions through a broker, what would your total loss be, including interest at 6 %?

28. Not allowing for the fluctuation in the market value of the stock, which is the better investment: stock quoted at $82\frac{1}{2}$, brokerage $\frac{1}{8}$ %, paying a 5 % dividend, or a mortgage and 6 % interest? Find the difference in yearly income from an amount sufficient to buy 250 shares.

BOND INVESTMENTS

215. Bonds. — A **bond** is a written agreement to pay, for value received, a certain sum of money at or within a certain fixed time, with a fixed rate of interest to be paid at stated times. That is, it is a *promissory note* issued by a corporation and offered for sale.

The bonds of a stock corporation, called **industrial bonds**, are secured by a general mortgage on the property of the company. The bonds of a city, country, state, or national government, called **tax bonds**, are paid by a tax, and are, therefore, considered the safest kind of bond investments.

While the denomination of the standard bonds is usually $500 or $1000, a *market quotation* of 104 means either 104 % of the par value, or $104 for every $100 in par value, as one wishes to interpret it.

216. Coupon bonds. — A **coupon bond** has as many dated interest coupons attached as there are interest payments to be made. These coupons are, in effect, negotiable notes given by the corporation promising to pay the interest on the bond as it becomes due. Such coupons, as they become due, may be detached from the bonds and collected personally or through a bank, like any other kind of commercial paper.

217. Registered bonds. — A **registered bond** is a bond registered in the holder's name on the books of the authority making it. When the interest is due, it is paid by check or draft sent directly to the registered holder.

218. Stability of bonds. — The price of bonds is not subject to as sudden fluctuations in value as stocks are. The chief factors regulating the price of bonds are:

1. The security back of the bonds.
2. The length of time the bond has to run.
3. The rate of interest the bond is paying compared with the general interest rates of money.
4. The confidence of the buying public in the stability and general earning power of the corporations issuing the bonds.

This will explain the variation in price of bonds paying the same rate of interest which you will find in the daily market reports. Thus in the report of Feb. 26, 1916, you will find the following quotations for 5 % bonds: Am. Cotton Oil 5s, $97\frac{1}{2}$; Bethlehem Steel 5s, 102; Chi. & W. Mich. 5s, 83; Indiana Steel 5s, $102\frac{3}{4}$; M. K. & O. K. 5s, 60; West'n Elec. 5s, $103\frac{1}{8}$; and Wabash 5s, $104\frac{1}{2}$. Also in the same report you will find N. Y. City 4s, due in 1958, quoted at $98\frac{5}{8}$, and N. Y. City $4\frac{1}{2}$s, due in 1963, quoted at $107\frac{3}{8}$.

219. Bonds as an investment. — Bonds usually are bought for the interest they yield and not for a rise in value. That is, they are bought as an *investment*, not for *speculation*. Tax bonds, as city, county, state, or government bonds, and also first mortgage railroad or industrial bonds, where the corporation is earning a reasonable per cent of its capitalization, are conservative investments. Such securities, on account of their safety, find a ready sale at a comparatively low rate of interest. One dependent upon the income from his investments for his support should select such bonds in preference to stocks.

STOCKS AND BONDS COMPARED AS INVESTMENTS

Stocks	Bonds
1. The dividends depend upon the earning power of the corporation.	1. The interest is a fixed rate.
2. The dividends are not due until they have been declared by the board of directors.	2. The interest is paid at regular fixed periods.
3. Subject to sudden fluctuations in value.	3. Only slight fluctuations in value.

TABLE SHOWING STABILITY OF BONDS

Name of Bond	Interest	Date of Maturity	1914		1915	
			High	Low	High	Low
N. Y. City	4%	May 1959	100⅛	95	99	94
Atchinson Gen.	4%	Oct. 1995	96½	90	95¾	89
Baltimore & Ohio	4%	July 1948	96	87⅝	92	83⅜
Cent. of N. J.	5%	July 1987	118	111½	118	110½
Canada So.	5%	Oct. 1962	106⅞	101	105¾	99⅞
Bethlehem Steel	5%	Jan. 1926	100¼	93½	103	98½
U. S. Steel	5%	Apr. 1963	103¼	99½	105	99¾

220. Rate of income on bond investments. — A bond, like any other promissory note, matures at its *face* or *par value*. Hence, if a bond is bought above par and held until maturity, it matures for less than it cost. This then gives a total net income less than the total gross income. On the other hand, if bought below par, it matures for more than it cost, and this gain must be considered in reckoning the rate of income.

Bankers do not have to make such computations, for tables have been worked out showing the rate of income a bond will

pay for all periods, rates, and prices that are liable to occur. A sample from such a book is shown on the following page.*

A method of finding the yield, however, is shown below.

PROBLEMS

1. Bonds costing 104, including brokerage, paying $4\frac{1}{2}\%$ interest annually, and maturing in 20 years, will yield the investor what rate of income?

SOLUTION AND EXPLANATION. — It may be considered that from each year's interest a "sinking fund" (see § 203) is taken that at the date of maturity will amount to the \$4 premium that was paid for the bond. Since the first interest is paid 19 years before maturity, we can use the table in § 203 and find the amount to which \$1 from each year's interest will amount, at say 4 %. This is \$28.778 from the first 19 payments. Adding \$1 from the last payment gives a total of \$29.778. But \$4, and not \$29.778, is needed, so $4 \div 29.778$, or .1343 of \$1, or \$.1343, is needed from each year's interest. This leaves a net yearly income of \$4.3657 on each \$104 invested. $\$4.3657 \div \$104 = 4.19\%$.

NOTE. — When the time to run is short, a very close approximation to the rate of income may be found without considering the compound interest in distributing over the whole period the loss or gain at maturity. Thus, 5 % bonds due in 3 years, bought at 97, including brokerage, if held until maturity, will mature at \$3 more than cost and thus add approximately \$1 per year to each year's interest of \$5. Hence a \$97 investment earns about 6.19 %.

2. Give the approximate yield of a 6 % bond bought at 104, including brokerage, if due in 4 years. If due in 6 years. If due in 8 years.

3. Why is a 5 % bond bought at 97 a better investment than a 6 % bond bought at 103 if both are due in 3 years?

4. Find the yield of a 6 % bond, interest payable semi-annually, bought at 112.55, due in 20 years, and compare the result with the table on next page. (In distributing the loss at maturity over 20 years allow 4 % payable annually.)

* From Montgomery Rollins' Table of Bond Values, copyright by Montgomery Rollins.

BOND BOOK TABLE—DUE IN 20 YEARS

(Interest payable semi-annually)

Per Cent Per Annum	3%	3½%	4%	4½%	5%	6%	7%
2.90	101.51	109.06	116.60	124.15	131.70	146.80	161.89
3.	100.00	107.48	114.96	122.44	129.92	144.87	159.83
3.10	98.52	105.93	113.34	120.75	128.16	142.98	157.81
3⅛	98.15	105.55	112.94	120.33	127.73	142.52	157.31
3.20	97.06	104.41	111.75	119.09	126.44	141.13	155.82
3¼	96.34	103.66	110.97	118.28	125.59	140.21	154.83
3.30	95.63	102.91	110.19	117.47	124.75	139.30	153.86
3.35	94.93	102.17	109.42	116.66	123.91	138.40	152.89
3⅜	94.58	101.81	109.04	116.27	123.49	137.95	152.41
3.40	94.23	101.44	108.66	115.87	123.08	137.51	151.93
3.45	93.54	100.72	107.90	115.08	122.26	136.62	150.98
3½	92.85	100.00	107.15	114.30	121.45	135.74	150.04
3.55	92.17	99.29	106.41	113.52	120.64	134.87	149.11
3.60	91.50	98.58	105.67	112.75	119.84	134.01	148.18
3⅝	91.16	98.23	105.30	112.37	119.44	133.58	147.72
3.65	90.83	97.88	104.94	111.99	119.04	133.15	147.26
3.70	90.17	97.19	104.21	111.24	118.26	132.30	146.35
3¾	89.51	96.50	103.50	110.49	117.48	131.46	145.44
3.80	88.86	95.82	102.78	109.74	116.70	130.63	144.55
3⅞	87.90	94.81	101.73	108.64	115.56	129.39	143.22
3.90	87.58	94.48	101.38	108.28	115.18	128.98	142.78
4.	86.32	93.16	100.00	106.84	113.68	127.36	141.03
4.10	85.09	91.86	98.64	105.42	112.20	125.76	139.32
4⅛	84.78	91.54	98.31	105.07	111.84	125.37	138.90
4.20	83.87	90.59	97.31	104.03	110.75	124.19	137.63
4¼	83.27	89.96	96.65	103.35	110.04	123.42	136.80
4.30	82.68	89.34	96.00	102.66	109.33	122.65	135.98
4⅜	81.80	88.42	95.04	101.65	108.27	121.51	134.75
4.40	81.51	88.11	94.72	101.32	107.93	121.14	134.35
4½	80.35	86.90	93.45	100.00	106.55	119.65	132.74
4.60	79.22	85.72	92.21	98.70	105.19	118.18	131.16
4⅝	78.94	85.42	91.90	98.38	104.86	117.82	130.77
4.70	78.11	84.55	90.99	97.43	103.86	116.74	129.61
4¾	77.57	83.98	90.39	96.80	103.20	116.02	128.84
4.80	77.02	83.40	89.79	96.17	102.55	115.32	128.08
4⅞	76.22	82.56	88.90	95.24	101.59	114.27	126.95
4.90	75.95	82.28	88.61	94.94	101.27	113.92	126.58
5.	74.90	81.17	87.45	93.72	100.00	112.55	125.10
5.10	73.86	80.09	86.31	92.53	98.76	111.20	123.65
5⅛	73.61	79.82	86.03	92.24	98.45	110.87	123.29
5.20	72.85	79.02	85.19	91.36	97.53	109 87	122.22
5¼	72.34	78.49	84.64	90.78	96.93	109.22	121.51
5.30	71.85	77.97	84.09	90.21	96.33	108.57	120.81
5⅜	71.11	77.19	83.27	89.36	95.44	107.60	119.77
5.40	70.87	76.94	83.01	89.07	95.14	107.28	119.42
5½	69.90	75.92	81.94	87.96	93.98	106.02	118.06
5⅝	68.72	74.68	80.64	86.59	92.55	104.47	116.38
5¾	67.57	73.46	79.36	85.26	91.15	102.95	114.74
5⅞	66 43	72.27	78.11	83.95	89.78	101.46	113.13
6.	65.33	71.11	76.89	82.66	88.44	100.00	111.56

PROBLEMS

1. If a $5000 bond is bought when quoted at $103\frac{7}{8}$, brokerage $\frac{1}{8}\%$, how much does it cost? What is it worth at maturity?

2. How much will $8000 in bonds cost at $93\frac{7}{8}$, including brokerage? How much are they worth at maturity?

3. Sometimes bonds by the same corporation and bearing the same rate of interest will sell for different prices when one is almost due and the other has a long period to run. Explain why this is so.

4. On Feb. 26, 1916, New York City $4\frac{1}{2}$s, due in 1965, sold for $107\frac{5}{8}$. If they were due in 1920, would you expect them to sell for more or less? Why?

5. On Feb. 26, 1916, Ill. Cent. 4s, due in 1953, sold for 86. If they were due in 1919 would you expect them to sell for more or less? Why?

6. Why could you not afford to pay 115 for a 6% bond due in 3 years?

7. Granting the security of the two as equal, which would you consider the better investment, 4% bonds at 90, due in 5 years, or 4% bonds at 80, due in 20 years? Why?

8. Why is a 3% bond at 90 due in 3 years a good investment, granting that the financial standing of the corporation is good?

9. A corporation has a bonded indebtedness of $216,000. The rate of interest is $4\frac{1}{2}\%$. What yearly interest does the corporation have to pay?

10. How many $1000, $4\frac{1}{2}\%$ bonds will you need to give you a yearly income of $1800?

11. If there is invested for you $30,000 in bonds at par, half of them 4s and the others 5s, what is your yearly income from them?

12. A man gave his daughter enough 5% bonds to give her an annual income of $850 per year. How much did they cost at $104\frac{1}{2}$, brokerage $\frac{1}{8}$%?

At sight, give the cost, without brokerage, of a $1000 bond in each of the following:

	Name	Price		Name	Price
13.	Rep. of Cuba 5s	$94\frac{1}{8}$	**17.**	Chile Copper 7s	$131\frac{5}{8}$
14.	City of New York 4s	$98\frac{5}{8}$	**18.**	Northern Pacific 3s	$66\frac{7}{8}$
15.	Adams Express 4s	$84\frac{1}{2}$	**19.**	Am. Smelting 6s	$110\frac{1}{2}$
16.	C. & N. W. $3\frac{1}{2}$s	$81\frac{3}{4}$	**20.**	N. Y. Telephone $4\frac{1}{2}$s	$98\frac{1}{2}$

21. Give the income from a $5000 bond of each of the kinds in the table given above.

22. From the bond table find how much a city is really paying for the use of money when it sells its $4\frac{1}{2}$%, semi-annual, 20-year bonds at 96.

TAXES

221. Taxes. — Most of the expenses of towns, cities, counties, and states are met by money called a **tax** levied by the proper officers upon the property of the town, city, county, or state.

222. Two kinds of property tax. — Property is divided into two classes for taxation: (1) **real estate**, regarded as immovable, as lands and buildings, including mines, quarries, railroads, etc.; and (2) **personal property**, which includes all movable property.

223. Personal tax. — A tax levied upon a person regardless of the property he owns is called a **poll tax**.

224. Assessors. — Persons appointed to estimate the value of each person's property and apportion the taxes in proportion to the value of the property owned are called **assessors**. The value they place upon the property is called the **assessed valuation**. The tax rate is given in various ways: sometimes as mills on the dollar; sometimes as dollars per $100; sometimes as dollars per $1000; and sometimes as a *per cent* of the assessed valuation.

A FORM OF TAX BILL

RATES $2.02

THIS BILL MUST BE RETURNED WHEN YOU PAY YOUR TAXES

Mr. John Doe	Page 123 Line 39
No. 56 N. Walnut St.	Map 3 Block C Lot No. 38

Real Estate	Personal Property	Total Valuation	State School, and County Tax		School Tax		Town Tax		Poll	Total Tax	
7200	900	8100	55	64	35	08	72	90		163	62

PROBLEMS

1. At 18 mills on $1, what is the rate per cent of taxation? What is the tax on an assessed valuation of $16,500?

2. At $1.96 per $100, what is the rate per cent of taxation? What is the tax upon property assessed at $35,700?

3. At $18.50 per $1000, what is the rate per cent of taxation? What is the tax upon property assessed at $10,300?

4. If the assessed valuation of the taxable property of a city is $135,000,000 and $1,687,500 is the tax to be raised, find the tax as a rate per cent; as the amount on $100; as the amount on $1000.

5. If taxes are increased from $1.76 to $2.12 per $100 in your town, how much is the increase in taxes for a man who owns property assessed at $8500?

6. If taxes are increased from $1.55 to $1.78 on $100, how much more does one pay whose property is assessed at $150,500?

Give the tax rate per $100:

	Assessed Valuation	Property Tax to be Raised		Assessed Valuation	Property Tax to be Raised
7.	$4,800,000	$36,000	12.	$135,000,000	$900,000
8.	$6,500,000	$130,000	13.	$490,000,000	$6,300,000
9.	$16,500,000	$288,750	14.	$215,000,000	$2,700,000
10.	$24,700,000	$308,750	15.	$356,000,000	$7,191,200
11.	$51,000,000	$750,000	16.	$645,000,000	$10,320,000

NATIONAL REVENUES

225. Indirect tax.—The people are not taxed directly upon the property they own to support the national government as they are to support state, county, and local governments. The expenses of the national government are over $2,000,000 per day. In 1915 the total expense was $731,399,759, distributed as follows: civil and miscellaneous, $207,169,824; war department, $172,973,092; navy department, $141,835,654; Indians, $22,130,351; pensions, $164,387,941; and interest of public debt, $22,902,897.

This expense is met chiefly by: (1) **tariffs, duties,** or **customs,** which are levied upon imported goods; (2) **internal revenue,** which is levied upon certain things made in this

country, as alcoholic liquors and tobacco products; and (3) an **income tax**, levied upon incomes in excess of certain fixed amounts.

For the year ending June 30, 1915, the incomes from these three sources were as follows: internal revenue, $415,669,876; duties or customs, $209,786,673; and income tax, $80,190,694, of which $41,046,165 was from individuals and the rest from corporations.

226. Duties or Customs. — Some imported goods are not subject to duty. Such goods are said to be on the **free list**. The duties are of two kinds: (1) **ad valorem duty**, which is a per cent of the invoiced price of the goods at the place of purchase; and (2) **specific duty**, which is a certain amount per unit, as bushel, barrel, yard, etc. Some goods are subject to one or the other and some to both.

Prior to 1913 there was no income tax, but the tariff rates were higher than under a new law passed that year, known as the *Underwood-Simmons tariff law.*

TABLE SHOWING A FEW CHANGES IN TARIFF RATES

Article	Payne-Aldrich Law (Old)	Underwood-Simmons Law (New)
Wool blankets	33¢ per lb. and 50%	25%
Beef, lamb, pork	25%	Free
Brooms	40%	15%
Cotton stockings	30%	20%
Woolen yarns	$27\frac{1}{2}$¢ per lb. and 35%	18%
Kitchen utensils	40%	25%

PROBLEMS

1. The duty on imported automobiles is 30%. What is the duty on a machine costing $4600 in Paris?

2. The duty on watch and clock movements is 30%. Find the duty on a watch movement costing $8.50 in Europe.

3. The total value of watch and clock movements imported in a recent year was $2,556,635. Find the duty.

4. The duty on feathers, flowers, etc., used for decorations is 60%. During a recent year the total value of these importations was $11,660,084. Find the duty.

5. The duty on copper wares was reduced by the Underwood-Simmons law from 45% to 20%. This country imported $29,271,540 worth during a recent year. By how much is the duty lessened upon such an amount by the new law?

6. The average duty upon jewelry is 60%. If this country imports yearly jewelry valued at $31,112,110, as it did in a recent year, how much income does the government get from this source?

7. Under the Payne-Aldrich tariff law the duty on combed wool or tops was $24\frac{3}{4}$¢ per pound and 30%. Under the new Underwood-Simmons law it is 8%. During a recent year we imported 266,409,304 pounds of wool, valued at $45,171,994. By how much is the income to the government lessened upon such an importation?

8. The duty upon Brussels carpet was 44¢ per square yard and 40% under the Payne-Aldrich law. It is 25% under the Underwood-Simmons law. Find the difference in duty upon a rug 9′ × 12′ valued at $36.80.

9. The duty upon Oriental and Axminster rugs was 90¢ per square yard and 40% under the old law, and is 50% under the new. Find the difference in duty on each of the following:

Three 8′ × $10\frac{1}{2}$′ Oriental rugs valued at $78 each;
Two 9′ × 13′ Oriental rugs valued at $198 each;
Four 9′ × 12′ Axminster rugs valued at $56 each.

227. Income tax. — An **income tax**, as its name implies, is a tax upon incomes. The present personal income tax consists of a **normal tax** and an **additional tax**.

The normal tax is 2% annually of the net income, above an exemption of $3000 if the person is unmarried and $4000 if married.

When the net income exceeds $20,000 the *additional tax* is:

1% on amounts over $20,000 and not exceeding $40,000.
2% on amounts over $40,000 and not exceeding $60,000.
3% on amounts over $60,000 and not exceeding $80,000.
4% on amounts over $80,000 and not exceeding $100,000.
5% on amounts over $100,000 and not exceeding $150,000.
6% on amounts over $150,000 and not exceeding $200,000.
7% on amounts over $200,000 and not exceeding $250,000.
8% on amounts over $250,000 and not exceeding $300,000.
9% on amounts over $300,000 and not exceeding $500,000.
10% on amounts over $500,000 and not exceeding $1,000,000.
11% on amounts over $1,000,000 and not exceeding $1,500,000.
12% on amounts over $1,500,000 and not exceeding $2,000,000.
13% on amounts over $2,000,000.

PROBLEMS

1. Find the income tax of an unmarried person having an annual income of $75,000.

SOLUTION

Normal tax, 2% of $72,000 ($75,000 − $3000) =	$1440
Additional tax, 1% of $20,000 ($40,000 − $20,000) =	200
Additional tax, 2% of $20,000 ($60,000 − $40,000) =	400
Additional tax, 3% of $15,000 ($75,000 − $60,000) =	450
Total tax =	$2490

2. Find the income tax of a married person whose income is $65,000 annually.

Find the income tax upon the following incomes of married people:

3. $9000.	6. $90,000.	9. $300,000.
4. $25,000.	7. $125,000.	10. $450,000.
5. $70,000.	8. $200,000.	11. $750,000.

FIRE INSURANCE

228. Insurance. — Insurance is an agreement by an **insurance company**, for a consideration called a **premium**, to indemnify the insured party for actual losses arising from certain stipulated causes. The agreement, or contract, between the parties is called the **policy**.

229. Fire Insurance. — Fire insurance is an agreement to indemnify the insured against actual losses from accidental fires. The "loss by fire" includes any damage resulting from chemicals or water used in extinguishing the fire. Fire caused by lightning is usually included under accidental fires.

230. Form of Policy. — The form of policy is regulated by the laws of the various states. It is a fixed form of agreement to do or not to do certain things for a fixed consideration.

231. Riders. — The **riders** are certain clauses attached to the policy granting certain privileges or making certain restrictions.

232. The co-insurance or average clause. — Among the standard "riders", the most important is the **average** or

co-insurance clause. This is an agreement on the part of the insured to carry a certain amount of insurance upon his property. Failing to do this, the insured becomes a co-insurer with the company for whatever amount his insurance lacks of the amount agreed upon. Under a co-insurance clause, then, the liability of the company is limited to no greater proportion of a loss or damage than the sum insured bears to a certain per cent of the actual value of the property at the time of the loss. The amount most commonly carried under a co-insurance clause is 80 % of the actual value.

Thus, if I accept an 80 % co-insurance clause as a part of my contract upon property worth \$10,000, I must carry \$8000 or, failing to do so, assume that part of the risk that the deficiency bears to \$8000. Thus if I insure for but \$6000, I must bear $\frac{1}{4}$ of any loss or damage.

233. Premium rates. — Rates vary depending upon the nature of the risk. Upon a building they depend very largely upon: (1) the location; (2) the construction; (3) the occupancy; (4) the exposure; (5) and whether or not there is co-insurance.

The rate is usually quoted as cents per \$100. Thus a premium on a policy of \$6000 at 24 ¢ per \$100 is 60 × 24 ¢, or \$14.40. Usually the premium for a 3-year policy is $2\frac{1}{2}$ times the premium on a 1-year policy.

234. Other kinds of property insurance. — Fire insurance has been discussed here, being one of the common types of *property insurance.* However, there are numerous kinds of insurances. Among the most common are tornado, lightning, burglary, live stock, marine, plate glass, steam boilers, transit, automobile, etc. The principles of all are similar. The contract between the insured and insurer is always

called the policy, and shows the conditions under which the insurer guarantees to indemnify for losses.

PROBLEMS

1. A man insured his property valued at \$8000 for but \$5000. In case of a \$2000 loss, how much will he get if no co-insurance clause is made part of the contract? How much with an 80 % co-insurance clause?

2. If a man has property valued at \$15,000, for how much should it be insured to satisfy the 80 % co-insurance clause? If insured for \$8000 in one company and \$4000 in another, how much will each pay upon a loss of \$1800?

3. A building valued at \$8400 is insured for \$7000 at 60¢ per \$100 for a 3-year term. Find the premium.

4. Had the property described in Problem 3 been insured under a 100 % co-insurance clause, what part of a loss would the owner have to sustain? In case of a loss of \$2800, how much would the insurance company have to pay?

5. Property valued at \$10,000 was insured in one company for \$3000 and in another for \$5000, no co-insurance clause. A loss by fire amounted to \$2400. How much must each pay?

6. Suppose the property described in Problem 5 had been insured for \$3000 in one company and but \$2000 in another, no co-insurance clause, how much of a \$2400 loss would each pay?

7. In Problems 5 and 6, had an 80 % co-insurance clause been made part of the contract, how much would each company have paid in each case?

8. On account of the fire protection of most towns and cities there is seldom a total loss. Study Problems 5, 6,

and 7 and show why a company can give a reduced rate when making a co-insurance clause part of the contract.

9. If a store and its contents are insured in three companies for $10,000, $15,000, and $25,000 respectively, and the property is damaged by fire and water to the amount of $6000, no co-insurance, how much will each company have to pay?

10. I insure my property for $6000, accepting as a part of the contract an 80% co-insurance clause. At the time of a $600 loss the actual value of the property is $9000. How much can I collect?

11. A policy may be canceled by the insurer by giving the insured 5 days' notice. In such cases, such part of the premium is returned as the unexpired term is of the whole term of insurance. If I have paid $140 for a 3-year policy and it is canceled by the company after 1 yr. 8 mo., how much of the premium should be returned to me?

12. If a policy is canceled by the insured, there is returned to him the difference between what he has paid and the premium for the term the policy has run at the company's "short-rates schedule". I insured my property for $16,000 for a 3-year term at $2\frac{1}{2}$ times the annual rate of 22¢ per $100. At the end of 180 days I surrender my policy. The "short rates schedule" for that period was 70% of the annual premium. How much should be returned to me?

LIFE INSURANCE

235. Life insurance. — A **life insurance policy** is a contract whereby the insurer, for a consideration called a **premium**, agrees to pay a specified sum of money at the death of the insured or at some fixed time.

236. Kinds of policies. — There are three general forms of policies: (1) **ordinary life**; (2) **limited life**; and (3) **endowment.**

In the *ordinary life policy*, the premiums are paid during the life of the insured, and the insurer agrees to pay a fixed sum to the heirs of the insured, or to some other party designated by him, at his death. The person named in the policy to receive the payment upon the death of the insured is called the **beneficiary.**

In the *limited life policy*, the premiums are paid for a fixed number of years but the sum guaranteed in the policy is not paid until the death of the insured.

In an *endowment policy*, the insurer agrees to pay the insured the whole amount named in the policy if he survives beyond a specified date, or to pay the amount as above if he dies before this date.

237. The elements of a premium. — The annual **gross premium** paid by the insured is made up of three items: (1) *mortality cost;* (2) *reserve;* and (3) *expense loading.* The first two of these items form the **net** or **mathematical premium.** These two items are determined by a given **mortality table** which shows the number of persons out of a given number of a given age that die during the year and each succeeding year thereafter as determined from actual experience extending over a long period of years.

The *American Mortality Tables* are compiled from the experience of the New York Mutual Life Insurance Company and are the tables prescribed by statute in most of the states as the basis upon which the reserve of life insurance companies shall be computed.

NOTE. — The kind of life insurance considered here is called "old line" insurance. In fraternal insurance, and other forms of assessment insurance, there is no reserve element.

AMERICAN TABLE OF MORTALITY

Completed Age	Number surviving at Each Age	Deaths in Each Year	Death Rate per 1000	Average Future Lifetime[1] Years	Completed Age	Number surviving at Each Age	Deaths in Each Year	Death Rate per 1000	Average Future Lifetime Years
10	100,000	749	7.490	48.7	55	64,563	1,199	18.571	17.4
11	99,251	746	7.516	48.1	56	63,364	1,260	19.885	16.7
12	98,505	743	7.543	47.5	57	62,104	1,325	21.335	16.1
13	97,762	740	7.569	46.8	58	60,779	1,394	22.936	15.4
14	97,022	737	7.596	46.2	59	59,385	1,468	24.720	14.7
15	96,285	735	7.634	45.5	60	57,917	1,546	26.693	14.1
16	95,550	732	7.661	44.9	61	56,371	1,628	28.880	13.5
17	94,818	729	7.688	44.2	62	54,743	1,713	31.292	12.9
18	94,089	727	7.727	43.5	63	53,030	1,800	33.943	12.3
19	93,362	725	7.765	42.9	64	51,230	1,889	36.873	11.7
20	92,637	723	7.805	42.2	65	49,341	1,980	40.129	11.1
21	91,914	722	7.855	41.5	66	47,361	2,070	43.707	10.5
22	91,192	721	7.906	40.9	67	45,291	2,158	47.647	10.0
23	90,471	720	7.958	40.2	68	43,133	2,243	52.002	9.5
24	89,751	719	8.011	39.5	69	40,890	2,321	56.762	9.0
25	89,032	718	8.065	38.8	70	38,569	2,391	61.993	8.5
26	88,314	718	8.130	38.1	71	36,178	2,448	67.665	8.0
27	87,596	718	8.197	37.4	72	33,730	2,487	73.733	7.6
28	86,878	718	8.264	36.7	73	31,243	2,505	80.178	7.1
29	86,160	719	8.345	36.0	74	28,738	2,501	87.028	6.7
30	85,441	720	8.427	35.3	75	26,237	2,476	94.371	6.3
31	84,721	721	8.510	34.6	76	23,761	2,431	102.311	5.9
32	84,000	723	8.607	33.9	77	21,330	2,369	111.064	5.5
33	83,277	726	8.718	33.2	78	18,961	2,291	120.827	5.1
34	82,551	729	8.831	32.5	79	16,670	2,196	131.734	4.7
35	81,822	732	8.946	31.8	80	14,474	2,091	144.466	4.4
36	81,090	737	9.089	31.1	81	12,383	1,964	158.605	4.1
37	80,353	742	9.234	30.4	82	10,419	1,816	174.297	3.7
38	79,611	749	9.408	29.6	83	8,603	1,648	191.561	3.4
39	78,862	756	9.586	28.9	84	6,955	1,470	211.359	3.1
40	78,106	765	9.794	28.2	85	5,485	1,292	235.552	2.8
41	77,341	774	10.008	27.5	86	4,193	1,114	265.681	2.5
42	76,567	785	10.252	26.7	87	3,079	933	303.020	2.2
43	75,782	797	10.517	26.0	88	2,146	744	346.692	1.9
44	74,985	812	10.829	25.3	89	1,402	555	395.863	1.7
45	74,173	828	11.163	24.5	90	847	385	454.545	1.4
46	73,345	848	11.562	23.8	91	462	246	532.466	1.2
47	72,497	870	12.000	23.1	92	216	137	634.259	1.0
48	71,627	896	12.509	22.4	93	79	58	734.177	.8
49	70,731	927	13.106	21.6	94	21	18	857.143	.6
50	69,804	962	13.781	20.9	95	3	3	1000.000	.5
51	68,842	1,001	14.541	20.2					
52	67,841	1,044	15.389	19.5					
53	66,797	1,091	16.333	18.8					
54	65,706	1,143	17.396	18.1					

[1] Average Future Lifetime is sometimes called "Expectation of Life."

238. The mortality cost. — The **mortality cost** is the amount necessary to collect at the beginning of each year to pay the death claim of the current year as determined by the mortality tables.

239. The reserve. — The **reserve element** of a premium is the amount which from each premium at a given rate of compound interest, usually 3% or $3\frac{1}{2}$%, will amount to the face of the policy at a given time. Just as a sinking fund (§ 203) builds up the amount needed to meet a certain obligation, so the reserve element of a life insurance premium amounts in a fixed time to the face of the policy. One, then, is gradually paying off his own policy, as it were, and if he lives beyond the time for which the rate was computed he has paid it in full and is helping to pay for those who die before the computed time.

The company's real *risk* at any time is the difference between the face of the policy and the amount of the reserve.

240. The expense loading. — The **expense loading** is a fixed per cent of the *mathematical premium* that is added to meet the expenses of the management. This loading ranges from $16\frac{2}{3}$% to 25% of the net premium.

241. The cash surrender value of a policy. — It was seen in § 239 that the reserve is creating a fund which in time amounts to the face of the policy. It is a sort of *savings bank account* with the company.

It really belongs to the insured, and the company will, upon the surrender of the policy, give the holder a cash payment which is practically the reserve on his policy. This is called the **cash surrender value** of a policy.

Instead of taking cash, one may take a paid-up policy for the amount the reserve will purchase; or take extended

insurance for the face value of the policy for as long a period as the reserve will purchase.

242. The surplus. — There are three main sources of **surplus** in a life insurance company, viz.:

1. A *saving from the expense loading in a premium;* for most companies provide for a greater expense than they actually use.

2. A *saving in mortality cost;* for by using care in selecting risks, the *actual* mortality is usually less than the *expected* mortality upon which the premiums were based.

3. *The interest earned on the reserve in excess of the rate paid upon it;* for just as a savings bank that pays 3% or $3\frac{1}{2}$% upon its deposits receives a rate in excess of this, so a life insurance company usually earns more than the rate upon which the required reserve element of the premium was computed.

243. The dividends. — That part of the surplus which is distributed among the policy holders is called a **dividend.** It is reckoned as a per cent of the total reserve of the company, hence each policy holder gets that per cent of the reserve that is credited to his policy. If the rate of dividend remains the same, the dividend on a policy increases from year to year during the life of the policy. Why?

The dividends may be used: (1) to reduce the next premium; (2) to purchase additional insurance, payable when the policy matures; or (3) to reduce the number of payments.

Those policies whose holders receive a dividend are called **participating policies.** All others are **non-participating.** Since the holders of non-participating policies do not receive

a dividend, which in reality is a return of an overcharge, they usually pay a smaller premium.

244. The problems of insurance. — The computation of the premiums, etc., upon the various policies is done by an *actuary*. Such computations are too difficult for this course. There are problems, however, that arise in the minds of buyers of insurance and those interested that are easily understood.

PROBLEMS

1. From the American Experience Table of Mortality, the company expects that of 78,106 persons of 40 years of age, 765 will die during the year. With a membership of that number each insured for $1000, how much must be collected from each at the beginning of the year to meet the claims, granting that the money bears 3% interest, and that the claims are due at the end of the year?

2. A man 30 years of age insured his life on the Ordinary Life Plan for $1000 paying $21.95 annually, paid at the beginning of each year. If death occurs at the end of the 6th year (before the 7th payment has been paid), what is the balance in favor of life insurance to his estate against placing each premium in a savings bank paying 4% interest compounded annually?

3. Had the man described in Problem 2 died at the end of the 20th year, compare the savings bank investment with life insurance.

4. A man at 42 years of age took out a 20-payment policy, non-participating, for $8000, paying $37.20 per $1000. Suppose he dies at the end of the 12th year, compare insurance with a 4% savings bank investment.

5. In Problem 4, make a similar comparison if he lives until the end of the 20th year.

6. A man at 25 years of age can take out a $1000 ordinary life policy for $20.14 or a 20-payment life for $30.12. At the end of 10 years the dividends will have averaged $3.05 per year in the first, and $4.50 per year in the second. The first will have a cash surrender value of $98.94, and the second of $208.95. Suppose that he may wish to surrender his policy at the end of the 10th year, which will be the better for him to have taken?

SOLUTION. — Subtracting the average dividends, the net premiums were $17.09 and $25.62, respectively. Thus, an excess of $8.53 annually in the 20-payment plan gives $110.01 greater cash value at the end of 10 years, or $12.90 for every $1. It is seen from the tables on page 246 that this is equivalent to over 4% and less than 5% compound interest. This is but a fair rate of interest, hence there is but little difference.

7. Suppose the man designated in Problem 6 had taken a 10-year endowment costing $101.85, average dividends $15 per year, compare with the 20-payment life, as in the solution above, at the end of 10 years.

SUGGESTION. — The cash value of the 10-year endowment at the end of 10 years is $1000.

8. In the same way, compare for the same time an ordinary life policy, from the data of Problem 6, with a 20-year endowment costing $48.15, average dividends $7.14, with a cash surrender value of $407.79.

9. At age 55 an ordinary life policy will cost $56.93, and a 20-payment life $62.68. It is estimated that the average yearly dividend for the first 10 years in each will be $8.81 and $9.67 respectively. The cash surrender value at the end of 10 years will be $290.50 in the first and $362.37 in the other. If surrendered at the end of 10 years, compare the 20-payment with the ordinary life.

SUGGESTION. — It will be found that the extra cash surrender value is equal to $14.70 per extra $1 paid in premiums. Since that is larger than any amount given in the table for 10 years, it is more than 6 %. By doubling the number of periods and the amount, it is seen that for 20 periods $29.40 is a little more than 3½ %. Hence the interest is more than 7 % payable semi-annually.

10. A 10-year endowment at age 55 costs $113.74. If the average dividend is $17.41, compare with ordinary life if surrendered at the end of 10 years.

11. A 20-year endowment at age 55 costs $66.36 per $1000. The average dividends are $10.33. The cash surrender value at the end of 10 years is $408.33. Compare with an ordinary life policy if surrendered at the end of 10 years.

Fill the following table showing the balance in favor of life insurance or savings bank for a man who, at 45, takes out ordinary life at $33.32 per $1000.

	IF DEATH OCCURS AT END OF	AMOUNT OF MONEY INVESTED		SAVINGS BANK ACCUMULATION AT 4% ANNUALLY		INSURANCE PAID		BALANCE IN FAVOR OF BANK OR INSURANCE	
12.	5 yr.								
13.	10 yr.								
14.	15 yr.								
15.	20 yr.								
16.	25 yr.								
17.	30 yr.								
18.	35 yr.								

19. A man, age 41, took out an ordinary life policy for $1000, paying $32.02 annually, less yearly dividends. At the end of the third year, his reserve credit was $40.94. If a 4 % dividend is declared, how much will it reduce his fourth premium? At the end of 15 years the reserve is $289.70. A 4½ % dividend will leave how much to pay on the sixteenth premium?

MISCELLANEOUS APPLICATIONS OF PERCENTAGE

1. The gross sales of a manufacturing company during a certain period amounted to $240,000. The gross profit was 25 % of the sales. The total cost of selling including rent, light, management, etc., equaled 19 % of the sales. What per cent of the sales was net profit? How much was the net profit?

2. The cost to manufacture a certain gasoline engine was found to be $38.49. The total cost of selling it was estimated to equal 40 % of the cost to manufacture. At what must it be listed to give a net profit of 15 % of the sales?

3. At what must the engine described in Problem 2 be listed to give a profit of 15 % of the list price after discounting it 20 %? To give 15 % of the net selling price after discounting the list price 20 %?

4. At what must the same engine be listed to give a profit of 20 % of the selling price after discounting the list price $33\frac{1}{3}$ %?

5. The total cost to the manufacturer of making and selling an automobile was $1258. If the manufacturer wishes to make 15 % of the price at which he sells to the dealer, and the dealer 20 % of the price at which he sells to the customer, what retail price must be put upon the automobile?

6. One year a dealer's gross sales were $86,480. The goods had all been sold at an average advance of 60 % of the cost. The total expenses that year equaled 40 % of the sales. Find the rate of loss or gain on the sales.

7. If an importer pays $28.50 abroad for an article, and pays a duty of 30 %, and $6.95 transportation charges, for what must he sell it to make a profit of 20 % on the total cost? To make 20 % of the selling price?

8. In a department of a retail business the average sales of a clerk getting a salary of \$22.50 per week were \$50 per day. The cost of management and other charges equaled $10\frac{1}{2}$ % of the sales. If the goods of that department were marked to gain $33\frac{1}{3}$ % on the delivered cost, the net profit per day was what per cent of the sales?

9. In Problem 8, what was the net weekly profit to the business from this clerk's sales?

10. By what must a merchant multiply the delivered cost of an article to find what his net gain will be when goods are marked 40 % above delivered cost and the total cost of doing business equals $12\frac{1}{2}$ % of the sales?

11. By what must a merchant multiply the delivered cost of an article to find what his net gain will be on a certain article when the gross gain averages 30 % of the sales and the total cost of doing business is 16 % of the sales?

12. By what must a merchant divide the delivered cost of an article to get a selling price that will net him 12 % of the sales after deducting 18 % of the sales for the expenses incurred in selling the goods?

13. Successive discounts of $33\frac{1}{3}$ %, 20 %, and 10 % on an invoice of goods are equal to what single discount on the invoice?

14. Find which is best for the buyer: successive discounts of 40 % and 40 %, 60 % and 20 %, or 70 % and 10 %.

15. It would seem from Problem 14 that when the sums of two successive discounts are the same the more the discounts differ from each other the better they are for the buyer. Try other discounts and see if the same thing holds true.

NOTE. — While what seems to be true from special cases in Problems 14 and 15 is true in general, to prove it would require a use of algebra that cannot be assumed here.

16. A man is living in a house costing him $8500 for house and lot. His taxes are $122.50 a year. The general upkeep amounts to $75 a year. If he could have loaned his money at 6 %, how much "rent" per month is it really costing him, considering that the value of the property is not changing?

17. A man bought a piece of unimproved real estate for $3600. He held it 5 years and sold it for $4750. The taxes were $35.50 each year. He paid an agent $2\frac{1}{2}$ % commission for selling the property. There were incidental expenses amounting to $23.75 connected with the transaction. His gain was how much more or less than a savings bank deposit of 4 % compounded semi-annually? (Do not consider interest on the taxes.)

18. A note of $1650 dated June 1, 1916, to run one year at 6 % was discounted at a bank on April 20, 1917, at 6 %. Find the proceeds on the note.

19. A merchant had an invoice of $3680 billed "net in 60 days", or 2 % off for cash in 10 days. How much can he save by borrowing the money at a bank at 6 % for 60 days and getting his cash discount on the invoice?

20. A man bought stock at $134\frac{1}{2}$, brokerage $\frac{1}{8}$ %, and sold in 14 months at $129\frac{1}{4}$, brokerage $\frac{1}{8}$ %, after having received an 8 % dividend. Compare his dividend, after having deducted his loss in selling, with interest on the investment at 6 %.

21. A man has 200 shares of stock paying a 10 % dividend which he can sell at 198, brokerage $\frac{1}{8}$ %. By how much will his income be increased or decreased if he sells his stock and loans the money at $5\frac{1}{2}$ %?

22. A man is offered $10,250 for a house which he is renting for $95 per month. If taxes, insurance, and upkeep

amount to \$285 per year, by how much will his net income be changed by selling and investing in non-taxable $4\frac{1}{2}\%$ bonds at $102\frac{1}{2}$, no brokerage?

23. If you insure your property valued at \$9000 for but \$5000 after accepting an 80% co-insurance clause as a part of the contract, how much will you collect in case of a \$2500 loss?

24. Property worth \$30,000 is insured in 4 companies for \$4000 in each, no co-insurance. In case of a \$12,000 loss, how much will each company have to pay?

25. In case of an 80% co-insurance clause in Problem 24 how much will each company have to pay? How much will the owner lose?

26. Goods costing me \$2600 were sold through an agent for \$3000. The agent's commission was 8% of the sales. Other charges amounted to \$45. My profit equaled what per cent of the cost?

27. A manufacturer made four types of gasoline engines costing him \$52.50, \$56, \$84, and \$94.50, respectively, to manufacture. He wishes to list them so as to make 25% of the list price after giving a discount of 5%. Find at what price he must list them.

28. The capital of a certain company is \$115,000,000. The sales one year were \$54,377,151. The total expenses were \$45,178,731. The net profit was what per cent of the sales? Find, to the nearest whole number, the dividend that can be paid and the surplus left undistributed.

29. A company has \$165,000,000 outstanding common stock, \$15,000,000 outstanding 7% preferred stock, and \$800,000 outstanding $4\frac{1}{2}\%$ bonds. One year the gross sales were \$94,424,841. The expenses, not including preferred

dividends and interest, were $79,498,222. Find the surplus and what per cent of the sales it is. Also find what dividend, to the largest whole number, can be paid on the common stock.

30. What would be a reasonable market value for the stock of the company described in Problem 29 in case the business seems to be a permanent one?

31. I insure property worth $24,000 for $15,000, accepting an 80% co-insurance clause as part of the contract. What per cent of the risk do I assume? In case of a loss of $2600, how much do I collect?

32. A man at 60 years of age takes out a 10-year $1000 endowment policy paying $121.68 annually. The average yearly dividends are estimated at $18.60. If he lives to receive the insurance, how much does he receive? How much has he actually invested? To what would his actual investment have amounted had he placed the premiums in a savings bank paying 4% compounded annually?

33. A merchant bought goods listed at $8565, at 25% and 10% off, "net 60 da." He can get a further discount of 5% for cash. How much can he save by borrowing the money at a bank at 6% and paying cash?

34. A wholesaler sold a bill of goods listed at $2750 less 25% and 20%, net 60 days, drawing upon his customer by a 60-day draft which is accepted 5 days after date? Find the proceeds if discounted at a bank at 6% on the day of acceptance.

35. How much yearly is necessary to create a 4% sinking fund to meet an obligation of $20,000 due in 20 years?

36. Why is the per cent of profit reckoned on the selling price a smaller rate than if reckoned upon the cost?

37. Why will loss reckoned upon the selling price give a larger rate than upon the total expenditures?

38. Show in what way a merchant may be deceived as to his rate of real gain when he reckons his gross gain upon the cost, and the cost of doing business upon the sales.

39. Illustrate Problem 38 by a concrete example.

40. A new billing clerk billed an invoice of \$345 less 35% and 10% as if it were 35% + 10%. Find the loss to the firm.

41. Common stock and preferred stock in General Motor Company ranged as given below from 1911 to 1915. How do you account for the fact that the preferred stock did not fluctuate as much as the common stock?

	1911		1912		1913		1914		1915	
	High	Low	High	Low	High	Low	High	Low	High	Low
Common	$51\frac{3}{4}$	35	$42\frac{7}{8}$	30	40	25	99	$37\frac{1}{2}$	558	82
Preferred	86	$74\frac{1}{2}$	$82\frac{3}{4}$	$70\frac{1}{4}$	$81\frac{3}{4}$	70	95	70	136	$90\frac{5}{8}$

42. Make two graphs on the same paper showing the variations of the stock listed in Problem 41.

43. The gross sales in a certain department were \$46,582. The delivered cost of the goods was \$32,465. The cost of doing business equaled $22\frac{1}{2}\%$ of the sales. The returned goods amounted to $2\frac{1}{2}\%$ of the sales. The net gain was what per cent of the *net* sales?

44. The analysis of a business one year showed the following: Gross gain 38% of the sales; cost of selling and delivery 21% of the sales; loss in bad debts 5% of the sales. The net gain was what per cent of the amount collected?

45. If a manufacturer lists his goods to give a profit of 40% of the list price but discounts the list price 20%, what per cent of the cost to manufacture is he making?

46. If the manufacturer had listed his goods 40 % above cost to manufacture and discounted them 20 %, what per cent of the net selling price would he have made?

47. At what price must an article costing $13.05 be listed so as to give a profit of 25 % of the net sales after allowing discounts of $33\frac{1}{3}$ % and 25 %?

48. A man carries the following insurance upon his property: $3000 in one company, $5000 in another, and $8000 in a third. At the time of a $6400 loss the value of the property is $27,500. Under an 80 % co-insurance clause, how much will each company pay and how much will the owner lose? Without the clause, how much will each company pay?

49. What per cent of his actual receipts is a merchant losing who sells his goods 50 % above cost, if the cost of doing business equals 30 % of the sales and there is a loss of 5 % of the sales in bad debts?

50. A merchant has his goods marked so as to make an average of 30 % of the sales. One fifth of the goods, however, must be sold out at the end of the season at a discount of 25 % of the marked price. What per cent of the actual receipts are made, not counting the cost of doing business?

51. How much must a man invest semi-annually for 20 years at 4 %, compounded semi-annually, to have a sum that at the expiration of the 20 years will net him $935 annually at $5\frac{1}{2}$ % simple interest?

52. If a man deposits $300 semi-annually for 15 years at 4 %, compounded semi-annually, and at the end of 15 years loans the amount thus accummulated at $5\frac{1}{2}$ % simple interest, find his income.

CHAPTER IX

RATIO, PROPORTION, AND VARIATION

245. Ratio. — When one number is divided by another number of the same kind, the quotient is the **ratio** of the first number to the second. Then from § 31, 1, it is seen that:

A ratio is always an abstract number.

The ratio of two numbers may be expressed in three ways. Thus, the ratio of 3 to 5 is written $\frac{3}{5}$, $3 \div 5$, or $3:5$. Of these forms the first two are in most common use.

246. Uses of ratio. — Need of ratio, or comparison, is found on every hand. In finding what per cent one number is of another, we find the *ratio* of one to the other and express it as per cent. The comparison of one area or volume with another is expressed by their ratio. In many of the solutions of problems, the ratio of the thing wanted to the thing given is used in getting the result. Thus, if 5 things cost \$20, 3 similar things should cost $\frac{3}{5}$ of \$20; that is, the ratio of 3 to 5 is used.

EXERCISES

Give the ratios of:

1. 3 to 7.

2. 7 to 9.

3. 6 to 14.

4. 9 to 21.

5. 16 to 56.

6. 25 to 10.

7. 35 to 14.

8. $\frac{1}{2}$ to $\frac{2}{3}$.

9. $\frac{3}{4}$ to $\frac{5}{6}$.

10. $\frac{7}{8}$ to $\frac{1}{2}$.

11. $\frac{5}{8}$ to $\frac{2}{3}$.

12. $\frac{4}{5}$ to $\frac{7}{8}$.

13. $\frac{3}{7}$ to $\frac{2}{3}$.

14. $1\frac{1}{2}$ to 5.

15. $1\frac{1}{2}$ ft. to 38 in.

16. 11 in. to $2\frac{1}{2}$ ft.

17. 3 pt. to 2 qt.

18. 84 sq. in. to 1 sq. ft.

19. 8 mo. to $1\frac{1}{2}$ yr.

20. 14 oz. to $2\frac{1}{2}$ lb.

21. 11 ft. to 6 yd.

22. What is the ratio of the area of a 3-ft. square to that of a 5-ft. square?

23. Compare the area of a 4-ft. circle with that of a 10-ft. circle.

24. What is the ratio of the volume of a 5-ft. sphere to that of a 15-ft. sphere?

25. Compare the area of a rectangle 6 ft. by 8 ft. with that of a 10-ft. square.

26. Compare the area of a triangle whose altitude and base are 6 ft. and 8 ft. respectively with that of a rectangle 8 ft. by 12 ft.

27. Compare the yearly income from \$2000 at 6 % interest with that from \$4000 at 5 %.

28. Compare a discount of 40 % with two successive discounts of 30 % and 30 %.

29. Compare the surface of a 6-in. sphere with the total surface of a cylinder 4 in. in diameter and 8 in. high.

30. What is the ratio of the area of a 12-ft. circle to the combined areas of a 6-ft. and a 10-ft. circle?

PROBLEMS

1. If a field 12 rd. by 20 rd. produces 270 bu. of potatoes, what should be the yield of a field 18 rd. square at the same rate?

SOLUTION

$$\frac{\overset{3}{\cancel{18}} \times \overset{9}{\cancel{18}}}{\underset{2}{\cancel{12}} \times \underset{10}{\cancel{20}}} \times \overset{27}{\cancel{270}} \text{ bu.} = 364\tfrac{1}{2} \text{ bu.}$$

EXPLANATION.— The ratio of what is wanted to what is given is the ratio of the areas, or the ratio of 18 × 18 to 12 × 20. Multiplying the 270 bu. by this ratio gives the result required.

2. If a walk 3 ft. wide and 200 ft. long costs $125, how much should one 5 ft. wide and 180 ft. long cost at the same rate?

3. The excavation for a cellar 34 ft. by 36 ft. and 6 ft. deep cost $120. At the same rate, what will one 36 ft. by 40 ft. and 6 ft. deep cost?

4. In a corporation whose capital stock is $8,000,000, there are $750,000 of outstanding bonds. What is the ratio of the bonds to the stock?

5. In a certain department the gross gain was 40 % of the sales and the expenses of doing business equaled 25 % of the sales. Compare the cost of doing business with the gross gain. With the net gain.

6. At $24, a suit sold at a gain of 20 % on the cost. What would the rate of gain have been had it sold for $30?

7. Compare the area of a 12-inch circle with that of a 12-inch square. If both are cut from the same sheet of metal, what will the circle weigh when the square weighs $2\frac{1}{2}$ lb.?

8. A cubic foot of water weighs 62.5 lb., and a cubic foot of ice weighs 56.25 lb. Compare the weight of a cubic foot of ice with that of a cubic foot of water.

9. A cubic inch of brass weighs 4.848 oz. Compare the weight of brass with the weight of an equal volume of water.

10. On the Fahrenheit thermometer the freezing point is 32° and the boiling point is 212°. On the Centigrade thermometer the freezing point is 0° and the boiling point is 100°. Compare a change in temperature as registered upon the Fahrenheit thermometer with the same change registered upon the Centigrade thermometer. A change of 36° F. is a change of how many degrees C.?

247. Application of ratio to specific gravity. — The **specific gravity** of a substance is the ratio of the weight of that substance to the weight of an equal volume of some substance taken as a standard. For the specific gravity of solids and liquids, distilled water is usually taken as the standard substance. For gases, either air or hydrogen is taken as the standard substance.

Thus, any volume of steel is 7.8 times as heavy as an equal volume of water, or the ratio of the weight of steel to that of an equal volume of water is 7.8. Hence the specific gravity of steel is 7.8.

NOTE. — The **density** of a substance is the weight of a unit of volume of the substance. In the metric system, the unit of volume is the *cubic centimeter*, and the unit of weight is the *gram*, which is the weight of a cubic centimeter of water. It follows that if the weight of a substance is expressed in grams and its volume in cubic centimeters, the specific gravity referred to water as the standard is numerically equal to the density.

SPECIFIC GRAVITIES, REFERRED TO WATER

Platinum	21.5	Brass	8.3	Sulphuric Acid	1.84
Gold	19.3	Steel	7.8	Sea Water	1.026
Mercury	13.6	Tin	7.3	Linseed Oil	0.94
Lead	11.3	Zinc	7.2	Ice	0.92
Silver	10.5	Cast Iron	7.	Alcohol	0.79
Bronze	8.9	Granite	2.7	Petroleum	0.7
Nickel	8.5	Glass	2.5	Oak	0.86
Copper	8.9	Sulphur	2.	Cork	0.24

The weight of a cubic foot of water is 62.5 *pounds.*

PROBLEMS

1. What is the weight of a cubic foot of lead? Of cast iron?

2. What is the weight of 5 cu. ft. of granite? Of 10 cu. ft. of oak?

3. What is the weight of 1 cu. ft. of copper? Of glass? Of bronze?

4. What is the weight of 1 cu. in. of gold? Of silver? Of platinum?

5. What is the weight of 3 cu. in. of mercury? Of 12 cu. in. of nickel?

6. What is the weight of a block of ice 12 in. thick, 18 in. wide, and 35 in. long?

7. How many tons of ice will an ice house hold that is 24 ft. by 30 ft. by 20 ft., allowing 1 ft. on all sides and above and below for sawdust?

8. The specific gravity of good milk is 1.032. If 520 cu. cm. of the milk taken from a can in a certain dairy wagon is found by the inspector to weigh 532 g., is the milk good?

9. One cubic foot of a certain kind of limestone is found to weigh 187.5 lb. What is the specific gravity?

10. How many ounces (avoirdupois) of alcohol will a "two-ounce" bottle hold?

11. How many grams of mercury will a glass tube contain whose cross-sectional area is 1 sq. cm. and length 20 cm.?

12. An object lighter than water will sink until the volume of water displaced equals the weight of the object. To what depth will an oak beam 12 inches square and 10 feet long sink?

13. A grindstone when new is 6 ft. in diameter and has a 14-in. face. What has it lost in weight when it is worn down 2 in.? (The specific gravity of sandstone is 2.42.)

14. What weight of brass is cut off in turning a piece 1 in. in diameter and 1 ft. long from a bar $1\frac{1}{4}$ in. square and 1 ft. long?

15. A tank car has a tank 7 ft. 9 in. inside diameter and 34 ft. long. What weight of petroleum will it carry?

16. A railroad track tank 7 in. deep, 19 in. wide, and 1200 ft. long (inside measures) is made of $\frac{3}{16}$-in. steel plate. Find its weight, no allowance being made for laps, joints, rivets, or stiffening. (The tank is open the entire length at the top to allow the tender scoop to take water while the train is in motion.)

PROPORTION

248. Proportion. — The statement that one ratio equals another is called a **proportion**.

Thus, $\frac{3}{5}=\frac{12}{20}$, or $\frac{a}{b}=\frac{c}{d}$, is a proportion.

In the proportion $\frac{a}{b}=\frac{c}{d}$, the numbers a, b, c, and d are called the **terms**; a and d are called the **extremes** and b and c the **means**.

If the two mean terms are alike, as in $\frac{a}{n}=\frac{n}{b}$, n is called the **mean proportional** between a and b.

In the past a proportion was written in the form $a:b=c:d$, or $a:b::c:d$. These forms are becoming obsolete.

249. Principles of proportion. — There are certain principles of proportion that may be derived by observing that if one number equals another, the products or quotients of these numbers obtained by multiplying or dividing them by equal numbers are equal. Thus, $8=8$, so $2\times 8=2\times 8$, or $8\div 2=8\div 2$. Or if $a=b$, then $na=nb$, and $\frac{a}{n}=\frac{b}{n}$.

Hence in the proportion $\frac{a}{b}=\frac{c}{d}$, if both ratios are multiplied by bd, we obtain $ad = bc$. That is,

I. *The product of the extremes equals the product of the means.*

Also from $ad = bc$, by dividing by d, $a=\frac{bc}{d}\cdot$ And by dividing by a, $d=\frac{bc}{a}\cdot$ That is,

II. *Either extreme equals the product of the means divided by the other extreme.*

Had $ad = bc$ been divided in turn by b and c, the results would have been $c=\frac{ad}{b}$, and $b=\frac{ad}{c}\cdot$ That is,

III. *Either mean equals the product of the extremes divided by the other mean.*

250. The uses of proportion. — In early times proportion, called "The Rule of Three", was used to solve many of the problems of arithmetic. But now its use is confined largely to expressing relations in practical geometry, and in physical science.

The simple problem, "If 5 cows cost \$400, how much will 12 cows cost at the same rate?" was solved by proportion as follows:

$$\frac{5}{12}=\frac{\$400}{x}.$$

$$x=\frac{12\times\$400}{5}=\$960.$$

The reasoning was as follows: The ratio of the number of cows must equal the ratio of their costs. The cost of 12, not being known, is called x. Then by Principle II, x is found.

EXERCISES

Find the value of x in:

1. $\frac{x}{5}=\frac{9}{15}$.
4. $\frac{3}{5}=\frac{8}{x}$.
7. $\frac{5}{x}=\frac{8}{9}$.
10. $\frac{3}{5}=\frac{x}{7}$.
2. $\frac{x}{8}=\frac{1}{12}$.
5. $\frac{2}{7}=\frac{9}{x}$.
8. $\frac{7}{x}=\frac{3}{5}$.
11. $\frac{7}{8}=\frac{x}{9}$.
3. $\frac{x}{6}=\frac{9}{16}$.
6. $\frac{3}{4}=\frac{10}{x}$.
9. $\frac{2}{x}=\frac{5}{8}$.
12. $\frac{2}{3}=\frac{x}{11}$.

Find the mean proportional between:

13. 3 and 12.
15. 3 and 27.
17. 8 and 32.
14. 4 and 25.
16. 5 and 45.
18. 8 and 18.

19. In a city whose population is 96,000, the number of Germans is 12,000. What is the ratio of the German population to the whole population?

20. If the ratio found in Exercise 19 does not change, how many Germans will there be when the total population is 120,000?

251. Partitive proportion. — The process of dividing a number into parts proportional to several given numbers is called **partitive proportion.**

Thus, divide \$24 among three boys so that their shares are proportional to 1, 2, and 3. Since there are 6 parts $(1 + 2 + 3)$, the first gets $\frac{1}{6}$, the second $\frac{2}{6}$, or $\frac{1}{3}$, and the third $\frac{3}{6}$, or $\frac{1}{2}$.

PROBLEMS

1. A man failing in business owes one man \$5000, another \$8000, and a third \$11,000. His resources are but \$1600. How much can he pay each?

2. Divide \$150 among three persons so that the shares are proportional to 3, 4, and 8.

3. The paving of a street costs \$4500. It is divided among four property holders in proportion to the frontages of their lots, which are 80 ft., 80 ft., 90 ft., and 110 ft., respectively. How much must each pay?

4. If \$60,000 is divided among four people so that the shares are proportional to 1, 3, 3, and 8, find how much each receives.

5. If A invests \$3000, B \$4000, and C \$5000 in a business, and there are net profits of \$3600, how much should each receive?

252. Direct and inverse proportion. — If four numbers, a, b, c, and d are in proportion, so that $\frac{a}{b}=\frac{c}{d}$, then a and b are said to be **directly proportional** to c and d. But if they form the proportion $\frac{a}{b}=\frac{d}{c}$, then a and b are said to be **inversely proportional** to c and d.

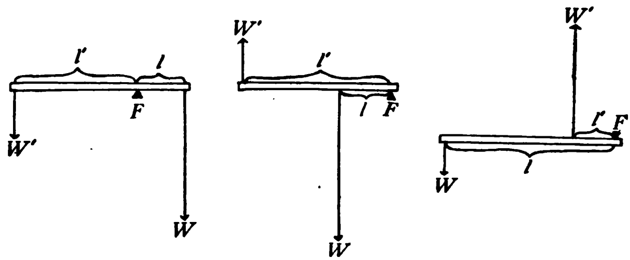
Thus, if a train runs 60 mi. in 2 hr., it should go 90 mi. in 3 hr. $\frac{60 \text{ mi.}}{90 \text{ mi.}}=\frac{2 \text{ hr.}}{3 \text{ hr.}}$. The distances are *directly* proportional to the lengths of time. Again, provisions that would last an army of 1000 men 20 da. should last an army of 4000 men only 5 da. $\frac{1000 \text{ men}}{4000 \text{ men}}=\frac{5 \text{ da.}}{20 \text{ da.}}$. The lengths of time the provisions would last are *inversely* proportional to the numbers of men.

Notice that in the illustration of *direct proportion*, as the time was increased the distances increased with it at the same rate, whereas, in the *inverse proportion*, as the number of men was increased the length of time decreased.

SIMPLE MACHINES

253. Applications of proportion to simple machines. — In every simple machine there are two forces involved: the **resistance**, or force to be overcome, and the **effort**, or force necessary to overcome the resistance. The relation between the resistance and effort depends upon the nature of the machine and upon the dimensions of its parts.

254. The lever. — In the lever, the resistance W and the effort W' are applied at different points of a rigid bar which revolves freely about a point of support called the *fulcrum*. There are three classes of the lever as shown in the figures. The distances l and l', from the fulcrum F to the points of the lever where the resistance and effort, respectively, are applied, are called the *arms* of the lever. It is shown that



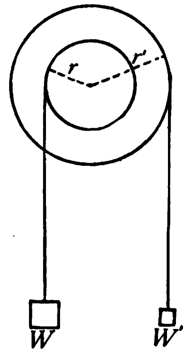
The resistance and effort are inversely proportional to their distances from the fulcrum; that is,

$$\frac{W}{W'}=\frac{l'}{l}.$$

The beam balance, common steelyard, scissors, pincers, crowbar, etc., are familiar examples of levers.

255. Wheel and axle. — This consists of a wheel and cylinder constituting a rigid body, and revolving about the same axis. The effort W' is applied to the circumference of

the wheel, and the resistance W to that of the cylinder by means of a cable winding around it in a direction opposite to the motion of the wheel. It is shown that



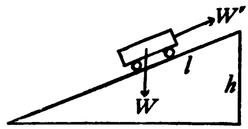
The resistance and effort are inversely proportional to the radii of the cylinder and wheel, respectively; that is,

$$\frac{W}{W'} = \frac{r'}{r}.$$

The windlass and capstan are familiar examples of the wheel and axle.

Modifications of the wheel and axle are wheels on separate axles, the motion being transmitted by means of belts, or by means of teeth in the circumferences of the wheels.

256. The inclined plane. — The inclined plane is a surface inclined at an angle to a horizonal surface. The weight W of an object on an inclined plane tends to cause the object to move down the incline. The force, or effort W', exerted up the incline, and sufficient to hold the object in position, depends upon the relation of the length l to the height h of the incline. It is shown that,

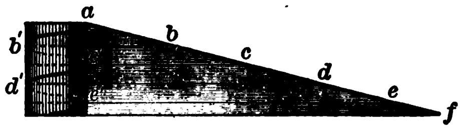


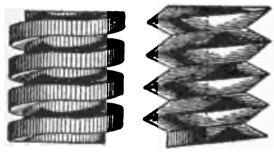
The weight and effort are directly proportional to the length and height of the incline; that is,

$$\frac{W}{W'} = \frac{l}{h}.$$

257. The screw. — The screw is a particular form of the inclined plane. This can be seen by winding a triangular piece of paper about a cylinder as shown in the figure.

The distance between two consecutive threads is called the *pitch* of the screw. The effort is applied at the end of a lever. The relation between the resistance W and the effort W' is the same as for the inclined plane. If the pitch is d, and the lever arm l, it is shown that





The weight and effort are directly proportional to the distance traversed by the effort in one revolution and the pitch of the screw; that is,

$$\frac{W}{W'}=\frac{2\pi l}{d}.$$

The screw is used for overcoming great resistances, such as raising buildings, propelling ships, in the letter-press, etc.

PROBLEMS

1. What effort is required to lift a weight of 800 lb. by means of a lever of which the weight-arm is 6 in. and the effort-arm 2 ft. 6 in.?

2. A pump handle is 3 ft. 8 in. long, and works on a pivot 4 in. from the end attached to the pump rod. What force is applied to the pump rod when the handle is pushed down with a force of 10 lb. weight?

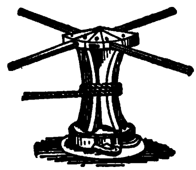
3. An oarsman, in rowing, uses an oar with which the distance from the rowlock to the water is 3 times the distance from the hand to the rowlock. What is the propelling force, if he pulls with a force of 48 lb.?

4. A man lifts a 1000-pound stone by means of a crowbar. The point of the crowbar is placed under the edge of the stone, and a small stone for fulcrum is placed 10 in. back from it. The crowbar is grasped at a distance of 6 ft. 8 in. from the fulcrum. What force is exerted in lifting the stone?

5. Two children play "teeter". One weighing 80 lb. sits 6 ft. from the point of support of the board, and the other 5 ft. Find the weight of second child.

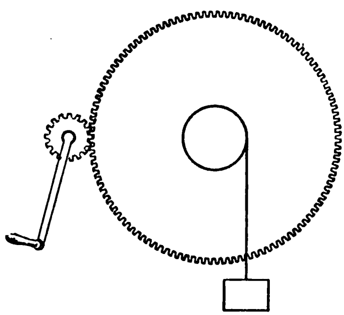
6. The length of the handles of a wheelbarrow is 5 ft., and the center of gravity of a load of 275 lb. is 18 in. from the hub of the wheel. What force is required to lift it?

7. The crank to the windlass of a well is 18 in. long, and the cylinder upon which the rope is wound 9 in. in diameter. What force is necessary to lift from the well a bucket of water weighing 48 lb.?



8. Two men working at a capstan walk in a circle 8 ft. in diameter, and each exerts a force of 60 lb. With each turn of the capstan 2 ft. of rope are pulled in. What pull is exerted along the rope?

9. A horse walking in a circle 15 ft. in diameter moves a house by means of a capstan 18 in. in diameter. The horse exerts a pull of 1200 lb. What is the resistance of the house?



10. A stone weighing 756 lb. is lifted into place in a wall by means of a cable wound round an axle of 5 in. radius. To the axle is rigidly attached a toothed wheel 20 in. in radius which is driven by another toothed wheel 4 in. in radius.

The latter wheel is turned by means of a crank 18 in. long. How much force must be applied to the crank?

11. A shaft has upon it two pulleys. The small one is 8 in. in diameter and drives machinery with a resistance of 196 lb. The shaft is turned by the other pulley, which is 22 in. in diameter. What effort must be applied to the large pulley?

12. If the wheels of an electric car are 2 ft., the axle cog-wheel 8 in., and the cog-wheel attached to the motor 12 in. in diameter, and if the resistance of the car is 1260 lb., what force must the motor supply to move the car?

13. There are four pulleys on a shaft driven by a fifth pulley. The driving pulley is 36 in. in diameter, and the others are 12 in., 10 in., 8 in., and 6 in. in diameter, respectively. The latter drive machinery with resistances of 50 lb., 36 lb., 48 lb., and 30 lb., respectively. What force must be communicated to the large pulley?

14. The current of water strikes an overshot water wheel with a force of 500 lb. On the shaft of this wheel, which is 8 ft. in diameter, is a pulley 2 ft. in diameter. The latter is connected by a belt to another pulley which turns a long shaft bearing many pulleys in a mill. How much force is communicated to this shaft?

15. The drive wheel of a certain lawn-mower is 10 in. in diameter, and contains 64 teeth. The little ratchet wheel, which propels the blades and which is driven by the first wheel, contains 10 teeth. The blades describe circles 6 in. in diameter. The effort at the blade is what part of that at the drive?

16. The drive-wheel of a locomotive 5 ft. in diameter has the connecting rod attached to a pivot 8 in. from the axle. What is the maximum force that tends to move the engine forward, if the force along the connecting rod due to the steam is 25,000 lb.?

17. If a barrel of flour weighs 200 lb., what force is required to roll it up an incline 8 ft. long into a wagon 3 ft. high? If it weighs 240 lb.?

18. If a stone weighs 850 lb., what force is required to pull it on rollers up an incline 12 ft. long, and place it upon a wagon 30 in. high?

19. What force is required to pull a loaded wagon weighing 3800 lb. up a slope 30 ft. high and 105 ft. long, neglecting friction?

20. A car weighing 15 tons stands on an inclined track 10 times as long as it is high. With what force does the car tend to run down the track?

21. Ice is pulled up an incline 50 ft. long and 20 ft. high, into an ice house. What force, neglecting friction, is necessary to pull up a block weighing 200 lb.?

22. Shavings from a spoke mill are carried up an incline 40 ft. long to a height of 10 ft. by means of a belt with slats on it, and thrown into wagons. If the shavings weigh 12 lb. to 10 ft. of the carrier, what force is required to run the carrier when loaded with shavings, if friction and the weight of the carrier are neglected?

23. The wheel of a letter-press is 14 in. in diameter, and the screw advances $\frac{1}{4}$ in. at each turn of the wheel. What pressure will a force of 15 lb. at the wheel exert upon the articles in the press?

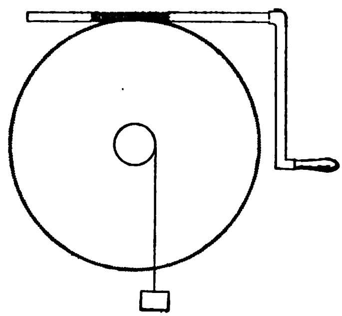
24. A jack-screw whose pitch is $\frac{3}{8}$ in. is operated by a lever 24 in. long. What force is required to lift the corner of a house with it, if the weight is 3 T.?

25. If the pitch of a jack-screw is $\frac{3}{4}$ in., and the lever arm 30 in., what weight may be lifted by it with a force of 100 lb.?

26. Two iron bars are bolted together by a bolt of which the distance between the threads is $\frac{1}{16}$ in. If the tap is tightened by means of a wrench of which the handle is 14 in. long, with a force of 25 lb., what is the strain on the threads? Neglect friction.

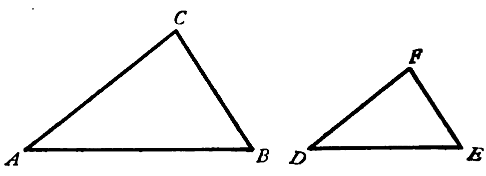
27. A carpenter wishes to make a bench-vise having a screw with a pitch of $\frac{3}{8}$ in., and a lever of such length that a force of 50 lb. applied at the lever will cause a pressure of 2200 lb. on a block in the vise. How long must the lever be?

28. The wheel of an endless screw which has four teeth to the inch is 24 in. in diameter and the axle 4 in. The crank of the screw is 15 in. long. What resistance will the machine overcome, if a force of 40 lb. is applied at the crank? What resistance will the same machine overcome if the crank of the screw is 20 in. and the effort 75 lb.?



SIMILAR FIGURES

258. Application of proportion to similar figures. — Two figures, such as two squares, two triangles whose angles are respectively equal, two circles, two cubes, two spheres, etc., which have the same shape are called **similar**.



In geometry it is shown that in two similar figures:

I. *Two corresponding lines are proportional to any other two corresponding lines;*

II. *The areas are proportional to the squares of any two corresponding lines;*

III. *The volumes are proportional to the cubes of any two corresponding lines.*

EXAMPLE. — The base and altitude of a triangle are 8 in. and 6 in., respectively, and the base of a similar triangle is 12 in. Find its altitude.

If x in. is the altitude, $\frac{x \text{ in.}}{6 \text{ in.}} = \frac{12 \text{ in.}}{8 \text{ in.}}$; x in. $= \frac{6 \times 12}{8}$ in., or 9 in.

Proportion is involved in the *drawing to a scale* of maps, plans for buildings, machine designs, etc. The map or plan of a thing is *similar* to the thing represented. Thus, the distances between points on a map of the United States are proportional to the actual distances which they represent.

PROBLEMS

1. Two corresponding sides of two similar triangles are 4 ft. and 12 ft., respectively. The other two sides of the first triangle are 5 ft. and 6 ft. Find the corresponding sides of the second triangle.

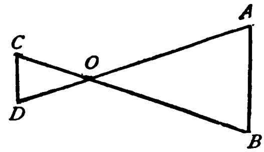
2. Two corresponding sides of two similar triangles are 18 yd. and 26 yd., respectively, and the area of the first is 104 sq. yd. Find the area of the second.

3. The diameters of two spherical steel balls are 6 in. and 8 in., respectively. If the smaller weighs 31.87 lb., what is the weight of the larger?

4. Of two similar iron castings, the smaller weighs 128 lb. Their lengths are 20 in. and 32 in., respectively. Find the weight of the larger.

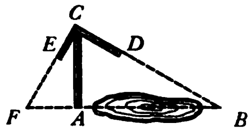
5. Sailors and others use the following method of estimating the distance OA to an object A. With the left eye

at C closed, the finger is pointed, at arm's length, at O, towards A. Then the right eye at D is closed and the left eye opened, when the object appears to have moved through the distance AB. The distance AB, being transverse to the line of sight, is estimated. The distance CD between the eyes of the average person is one tenth of the distance DO to the end of the outstretched finger. If AB is 500 ft., what is the distance from O to A? If AB is apparently 12 ft.? Estimate in this way the distances to objects about you.

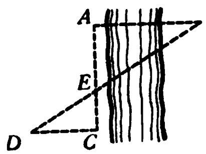


6. A rod 6 ft. high casts a shadow 4 ft. 6 in. long, and at the same time a tree casts a shadow 51 ft. long. How high is the tree?

7. Several centuries ago the distance from A to the inaccessible point B was determined by erecting a vertical staff AC, and placing upon this an instrument resembling a carpenter's square, directing one arm towards B, and noting on the ground the point F toward which the other arm pointed. By measuring AC and FA, and using similar triangles, AB was estimated. If $AC = 5$ ft. and $FA = 6$ in., what is AB? If $AC = 6$ ft., and $FA = 4$ in., what is AB?

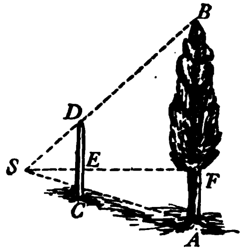


8. The distance AB across a stream may be obtained as follows: A line AC is measured off at right angles to AB, along the shore. From C the line CD is measured off at right angles to AC. The point E of AC which is in line with D and B is then located. AE and EC are measured. AB is then found by

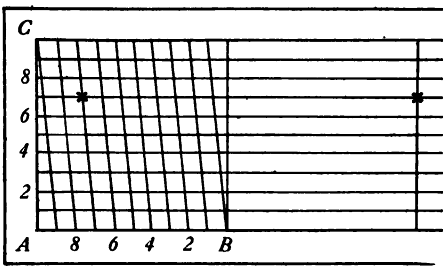


proportion. Write the proportion between AB and the measured distances. If $AE = 120$ ft., $EC = 40$ ft., and $CD = 60$ ft., find AB.

9. In forestry, when shadows cannot be used, the height AB of a tree is found as follows: A staff is held in an upright position CD. A man at S sights across the staff to the foot and to the top of the tree. An assistant notes the points C and D where the lines of vision cross the staff, and measures CD. The distances SE and SF are measured. If $CD = 4$ ft., $SE = 3$ ft., and $SF = 49$ ft. 6 in., find AB. If $CD = 4$ ft. 2 in., $SE = 2$ ft. 8 in., and $SF = 37$ ft. 5 in., find AB.



10. The instrument shown in this drawing is called a *diagonal scale*. It is used by draftsmen for measuring distances to one hundredth of an inch. $AB = 1$ in. Find .01 in. on this scale; find .02 in.; .03 in.; .12 in.; .45 in.; .96 in.; 1.25 in.; 1.84 in.



What is the distance between the points marked $x \cdot \cdot \cdot x$ on the diagonal scale?

11. The areas of two similar rectangles are 165 sq. in. and $371\frac{1}{4}$ sq. in., respectively. The sides of the smaller are 11 in. and 15 in., respectively. Find the sides of the larger.

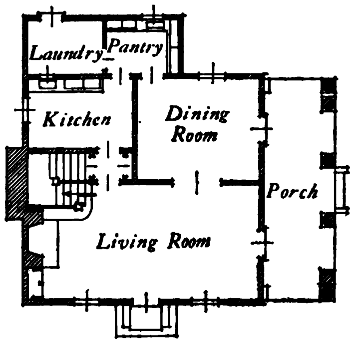
12. If, in a map, the distance between two cities 250 mi. apart is represented by a distance of 2.5 in., what should represent the distance between two places 372 mi. apart?

13. Consult a map of the United States. Find the scale to which it was drawn. Find from this map the number of miles from Boston to San Francisco.

14. On a map drawn to a scale of 240 mi. to an inch, the distance from Chicago to Denver is $8\frac{9}{16}$ in. How many miles is it from Chicago to Denver?

15. The room of a house is 16 ft. by 18 ft. If, in the plan, the width is represented by a line .8 in. long, what should represent the length?

16. In the house of which the floor plan is shown here the width of the living room is 15 ft. By measuring the corresponding width in the plan, find the scale to which the plan is drawn.



17. From the scale found in Problem 16, find the length and width of the dining room.

18. The dimensions of a rectangular piece of land are 220 yd. by 300 yd. If, in a plot of it, the width is made $1\frac{3}{4}$ in., what will be the length?

19. The surfaces of two similar bodies are 256 sq. cm. and 324 sq. cm., respectively. The volume of the first is 64 cu. cm. Find the volume of the second.

VARIATION

259. Variation. — A quantity whose value changes is called a **variable**. If two variable quantities have a constant ratio, one is said to *vary as* the other.

Thus, if the diameter of a circle increases, the circumference increases also, but the ratio of the circumference to the diameter is constant, and

equals π. This is expressed by saying that the *circumference of a circle varies as the diameter.*

If one quantity varies as another, any two values of the first will be directly proportional to the corresponding values of the second.

E.g., *the weight of a substance varies as its volume.* If the volume is doubled, the weight is doubled also. Hence, if w and v, the weight and volume, respectively, of a substance, change to w' and v', respectively,

then
$$\frac{w}{w'} = \frac{v}{v'}.$$

If two variable quantities have a constant product, one is said *to vary inversely as* the other. One increases as the other decreases.

Thus, the time required for a railway train to travel 200 mi. varies inversely as its speed; because, if t and r represent the time and rate, respectively, then $t \times r = 200$.

If one quantity varies inversely as another, any two values of the first will be inversely proportional to the corresponding values of the second.

E.g., if a varies inversely as b, and when a changes to a', b changes to b',

$$\frac{a}{a'} = \frac{b'}{b}.$$

Certain simple applications of the subject of variation will be discussed in the following sections.

260. The application of variation to falling bodies. — It is proved in mechanics that if a body is let fall from rest, its velocity increases from second to second, and that

I. *The velocity at any time varies directly as the number of units of time (seconds) that have passed.*

II. *The distance passed through at any time varies directly as the square of the number of units of time (seconds) that have passed.*

It is found experimentally that a body starting from rest falls approximately 16 feet the first second, and that the velocity at the end of the first second is approximately 32 feet per second.

PROBLEMS

1. If a body starting from rest falls 16 ft. the first second, how far will it fall in 2 sec.? In 3 sec.? In 4 sec.? In 5 sec.?

2. If a body starting from rest falls 16 ft. the first second, how far does it fall during the second second? The third second? The fourth second?

3. How far will a body starting from rest fall in 10 seconds? How far during the tenth second? How far in 1 minute?

4. If a falling body starts from rest, and has a velocity of 32 ft. per second at the end of the first second, what is its velocity at the end of the second second? At the end of the third second? At the end of the tenth second? At the end of a minute?

5. For how many seconds has a body fallen if it has a velocity of 192 ft. per second?

6. How long does it require an object starting from rest to fall 144 ft.? $\left(\frac{x^2}{1^2} = \frac{144}{16}.\right)$

7. A brick dislodged from the wall of a building strikes the ground in $2\frac{1}{2}$ sec. How high is the wall?

8. A stone thrown just over the top of the tree strikes the ground in 6 seconds from the time that it starts upward. Find the approximate height of the tree.

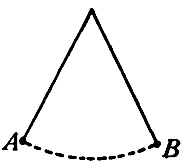
NOTE. — The time going up equals the time coming down.

9. With what vertical velocity must a bullet have left a gun, if it fell back to its starting place in just 9 seconds after it left the gun? How high did it go? (See note, Ex. 8.)

10. An object dropped from an aëroplane struck the ground in eight seconds. Find the height from which it was dropped. Had the aëroplane been 576 ft. high, with what velocity would the object have struck the ground?

11. An object dropped from a balloon strikes the ground in 6 seconds. At what velocity is the object falling when it strikes the ground, and how high is the balloon?

261. The application of variation to the vibration of the pendulum. — The time of vibration of a pendulum is that required for it to swing from A to B. It depends upon the length of the pendulum, and the latitude of the place.



The time of vibration of a pendulum at any given place varies directly as the square root of the length.

Thus, at New York a pendulum 4 ft. long requires 1.1 sec. for a vibration; one 1 ft. long requires 0.55 sec. for a vibration.

PROBLEMS

1. The length of a seconds pendulum (making a vibration in 1 sec.) in the latitude of New York is 39.1 in. What time is required for the vibration of a pendulum 9.78 in. long?

2. How long is a pendulum which vibrates three times in a second?

3. What is the length of the pendulum which vibrates 4 times in 3 seconds?

4. A pendulum 94 ft. long is made by suspending a cannon ball by means of a wire from the dome of a large building. What is the time of its vibration?

5. A clock whose pendulum beats seconds gains 2 minutes a day. How much too short is the pendulum?

6. A clock whose pendulum vibrates in a half second is found to gain 30 seconds a day in winter. How much has the pendulum contracted?

7. A clock whose pendulum is 20 in. long loses 10 min. a day. How much must the pendulum be shortened?

262. The application of variation to Boyle's Law. — The volume of a gas depends upon the amount of pressure upon it, the greater the pressure the less the volume. It was shown by Robert Boyle, an English physicist in the seventeenth century, that, if subjected to pressure:

The volume of a gas varies inversely as the pressure upon it.

If V is the volume of a gas under the pressure P, and the volume changes to V' when the pressure changes to P', then

$$\frac{V}{V'} = \frac{P'}{P}.$$

PROBLEMS

1. A room has a capacity of 150 cu. yd. The barometric pressure rises from 28 in. to 30 in. How many cubic yards of air in the room at the higher pressure have entered during the rise?

2. Twenty-four cubic inches of air under a pressure of 40 lb. will have what volume when the pressure is increased to 100 lb.?

3. When the atmospheric pressure shown by the barometer is 76 cm., 1000 cu. cm. of air weigh 1.293 g. What is the weight when the atmospheric pressure is 80 cm.?

4. If a toy balloon contains 2.5 quarts of gas when under a pressure of 1 atmosphere, to what size will it shrink if subjected to a pressure of 5 atmospheres?

5. The pressure of the atmosphere on a square inch of surface is approximately 15 lb. The bore of a popgun is $\frac{1}{10}$ sq. in. in cross-sectional area. The air in the popgun is compressed to $\frac{1}{12}$ of its original volume before the pellet is discharged. With what force is the pellet discharged?

6. Air in a cylinder at atmospheric pressure is compressed to $\frac{1}{10}$ of its volume. What pressure does it exert per square inch on the walls of the cylinder?

263. Intensity of light. — The amount of illumination, or intensity of light thrown upon a surface from a given source, depends upon the distance from the source. It is shown that:

The intensity of the illumination varies inversely as the square of the distance from the source of light.

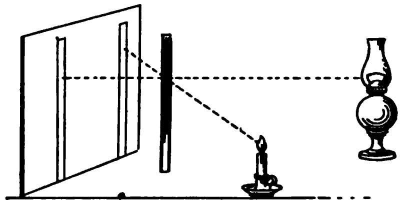
Thus, a standard candle placed 2 ft. from an open book will illuminate it with 4 times the intensity that it will when placed 4 ft. away. That is,

$$\frac{1}{\frac{1}{4}} = \frac{4^2}{2^2}.$$

This principle is used in determining the "candle power" of light.

PROBLEMS

1. *The candle power of a lamp is the ratio of its illuminating power to that of a standard candle.* A kerosene lamp and a standard candle are placed so that they cause a rod to cast shadows upon a surface or screen. The shadows are of equal intensity when the candle is 18 in. and the lamp 36 in. from the screen. What is the candle power of the lamp?



2. A kerosene lamp and a standard candle cast shadows of equal intensity upon a screen when the candle is 15 in. and the lamp 48 in. from the screen. Find the candle power of the lamp.

3. If a standard candle is placed 20 in. from the screen, how far from the screen will a lamp have to be placed, if its candle power is 9, in order to illuminate the screen with equal intensity?

4. A standard candle 1 ft., and an incandescent electric light 4 ft., from a screen, illuminate it with equal intensities. What is the candle power of the incandescent light?

5. A 16-candle power incandescent light at 25 in., and an oil lamp at 8 in., from a screen, illuminate it with equal intensities. What is the candle power of the lamp?

6. A cluster of twelve 16-candle power incandescent lights 10 ft., and an arc light 25 ft., from a wall, illuminate it with equal intensities. Find the candle power of the arc light.

7. How many incandescent lamps will be required to produce the same intensity of illumination at 6 ft. distance that is produced by 1 such lamp at 10 in. distance?

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ANSWERS

STONE–MILLIS HIGHER ARITHMETIC

Note. When answers are only approximate, the last figure is not necessarily the "next" figure of the computation, but it represents the *nearest* value.

Thus, .4249 to hundredths is .42 while .4251 is .43.

Pages 11–13

1. 66,945.
2. 78,297.
3. 68,733.
4. 57,239.
5. 52,538.
6. 48,721.
7. 39,977.
8. 52,301.
9. 52,554.
10. 57,699.
11. 73,708.
12. 63,284.
13. 67,152.
14. 58,036.
15. 56,168.
16. 53,280.
17. 57,251.
18. 49,456.
19. 45,695.
20. 38,630.
21. 49,461.
22. 43,444.
23. 54,798.
24. 51,029.
25. 53,750.
26. 51,334.
27. 46,794.
28. 57,454.
29. 54,582.
30. 19,071,779.
31. 20,282,742.
32. 17,958,339.
33. 18,700,316.

Pages 14–15

1. 84.
2. 87.
3. 87.
4. 92.
5. 91.
6. 800.
7. 800.
8. 791.
9. 753.
10. 732.
11. 6309.
12. 6256.
13. 7279.
14. 7631.
15. 5755.
16. 20,109.
17. 8199.
18. 13,864.
19. 14,133.
20. 8404.
21. *a.* $50,466.
b. 53,898.
c. 47,746.
d. 46,932.
e. 52,848.
f. 45,285.
Total $297,175.
A. $38,919.
B. 49,453.
C. 44,250.
D. 49,807.
E. 43,689.
F. 36,828.
G. 34,229.
Total $297,175.

22. Daily Totals
Mon. $9534.95.
Tue. $9217.42.
Wed. $8896.00.
Thur. $9068.33.
Fri. $9820.90.
Sat. $9744.50.
For wk. $56,282.10.
Brown $3902.27.
Clark $4261.52.
Cole $4259.79.
Davis $3619.17.
Day $3932.37.
Dole $4419.58.
Fales $4034.46.
Hart $4285.16.
Moore $4122.41.
Smith $4799.79.
Sloan $4914.40.
Stone $4828.37.
Young $4902.81.
Total $56,282.10.

Page 19

41. 114.
42. 444.
43. 557.
44. 438.
45. 448.
46. 39.
47. 54.
48. 114.
49. 119.

50. 316.
51. 317.
52. 416.
53. 227.
54. 407.
55. 3318.
56. 1138.
57. 318.
58. 319.
59. 4019.
60. 3303.
61. 1305.
62. 47,032.
63. 53,604.
64. 72,646.
65. 46,211.
66. 26,202.
67. 37,232.

Pages 19–20

1. 1,668,554.
2. 3,737,357.
3. 5,297,865.
4. 1,228,859.
5. 3,681,817.
6. 2,846,404.
7. 4,843,199.
8. 2,252,413.
9. 4,587,746.
10. 8,192,537.
11. 14,840,257.
12. 11,865,969.
13. 21,293,174.
14. 40,758,867.
15. 54,176,609.
16. 44,650,757.
17. 127,907,391,-873,544,617.
18. 558,868,257,-636,243,127.
19. 730,187,539,-387,773,744.
20. 613,973,740,-737,900,863.

Page 21

1. \$192.95.
2. \$484.05.
3. \$527.34.
4. \$548.05.
5. \$551.84.
6. \$655.38.
7. \$1040.83.
8. \$494.16.
9. \$908.28.
10. \$638.65.
11. \$1062.92.
12. \$442.38.
13. \$1006.04.
14. \$414.17.
15. \$881.49.
16. \$1051.92.
17. \$942.41.
18. \$878.78.
19. \$652.19.
20. \$949.69.
21. \$654.75.
22. \$987.28.
23. \$694.81.
24. \$1149.77.
25. \$708.15.

Page 23

1. \$40.20.
2. \$390.25.
3. \$269.75.
4. \$18.43.
5. \$827.44.

Pages 25–26

1. \$13.44.
2. \$500.78.
3. \$8458.25.
4. \$15,999.60.
5. \$7156.52.
6. \$38.09.
7. \$44.72.
8. \$584.37.
9. \$956.87.
10. \$4043.28.
11. \$550.08.
12. \$632.73.

Page 30

1. 241,395.
2. 507,327.
3. 816,314.
4. 336,000.
5. 711,708.
6. 256,088.
7. 600,416.
8. 239,512.
9. 355,152.
10. 645,414.
11. 235,776.
12. 667,644.
13. 195,684.
14. 599,814.
15. 199,082.
16. 676,347.
17. 215,586.
18. 617,376.
19. 540,265.
20. 591,306.
21. 806,400.
22. 322,500.
23. 623,200.
24. 595,200.
25. 578,880.
26. 609,700.
27. 174,800.
28. 669,600.
29. 205,200.
30. 658,600.
31. 184,978,570,-688.
32. 355,456,532,-374.
33. 845,053,317,-732.
34. 537,007,545,-108.
35. 264,839,374,-632.
36. 269,712,505,-650.

Page 33

1. $9736\frac{31}{39}$.
2. $13,016\frac{7}{16}$.
3. $3199\frac{4}{57}$.
4. $993\frac{22}{97}$.
5. $2872\frac{38}{67}$.
6. $3071\frac{8}{77}$.
7. $2763\frac{47}{68}$.
8. $2286\frac{7}{13}$.
9. $2904\frac{11}{98}$.
10. $2023\frac{1}{7}$.
11. $1899\frac{7}{19}$.
12. $4026\frac{35}{86}$.
13. $5751\frac{28}{163}$.
14. $2915\frac{182}{269}$.
15. $1957\frac{8}{47}$.
16. $3015\frac{51}{88}$.
17. $1337\frac{1}{39}$.
18. $1934\frac{19}{72}$.
19. $3004\frac{1}{2}$.
20. $3207\frac{89}{96}$.
21. $2955\frac{16}{67}$.

Page 34

1. A. 392.
   B. 789.
   C. 691.
   D. 849.

E. 926;
173 rem.
F. 592.
2. A. 519.
B. 827.
C. 793.
D. 839.
E. 768.
F. 927.
3. A. 845.
B. 956.
C. 892.
D. 781.
E. 398.
F. 483.
4. A. 798.
B. 629.
C. 918.
D. 718.
E. 961.
F. 582.
5. A. 539.
B. 618.
C. 729.
D. 397.
E. 846.
F. 589.

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14. 5, 5, 5, 5, 3, 3.
15. 5, 5, 257.
16. 2, 3, 3, 5, 7, 7.
17. 3, 3, 5, 163.
18. 2, 3, 3, 3, 3, 3, 11.
19. 3, 7, 11, 17.
20. 2, 3, 3, 5, 7, 11.
21. 2, 3, 3, 5, 163.

Page 38

1. 37.
2. 71.
3. 59.
4. 67.
5. 83.
6. 37.
7. 97.
8. 41.
9. 1.
10. 113.
11. 59.
12. 197.

Page 39

1. $22\frac{1}{2}$.
2. 2592.
3. $106\frac{2}{3}$.
4. 28.
5. $13\frac{5}{7}$.
6. 126.
7. $22\frac{1}{2}$.
8. 294.
9. $39\frac{3}{8}$.
10. 168.
11. 84.
12. $226\frac{2}{13}$.
13. 378.
14. 280.

Page 43

1. $\frac{2}{3}$.
2. $\frac{6}{7}$.
3. $\frac{24}{31}$.
4. $\frac{8}{7}$.
5. $\frac{2}{3}$.
6. $\frac{7}{9}$.
7. $\frac{5}{7}$.
8. $\frac{6}{11}$.
9. $\frac{7}{9}$.
10. $\frac{7}{10}$.
11. $\frac{8}{5}$.
12. $\frac{5}{7}$.
13. $\frac{2}{5}$.
14. $\frac{3}{8}$.
15. $\frac{3}{5}$.

Page 44

1. $2\frac{3}{16}$.
2. $2\frac{1}{42}$.
3. $1\frac{13}{32}$.
4. $2\frac{7}{60}$.
5. $2\frac{11}{36}$.
6. $1\frac{25}{48}$.
7. $2\frac{11}{42}$.
8. $2\frac{1}{4}$.
9. $1\frac{1}{6}$.
10. $2\frac{3}{10}$.
11. $1\frac{13}{15}$.
12. 2.
13. $\frac{1}{15}$.
14. $\frac{7}{16}$.
15. $\frac{1}{10}$.
16. $\frac{19}{54}$.
17. $\frac{7}{9}$.
18. $\frac{1}{6}$.
19. $\frac{7}{20}$.
20. $\frac{13}{56}$.
21. $\frac{23}{72}$.

Pages 44–45

2. $85\frac{11}{12}$.
3. $186\frac{5}{8}$.
4. $101\frac{1}{16}$.
5. $69\frac{7}{12}$.
6. $115\frac{5}{18}$.
7. $91\frac{11}{16}$.
8. $113\frac{1}{6}$.
9. $138\frac{5}{12}$.
10. $99\frac{3}{10}$.
11. $131\frac{5}{8}$.
12. $291\frac{11}{16}$.
13. $323\frac{1}{4}$.
14. $367\frac{1}{6}$.
15. $303\frac{1}{20}$.

Pages 46–47

3. $62\frac{1}{12}$.
4. $61\frac{3}{8}$.
5. $451\frac{11}{15}$.
6. $12\frac{5}{12}$.
7. $33\frac{7}{12}$.
8. $48\frac{7}{8}$.
9. $10\frac{5}{12}$.
10. $16\frac{8}{15}$.
11. $12\frac{13}{15}$.
12. $121\frac{2}{3}$.
13. $145\frac{3}{8}$.
14. $172\frac{1}{6}$.
15. $243\frac{17}{24}$.
16. $157\frac{8}{15}$.
17. $224\frac{9}{20}$.
18. $206\frac{17}{24}$.
19. $842\frac{1}{3}$.
20. $176\frac{1}{3}$.
21. $110\frac{3}{4}$.
22. $104\frac{23}{24}$.
23. $11\frac{5}{24}$.
24. $77\frac{8}{15}$.
25. $1427\frac{1}{24}$.
26. $65\frac{8}{15}$.
27. $82\frac{7}{12}$.
28. $891\frac{5}{8}$.
29. $3356\frac{3}{4}$.
30. $4016\frac{1}{21}$.
31. $3904\frac{13}{16}$.
32. $2932\frac{5}{8}$.
33. $2502\frac{1}{2}$.
34. $487\frac{13}{16}$.
35. $160\frac{13}{20}$.
36. $471\frac{23}{36}$.
37. $237\frac{13}{22}$.
38. $171\frac{53}{60}$.

39. $317\frac{13}{15}$.
40. $20\frac{19}{24}$.
41. $105\frac{1}{12}$.
42. $153\frac{3}{16}$.
43. $127\frac{1}{24}$.
44. $66\frac{7}{12}$.
45. $123\frac{3}{4}$.
46. $241\frac{7}{15}$.
47. $534\frac{7}{12}$.

Pages 49–50

2. 54.
3. 144.
4. 210.
5. 144.
6. 504.
7. 2944.
8. 7875.
9. 5472.
10. 4032.
11. 3680.
13. $\frac{23}{35}$.
14. $1\frac{175}{198}$.
15. $\frac{137}{168}$.
16. $1\frac{23}{32}$.
17. $1\frac{23}{60}$.
18. $\frac{631}{640}$.
19. $1\frac{1203}{1584}$.
20. $1\frac{775}{1134}$.
21. $1\frac{229}{408}$.
22. $1\frac{2}{3}$.
23. $2\frac{23}{45}$.
24. $1\frac{6814}{7875}$.
25. $\frac{29}{1132}$.
26. $\frac{16}{45}$.
27. $\frac{251}{312}$.
28. $\frac{29}{112}$.
29. $\frac{373}{600}$.
30. $\frac{125}{1134}$.
31. $\frac{71}{231}$.
32. $\frac{67}{810}$.
33. $\frac{43}{224}$.

Page 51

1. $\frac{9}{17}$.
2. $\frac{3}{8}$.
3. $\frac{3}{5}$.
4. $\frac{3}{4}$.
5. $\frac{7}{11}$.
6. $\frac{2}{3}$.
7. $\frac{1}{4}$.
8. $\frac{3}{5}$.
9. $\frac{4}{5}$.
10. $\frac{3}{5}$.
11. $\frac{2}{3}$.
12. $\frac{16}{25}$.

Pages 52–53

2. $561\frac{3}{4}$.
3. 243.
4. $808\frac{1}{3}$.
5. $638\frac{2}{5}$.
6. $576\frac{1}{5}$.
7. $222\frac{4}{7}$.
8. $906\frac{19}{20}$.
9. $294\frac{4}{9}$.
10. $312\frac{13}{16}$.
11. $291\frac{11}{12}$.
12. $95\frac{5}{8}$.
13. $171\frac{1}{2}$.
14. $384\frac{1}{4}$.
15. $501\frac{1}{3}$.
16. $384\frac{11}{18}$.
17. 392.
18. $151\frac{2}{3}$.
19. $232\frac{7}{8}$.
20. $779\frac{1}{6}$.
21. $1413\frac{3}{5}$.
22. $287\frac{1}{2}$.
23. $264\frac{11}{16}$.
24. $723\frac{1}{3}$.
25. $1059\frac{1}{2}$.

Page 55

68. $8124\frac{1}{5}$.
69. $3634\frac{5}{16}$.
70. $2903\frac{1}{25}$.
71. $309\frac{1}{49}$.
72. $777\frac{7}{9}$.
73. $989\frac{1}{16}$.
74. $721\frac{1}{36}$.
75. $2544\frac{9}{25}$.
76. $453\frac{4}{9}$.
77. $497\frac{7}{9}$.
78. $1153\frac{4}{25}$.
79. $1404\frac{9}{16}$.
80. $1148\frac{4}{25}$.
81. $1186\frac{1}{49}$.

Page 56

39. $\frac{2}{33}$
40. $\frac{3}{26}$.
41. $\frac{8}{51}$.
42. $\frac{3}{50}$.
43. $\frac{16}{297}$.
44. $\frac{3}{20}$.
45. $\frac{2}{33}$.
46. $\frac{4}{39}$.
47. $\frac{2}{45}$.
48. $\frac{3}{44}$.
49. $\frac{3}{34}$.
50. $\frac{3}{34}$.
51. $\frac{3}{38}$.
52. $\frac{3}{56}$.
53. $\frac{21}{437}$.
54. $\frac{4}{87}$.

Page 57

1. $1\frac{1}{3}$.
2. $\frac{9}{10}$.
3. $\frac{24}{25}$.
4. $\frac{24}{35}$.
5. $1\frac{19}{35}$.
6. $\frac{3}{4}$.
7. $\frac{5}{6}$.
8. $2\frac{1}{4}$.
9. $1\frac{1}{5}$.
10. $2\frac{5}{8}$.
11. $1\frac{1}{3}$.
12. $5\frac{1}{4}$.
13. $4\frac{2}{5}$.
14. $2\frac{26}{27}$.
15. $1\frac{17}{18}$.
16. $4\frac{8}{9}$.
17. $2\frac{1}{7}$.
18. $2\frac{2}{27}$.
19. $\frac{20}{33}$.
20. $\frac{16}{21}$.
21. $\frac{4}{7}$.
22. $\frac{9}{14}$.
23. $\frac{5}{6}$.
24. $\frac{9}{20}$.
25. $2\frac{4}{5}$.
26. $1\frac{11}{24}$.
27. $1\frac{3}{5}$.
28. $1\frac{13}{15}$.
29. $1\frac{1}{14}$.
30. $\frac{5}{8}$.
31. $2\frac{4}{7}$.
32. $1\frac{1}{7}$.

Page 58, § 62

2. $26\frac{47}{54}$.
3. $40\frac{134}{325}$.
4. $29\frac{51}{135}$.
5. $40\frac{1}{18}$.
6. $37\frac{63}{158}$.
7. $60\frac{1}{4}$.
8. $80\frac{110}{131}$.
9. $64\frac{61}{236}$.
10. $478\frac{52}{53}$.
11. $125\frac{5}{7}$.
12. $92\frac{36}{71}$.
13. $89\frac{23}{53}$.

Page 58, § 63

2. $480\frac{4}{5}$.
3. $1522\frac{7}{10}$.
4. $1236\frac{13}{15}$.
5. $1193\frac{23}{24}$.
6. $283\frac{17}{21}$.
7. $461\frac{17}{30}$.
8. $384\frac{13}{18}$.
9. $84\frac{5}{6}$.
10. $85\frac{29}{63}$.
11. $120\frac{53}{56}$.
12. $85\frac{35}{72}$.
13. $62\frac{3}{44}$.
14. $25\frac{48}{75}$.
15. $48\frac{31}{144}$.
16. $89\frac{1}{42}$.
17. $44\frac{14}{171}$.
18. $33\frac{41}{81}$.
19. $28\frac{1}{45}$.

Pages 59–60

1. $\frac{13}{20}$.
2. $\frac{39}{44}$.
3. $\frac{68}{105}$.
4. $\frac{8}{9}$.
5. $\frac{48}{55}$.
6. $\frac{6}{7}$.
7. $\frac{5}{12}$.
8. $\frac{9}{32}$.
9. $\frac{9}{20}$.
10. $19\frac{1}{4}$.
11. $8\frac{2}{3}$.
12. $\frac{7}{76}$.
13. $\frac{4}{49}$.
14. $\frac{91}{128}$
15. $32\frac{13}{14}$.
16. $134\frac{11}{81}$.
17. $2\frac{2}{5}$.
18. $6\frac{4}{5}$.
19. $3\frac{1}{26}$.
20. $5\frac{1}{3}$.
21. $1\frac{67}{98}$.
22. $1\frac{20}{27}$.
23. $2\frac{21}{32}$.
24. $1\frac{287}{550}$.
25. $2\frac{3}{8}$.
26. $\frac{2}{3}$.
27. $\frac{8}{35}$.
28. $21\frac{1}{3}$.
29. $\frac{713}{800}$.
30. $2\frac{1}{15}$.
31. $\frac{9}{25}$.
32. 25.
33. $\frac{123}{320}$.
34. $9\frac{29}{51}$.
35. 1.
36. $\frac{44}{135}$.

Pages 60–64

1. \$.72.
2. \$3.36.
3. \$30.
4. \$160.
5. \$12.
6. \$128; \$120.
7. $\frac{1}{7}$.
8. $\frac{1}{3}$.
9. $\frac{1}{5}$; $\frac{1}{6}$.
10. $637\frac{1}{2}$ bu.
11. $21\frac{3}{8}$ mi.
12. $\frac{1}{4}$; $\frac{1}{5}$.
13. \$35.
14. \$60.
15. \$24.
16. $\frac{4}{5}$; $\frac{4}{9}$.
17. 2680 gal.
18. $\frac{1}{5}$.
19. $\frac{1}{5}$.
20. \$9; \$6.
21. 300 yd.
22. \$40.
23. \$900.
24. \$35.
25. 210 bu.
26. 720 girls.
27. \$150.
28. $\frac{1}{5}$; $\frac{2}{7}$; $\frac{4}{7}$; $\frac{8}{35}$.
29. \$45; \$64.29; \$96.43; \$19.28.
30. \$40; \$36.
31. $\frac{2}{3}$; $\frac{1}{3}$.
32. \$2880.
33. \$480.
34. \$1000.
35. \$375.
36. $\frac{1}{8}$; 224 men.
37. \$1200.
38. \$6000.
39. \$2.40; \$4.00.

Pages 64–66

1. 630 bu.
2. \$616.67.
3. 14.
4. 7210 lb.
5. 125 da.
6. \$312; \$507.
7. $\frac{2}{3}$; \$75.
8. \$240.
9. \$66.40.
10. \$13,760; \$1400; \$1500.
11. $56\frac{1}{4}$ mi.
12. 240; 270.
13. $\frac{3}{5}$.
14. 108.
15. $\frac{3}{4}$; $\frac{1}{10}$; $\frac{3}{20}$.
16. $\frac{4}{9}$; $\frac{4}{13}$.
17. \$1608.
18. \$1398.
19. \$432.
20. \$269.
21. \$2.87.
22. $31\frac{1}{2}$ lb.
23. \$21.56.
24. \$1318.13.
25. $\frac{1}{12}$.
26. \$3.75; \$14.76; \$8.70.
27. \$5.74; \$22.96; \$13.53.
28. 595 bu.
29. $46\frac{19}{27}$ bu; 2522 bu.
30. $5\frac{7}{128}$ lb.

Page 68

1. 143.107.
2. 602.176.
3. 241.046.
4. 42.018.
5. 103.059
6. 325.6793.
7. 26.396.
8. 4.433.
9. 23.088.
10. 34.057.
11. 54.122.
12. 49.916.
13. 16.032.
14. 25.627.
15. 13.694.
16. 178.692.

Pages 69–70

1. 15.92.
2. 46.98.
3. 468.5.
4. 34.88.

5. 15.804.
6. 203.04.
7. 296.64.
8. 53.952.
9. 65.504.
10. 41.971.
11. 4.608.
12. 3.741.
13. 6.882.
14. 8.051.
15. 3.276.

Page 70

2. 7.68.
3. .238
4. 15.48.
5. 7.704.
6. 4.404.
7. 13.6.
8. 16.32.
9. 18.24.
10. 3.9923.
11. 5.1264.
12. 3.879.
13. 1.5665.
14. 4.097.
15. 5.94672.
16. 2.61464.

Page 71

2. 1329.3+.
3. 791.3−.
4. 197.8−.
5. 59.5−.
6. 1813.3−.
7. 84.3−.
8. 1523.5−.
9. 125.9+.
10. 137.2+.
11. 6672.0+.

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2. .739.
3. 4.23.
4. .0403.
5. 14.7.
6. .188.
7. 1.91.
8. .136.
9. .0117.
10. .097.
11. .0084.
12. .63.
13. 129.3.
14. .876.
15. .0994.
16. .00648.
17. .02322.
18. .00382.
19. .00151.
20. .494.
21. 1.57.
22. .00012.
23. .00613.

Page 73

2. 10.5.
3. 30.4.
4. 4700.
5. 33.
6. 93.
7. 1.23.
8. 1202.
9. 100.
10. 27.6.
11. 43.6.
12. .0092.
13. 970.
14. 75.3.
15. .106.
16. 5600.
17. 4030.
18. 231,900.
19. 3.82.

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3. 2.2+.
4. 126.1+.
5. 8.8−.
6. 22.5−.
7. 241.2−.
8. 53.4−.
9. 6.9−.
10. 4.5−.
11. 81.5−.
12. 49.1−.
13. 54.4+.
14. 17.0+.
15. 8.5+.
16. 160.
17. .4+.
18. 21.0+.

Page 76, § 78

1. $.555\frac{5}{9}$.
2. $.866\frac{2}{3}$.
3. $.642\frac{6}{7}$.
4. $.916\frac{2}{3}$.
5. $.566\frac{2}{3}$.
6. $.842\frac{2}{19}$.
7. $.944\frac{4}{9}$.
8. $.294\frac{2}{17}$.
9. $.136\frac{4}{11}$.
10. $.470\frac{10}{17}$.
11. $.575\frac{25}{33}$.
12. $.518\frac{14}{27}$.
13. $.516\frac{4}{31}$.
14. $.404\frac{16}{21}$.
15. $.301\frac{47}{53}$.
16. $.382\frac{46}{47}$.
17. .583+.
18. .529+.
19. .278−.
20. .467−.
21. .364−.
22. .105+.
23. .186+.
24. .762−.
25. .514+.
26. .405−.
27. .905−.
28. .977−.

Page 76, § 79

1. $\frac{14}{25}$.
2. $\frac{9}{20}$.
3. $\frac{9}{25}$.
4. $\frac{1}{8}$.
5. $\frac{77}{200}$.
6. $\frac{17}{40}$.
7. $\frac{1}{3}$.
8. $\frac{7}{15}$.
9. $\frac{19}{35}$.
10. $\frac{23}{60}$.
11. $\frac{419}{900}$.
12. $\frac{96}{275}$.

Page 77, § 80

1. $.\dot{2}\dot{7}$.
2. $.\dot{7}\dot{2}$.
3. $.8\dot{3}$.
4. $.58\dot{3}$.
5. $.541\dot{6}$.
6. $.7\dot{3}$.
7. $.\dot{0}\dot{7}$.
8. $.1\dot{4}\dot{5}$.
9. $.1\dot{3}\dot{6}$.
10. $.\dot{2}7\dot{9}$.
11. $.313\dot{8}$.
12. $.3\dot{2}\dot{8}$.
13. $.72\dot{2}\dot{7}$.
14. $.32\dot{4}\dot{7}$.
15. $.26\dot{4}\dot{8}$.
16. $.2\dot{3}\dot{4}$.

Page 77, § 81

1. $\frac{5}{9}$.
2. $\frac{2}{3}$.
3. $\frac{1}{6}$.
4. $\frac{26}{45}$.
5. $\frac{8}{11}$.
6. $\frac{28}{33}$.
7. $\frac{19}{90}$.
8. $\frac{31}{198}$.
9. $\frac{19}{66}$.
10. $\frac{65}{198}$.
11. $\frac{11}{37}$.
12. $\frac{16}{999}$.

Pages 78–80

1. 487.5 lb.
2. 8.333 lb.
3. 3.
4. 362 lb.
5. $23.85.
6. $23.06.
7. 1.54 lb.
8. 2.26 qt.
9. 3.33 times.
10. 2.15 lb.; 1.94 lb.
11. Beef, .41 lb. Pork, .49 lb. Fowl, .46 lb. Bread, .82 lb. Beans, .37 lb. Eggs, .50 lb.
12. 68.32¢.
13. 7.15¢.
15. 960.9 gal.
16. 27.27 cu. ft.
17. .80 bu.
18. $2.59.
19. .3183.
20. .1313 oz.
21. 7.615 in.
22. 2.55 in.
23. 63.5225 lb.

Page 82

1. 648.
2. 893.
3. 936.
4. 735.
5. 841.
6. 826.
7. 632.
8. 7722.
9. 8557.
10. 7176.
11. 6272.
12. 7182.
13. 8483.
14. 9584.
15. 38,016.
16. 27,048.
17. 37,345.
18. 66,836.
19. 12,000.
20. 74,774.
21. 82,080.
22. 174,825.
23. 373,252.
24. 245,508.
25. 678,957.
26. 672,975.
27. 842,765.
28. 344,616.

Page 83

21. 1,445,000.
22. 2,664,000.
23. 4,644,000.
24. 3,648,000.
25. 2,886,000.
26. 6,308,000.
27. 28,120,000.
28. 55,680,000.
29. 49,770,000.

Page 84, § 88

1. 4235.
2. 2115.
3. 2460.
4. 2420.
5. 1780.
6. 2260.
7. 1150.
8. 2175.
9. 1475.
10. 900.
11. 1175.
12. 300.
13. 3200.
14. 2800.
15. 2500.
16. 900.
17. $983\frac{1}{3}$.
18. 600.
19. 9200.
20. 32,800.
21. 20,650.
22. $12,833\frac{1}{3}$.
23. $15,566\frac{2}{3}$.
24. $9187\frac{1}{2}$.
25. 95,625.
26. 463,750.
27. 560,000.

Page 84, § 89

1. 6448.
2. 7416.
3. 9828.
4. 10,416.
5. 9512.
6. 9888.
7. 16,975.
8. 5076.
9. 4071.
10. 31,185.
11. 13,122.
12. 15,849.
13. 12,545.
14. 5728.
15. 9168.
16. 12,636.
17. 14,768.
18. 9423.
19. 14,375.
20. 22,572.
21. 120,032.
22. 106,092.
23. 93,972.
24. 125,349.
25. 286,708.
26. 359,424.
27. 609,444.
28. 527,058.
29. 601,195.
30. 463,353.

Page 85

1. 13,734.
2. 38,925.
3. 47,575.
4. 14,124.
5. 17,512.
6. 43,515.
7. 53,185.
8. 13,464.
9. 17,820.
10. 19,395.
11. 27,940.
12. 13,255.
13. 39,375.
14. 54,120.
15. 28,820.
16. 21,510.

Page 86

9. 60,710.
10. 1058.4.
11. 717.6.
12. 10,188.
13. 870.3.
14. 15,488.
15. 6132.
16. 592.2.
17. 960.7.
18. 1224.
19. 1146.
20. 679.
21. 729.
22. 129.

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19. .926.
20. .326.
21. 1.92.
22. .276.
23. .353.
24. 5.201.
25. 8.929.
26. 14.574.
27. 7.434.
28. 14.491.
29. 25.564.
30. 10.458.
31. 7.015.
32. 13.529.
33. 4.810.

Page 88

1. 14,025.
2. 9625.
3. 12,025.
4. 20,425.
5. 27,625.
6. 29,325.
7. 18,375.
8. 15,675.
9. 17,575.
10. 18,275.
11. 24,375.
12. 14,875.
13. 64,025.
14. 62,625.
15. 82,025.
16. 70,775.
17. 53,975.
18. 59,375.
19. 22,425.
20. 63,325.
21. 72,375.
22. 71,825.
23. 23,625.
24. 23,925.
25. 32,375.
26. 41,925.
27. 60,125.

Page 89, § 96

1. 1225.
2. 3025.
3. 9025.
4. 12.25.
5. 6.25.
6. 72.25.
7. 5625.
8. 20.25.
9. 13,225.
10. 15,625.
11. 182.25.
12. 306.25.
13. 119,025.
14. 75,625.
15. 27,225.
16. 38,025.
17. 42,025.
18. 1056.25.
19. 172,225.
20. 403,225.
21. 34,225.
22. 99,225.
23. 164,025.
24. 416,025.

Page 89, § 97

1. 62.56.
2. 230.76.
3. 157.04.
4. 62.912.
5. 69.12.
6. 27.3.
7. 65.8.
8. 624.48.
9. 1071.6.
10. 117.9.
11. 13.832.
12. 10.782.
13. 10.7232.
14. 7.054.
15. 3.7344.
16. .5052.
17. 7.0104.
18. 4.6158.
19. 6.7496.
20. .50736.
21. 5.625.
22. 5.1828.
23. 34.772.
24. 6.9544.
25. 6.1072.
26. .6768.
27. 3.264.
28. 25.08.
29. .612.
30. .2808.

Page 92

1. $596\frac{1}{4}$.
2. 564.
3. $309\frac{3}{5}$.
4. $842\frac{5}{8}$.
5. 820.
6. 546.
7. $748\frac{4}{5}$.
8. $715\frac{5}{7}$.
9. 504.
10. $302\frac{3}{4}$.
11. $316\frac{1}{4}$.
12. $712\frac{8}{11}$.
13. $750\frac{3}{5}$.
14. $785\frac{4}{7}$.
15. $903\frac{7}{15}$.
16. $714\frac{3}{8}$.
17. $895\frac{1}{7}$.
18. $723\frac{9}{13}$.
19. $562\frac{5}{9}$.
20. $722\frac{16}{19}$.

Page 96

31. 24,336.
32. 18,816.
33. $22,282\frac{1}{3}$.
34. $24,185\frac{1}{3}$.
35. 26,595.
36. 17,595.
37. 96,968.
38. 118,188.
39. 71,832.
40. 80,264.
41. 9648.
42. 16,704.
43. 15,552.
44. 35,370.
45. 42,120.
46. 48,070.
47. 51,480.
48. 17,358.
49. 14,036.
50. 20,295.
51. 23,320.

52. $17{,}517\frac{1}{2}$.
53. 8010.
54. $17{,}752\frac{1}{2}$.
55. $14{,}512\frac{1}{2}$.
56. $28{,}746\frac{2}{3}$.
57. 20,680.
58. $11{,}632\frac{1}{2}$.
59. $9982\frac{1}{2}$.
60. $11{,}687\frac{1}{2}$.

Pages 103–104

1. $63\frac{1}{2}$ ft.
2. 236 oz.
3. 605 qt.
4. 143 pt.
5. 21,792 sq. in.
6. 1003 sec.
7. 13,996 sec.
8. 1376 rd.
9. 7458 sq. rd.
10. 213 in.
12. 750 lb.
13. 90 sq. rd.
14. 280 rd.
15. 10 in.
16. 2 pk. 6 qt.
17. 8 sq. ft. 72 sq. in.
18. 2 qt. $3\frac{1}{5}$ gi.
19. 4 yd. 2 ft. 8 in.
20. 100 sq. rd.
21. 52 min.
22. 2 ft. $9\frac{3}{4}$ in.
23. 2 ft. $9\frac{3}{5}$ in.
24. 16 gal.
25. 10 yd. 5 in.
26. 1 hr. 12 min. 40 sec.
27. 4 sq. yd. 4 sq. ft. 136 sq. in.
28. 3 da. 17 hr. 40 min.
29. 263 rd. 3 yd. $1\frac{1}{2}$ ft.
30. 102 lb. 6 oz.
31. 35 sq. rd. 12 sq. yd. $1\frac{1}{4}$ sq. ft.
32. 81 rd. 3 yd. $2\frac{1}{2}$ ft.
33. 691 bu. 1 pk.
34. 5 da. 19 hr. 5 min.
35. 5 mi. 153 rd. $1\frac{1}{2}$ ft.
37. $\frac{2}{3}$ da.
38. $\frac{21}{32}$ bu.
39. .875 gal.
40. $.531\frac{2}{33}$ mi.

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1. 36 bu. 2 pk. 2 qt.
2. 29 lb. 7 oz.
3. 38 rd. 3 yd. $1\frac{1}{2}$ ft.
4. 4 yd. 1 ft. 10 in.
5. 2 bu. 1 pk. 6 qt.
6. 19 sq. yd. 6 sq. ft. 124 sq. in.
7. 24 yd. 1 ft. 8 in.
8. 378 ft.
9. 62 ft. 4 in.
10. 43 ft. 4 in.

Pages 106–107

1. 17 yd. 1 ft. 6 in.
2. 59 gal. 1 qt.
3. 26 hr. 6 min. 40 sec.
4. 99 lb. 12 oz.
5. 82 bu. 1 pk. 2 qt.
6. 93 sq. ft. 108 sq. in.
7. 123 gal. 1 qt. 1 pt.
8. 43 hr. 52 min. 55 sec.
9. 11 lb. $11\frac{1}{2}$ oz.
10. 14 gal. $\frac{1}{3}$ qt.
11. 3 hr. 21 min. 36 sec.
12. 14 bu. 1 pk. 3 qt. 1 pt.
13. 1 lb. $14\frac{3}{5}$ oz.
14. 5 ft. $2\frac{2}{3}$ in.
15. 7 gal. $\frac{3}{7}$ qt.
16. 16 min. $55\frac{3}{8}$ sec.
17. 154 rd. 3 yd. 2 ft.
18. 1 ft. 8 in.
19. 8 ft. $7\frac{1}{2}$ in.
20. 49 posts.
21. 12 posts; 33 cross pieces.
22. 535 ft. 5 in.
23. 1 ft. 11 in.
24. 8 ft. 5 in.
25. 6 yd. 2 ft.

Pages 113–115

1. $238.40.
2. $3.21.
3. $2.02.
4. 198.84 mi.
5. 30 ft. 8.7 in.
6. $101.57.
7. 136.79 km.
8. $32.18.
9. $1944.02.
10. 271.9 ft.
11. 300.74 yd.
12. 42.25 mi.
13. 29,009.65 ft.; 5.49 mi.
14. $2.48.
15. 67.7¢.
16. 59 ft., .67 in.
17. 55.476 gal.
18. 6340 qt.
19. 1873.9 lb.
20. 362,960 kg.; 800,181.6 lb.
21. 3.195 qt.
22. 36.14 mi.
23. 92.1 mi.
24. 7 hr. 11 min.
25. 3.06¢.

Pages 117–118

1. 10° 10′.
2. 25° 50′.
3. 9° 50′.
4. 36° 10′.
5. 42° 30′.
6. 2 hr.
7. 2 hr. 40 min.
8. 40 min.
9. 3 hr. 20 min.
10. 20 min.
11. 1 hr. 2 min.
12. 42 min.
13. 30 min. 40 sec.
14. 32 min. 40 sec.
15. 2 hr. 2 min. 40 sec.
16. 6 hr. 2 min.
17. 5 hr. 3 min.
18. 30°.
19. 60°.

20. 75°.
21. 90°.
22. 55°.
23. 22° 30′.
24. 92° 30′.
25. 153° 45′.
26. 27°.
27. 38°.
28. 28° 30′.
29. 49°.
30. C. 7 A.M.; M. 6 A.M.; P. 5 A.M.
31. 6 P.M.

Page 119

1. 160 A.
2. $5200.
3. 40 A.
5. $\frac{1}{4}$ sec.; $8960.
6. 40 A.
7. $5200.
8. $3840.

Pages 122–125

1. $230.40.
2. $36,666.67.
3. $192.
4. 1250.
5. $17\frac{1}{2}$¢.
6. $60 by front ft.
7. $64.80.
8. $115\frac{1}{5}$ T.
9. 180 bu.
10. $107.47.
11. 16.08 bu.
12. 7.346 A.
13. 2240 sq. ft.
14. 1532 sq. ft.
15. $8.55.
16. $17.28.
17. $70.20.
18. $27.36.
19. 528,000.
20. $16\frac{1}{4}$ A.
21. 264 sq. in.
22. $14\frac{9}{16}$ A.
23. 7050 sq. rd.
24. $83.20.
25. 107,375 sq. yd.
26. 229,375 sq. ft.

Pages 126–128

1. 2.546 ft.
2. 593.
3. 21.42 mi.
4. Add $\frac{1}{34}$.
5. 43.854 ft.
6. 916.3 ft. per min.
7. 199.
8. 327.
9. $16\frac{2}{3}$ in.
10. 288.
11. 31.416 ft.
12. 1.91 in.
13. $\frac{4}{9}$ as large.
14. $737.18.
15. $54.67.
16. 10,857.37 lb.
17. 209.1 sq. in.
18. 5890.5 sq. ft.
19. 1963.5 sq. ft.
20. 6283.2 sq. ft.

Pages 131–132

1. 28.
2. 58.
3. 92.
4. 56.
5. 83.
6. 52.
7. 73.
8. 67.
9. 99.
10. 87.
11. 54.
12. 97.
14. 532.
15. 547.
16. 636.
17. 746.
18. 869.
19. 2453.
20. 728.
21. 696.
22. 799.
23. 852.
24. 543.
25. 1319.
29. .75.
30. .96.
31. 6.498+.
32. .884+.
33. .943+.
34. 4.412+.
35. 28.721+.
36. .8.
37. .252+.
38. 43.959+.
39. 15.03.
40. .894+.
41. 1.41+.
42. 1.73+.
43. 2.23+.
44. 2.64+.
45. 3.16+.
46. 4.24+.
47. 4.89+.
48. 6.24+.

Pages 133–134

1. 52.6 ft.
2. 143.1 rd.
3. 2.9 ft.
4. 31.2 in.
5. 21.5 ft.
6. 13.5 rd.
7. 15.6 in.
8. 27.0 in.
9. Yes; 4.
10. 5.7 in.
11. 22.8 in.

Page 136

1. 67.99+.
2. 76.03+.
3. 93.52+.
4. 43.87+.
5. 38.44+.
6. 58.83+.
7. 30.62+.
8. 26.88+.
9. 25.17+.
10. 28.56+.
11. 27.16+.
12. 31.17+.
13. 8.72+.
14. 9.19+.
15. 6.83+.
16. 8.69+.
17. 5.93+.
18. 5.35+.
19. .62+.
20. .76−.
21. .97+.
22. .92+.
23. .87+.
24. .71−.

Pages 137–139

1. 60 ft.
2. 68 ft.
3. 57 ft.
4. 115 ft.

5. 42.426 ft.
6. 25.6 in.
7. 21.63 mi.
8. 223.6 rd.
9. 19 ft. nearly.
10. $33\frac{1}{3}$ yd.
11. 127.27 ft.
12. 207.385 ft.
13. 29 in. nearly.
14. .35 in.
15. 11.66 in.
16. 13.75 in.
17. 68.18 sq. in.
18. 20.78 in.
19. 43.3 sq. in.
20. .693 A.

Pages 140–142

1. 216 bd. ft.
2. 61,200 bd. ft.
3. 40; $60\frac{3}{4}$; $87\frac{1}{2}$; 121; 162; $211\frac{1}{4}$; $269\frac{1}{2}$; $337\frac{1}{2}$; 416; $505\frac{3}{4}$; $607\frac{1}{2}$.
4. 8 bd. ft.
5. $4\frac{1}{2}$ bd. ft.
6. 3 bd. ft.
7. 9 bd. ft.
8. 20 bd. ft.
9. 16 bd. ft.
10. 16 bd. ft.
11. $37\frac{1}{3}$ bd. ft.
12. 15 bd. ft.
13. 120 bd. ft.
14. 256 bd. ft.
15. 216 bd. ft.
16. $348\frac{3}{4}$ bd. ft.; $7.67.
17. $11.57.
18. 1920 bd. ft.
19. 664 bd. ft.; $18.59.
20. $471.50.
21. $\frac{1}{3}$ of area.
22. $76.80.
23. $\frac{1}{7}$ of area.
24. $28.42.
25. $16.13.
26. $26.
27. $207.36.
28. $58.37.
29. 29,376 bd. ft.

Pages 144–147

1. $1\frac{1}{3}$ cu. yd.
2. 5 ft.
3. 1224 bu.; 73,440 lb.
4. $248\frac{8}{9}$ cu. yd.; $87.11.
5. $95.41.
6. $33\frac{3}{4}$ cd.
7. $514\frac{2}{7}$ gal.
8. 329.175 T.
9. 1445 sq. ft.; 47.414 T.
10. 350 lb.
11. 41.568 cu. in.
12. 654.696 cu. in.
13. 720,000 cu. ft.
14. $8371\frac{7}{9}$ cu. yd.
15. 80.64 T.
16. 31,500 cu. ft.
17. $5\frac{5}{8}$ in.
18. 1.76 ft.
19. 4296.3 cu. yd.
20. 60.24 T.
22. 76 sq. ft.
23. 375 gal.
24. 614.4 bu.
25. $284\frac{4}{9}$ cu. yd.
26. 31.4 ft.
27. 269.0 ft.
28. 179.35 ft.
30. 540,000 cu. ft.

Pages 149–150

1. 16.36 qt.
2. 52.36 gal.
3. 4523.9 gal.
4. 646.27 bbl.
5. 3233.88 gal.
6. $136.07.
7. 1256.64 sq. ft.
8. $9.42.
9. .039 in.
10. .2146 of it.
11. 34.47 min.
12. 1.286 in.
13. 42.02 in.
14. 7833.6 gal.

Pages 150–151

1. 1910.09 sq. ft.
2. $33.43.
3. 213.75 T.
4. 96 T.
5. 31.17 ft.
6. 201 da.
7. 9.75 A.
8. 779.1 bd. ft.
9. 39¢.
11. 40; 63.

Page 154

1. 360 cu. ft.
2. 75 cu. ft.
3. 120 sq. ft.
4. 188.496 sq. in.
5. 150.7968 cu. ft.
6. 80.42 bu.
7. 58.9 bu.
8. 120.64 bu.
9. 384 cu. in.
10. 50.59 sq. in.
11. 41.568 cu. in.
12. 850.49 cu. in.
13. 108.57 bu.

Pages 156–157

1. 2144.67 cu. ft.
2. 201,062,400 sq. mi.
3. 130,289 tiles.
4. 1413.72 sq. ft.
5. 3619.12 sq. ft.
6. 402.12 sq. ft.
7. 148.48 lb.
8. 56.55 cu. in.
9. 52.63 lb.
10. 125.3 gal.
11. 5.236 T.
12. .301 lb.

Pages 157–161

1. $1166\frac{2}{3}$ cu. yd.
2. 73,440 lb.; 6560 lb.
3. $\frac{2}{3}$; 14.1372 cu. in.
4. 4771.3 gal.; 151.47 bbl.
5. 5513.5 gal.
6. 5.89 lb.
7. 14,100.48 gal.
8. 1151.5 gal.
9. 90.48 bu.
10. 9.27 gal.
11. $\frac{4}{9}$ in.
12. 214.5 bu.
13. 3.8 T.
14. 590.62 sq. ft.
15. 9047.8 gal.

16. The large one twice as much.
17. 544 bu.
18. 45.7 T.
19. 18.9 gal.
20. $1511\frac{1}{3}$ cu. yd.
21. 1080 cu. ft.
22. $154\frac{2}{7}$.
23. Nearly 191.
24. 296.6.
25. 15 in.
26. 11.1 in.
27. 279.154 gal.
28. .3794 of it.
29. 27.49 lb.
30. 6.827 in.
31. 13.857 in.
32. 9.4248 ft.
33. 37 in.
34. 3.527 ft.
35. 8.45 in.
36. 18.
37. 1,142,138.88 gal.
38. 11,388 gal. approx.
39. 1.6 squares.

Pages 162–166

1. $A = lw$.
2. 127.
3. $A = \frac{1}{2}hb$.
4. 96.
5. $C = \pi d$.
6. $C = 2\pi r$.
7. 50.2656.
8 175.9296.
9. $A = \pi r^2$.
10. 804.2496.
11. $A = \frac{\pi d^2}{4}$.
12. 201.0624.
13. $V = hlw$.
14. 350.
15. $V = bh$.
16. 96.
17. $V = \frac{1}{3}bh$.
18. 75.
19. Vol. of cyl.
20. 452.3904.
21. Sur. of sphere.
22. Vol. of sphere.
23. 17,157.3248.
24. 1809.5616.
26. 48.
27. 70.
28. 60.
29. 37.44.
30. 48.
31. 360 nearly.
32. 76 ; 195.
33. 108.1 lb.
34. 480 lb.
35. .25 in.
36. $\frac{1}{60{,}000}$ in.
37. 2.08 ohms.
38. 6487 ft.
39. 846.0 gal.

Pages 177–184

1. \$550 ; \$1237.50 ; \$220 ; 27 %.
2. \$33.29.
3. 97.15 lb.
4. \$25,447.50.
5. \$431.08.
6. 6.25 %.
7. $33\frac{1}{3}$ %.
8. Farms = 124.15 % of factories ; Factories = 80.55 % of farms ; Farms 55.39 % of total ; Factories 44.61 % of total.
9. 3212.48 % G. ; 16.85 % L. ; 41.59 % G. ; .97 % G. ; 18.55 % G.
10. 1788.76 %.
11. 1695.96 %.
12. War 23.65 % ; Navy 19.39 % ; Pen. 22.48 %.
13. 63.34 % ; 175.48 % ; 214.01 % ; 184.65 %.
14. 19.12 %.
15. \$147,330 ; 10.43 %.
16. 11.27 % ; 12.69 %.
17. 12 % ; 15.38 %.
18. 6.94 % ; 9.34 %.
19. \$163 ; \$156.48.
20. $3\frac{1}{3}$ %.
21. 38 % ; 62 % ; \$121,830.
22. 85.04 % ; \$4284.54.
23. \$12 ; \$3.60 ; \$4.05.
24. \$9.60 ; \$3.49 ; \$3.88.
25. 23.29 ¢ ; 23.29 ¢.
26. 5.41 %.
27. 4.11 %.
28. $12\frac{1}{2}$ %.
29. 20 % ; $16\frac{2}{3}$ %.
30. $37\frac{1}{2}$ % ; $27\frac{3}{11}$ %.
31. 33 ¢ ; 35 ¢.
32. \$375.
33. \$500.
34. \$300.
35. 13.82 %.
36. \$2252.17.
37. \$2860.05.
38. 16 %.
39. \$7200.
40. 20 %.
41. 20 % ; $16\frac{2}{3}$ %.
42. \$1905.
43. \$62.50.
44. \$17.28.
45. \$680.
46. \$3928.40.
47. 5 % ; $3\frac{4}{7}$ %.
48. 44 ¢ nearly.
49. $2\frac{7}{9}$ ¢.
50. \$68.

Pages 187–190

2. \$756.
3. \$51.84.
4. \$48.45.
5. \$13.76.
6. \$19.47.
7. \$62.05.
8. \$361.25.
9. \$229.16.
10. \$424.69 ; \$336.71 ; 79.28 % ; 44.22 %.

11. 50 % of it.
12. 30 % of it.
13. 30 % of it.
14. 60 % of it.
15. 50 % of it.
16. 30 % of it.
17. 50 % of it.
18. 70 % of it.
19. $825.
20. $1050.
21. $1260.
22. $900.
23. $970.
24. $1200.
25. $1800.
26. $682.50.
27. $895.
28. $3500.
29. 32.07 %.
30. 33.7 %; 27$\frac{1}{7}$%.
31. $37.80.
32. $510.72; $25.54.
33. $1.66; $4.16.
34. First; 50¢ over 2d; $2.50 over 3d.

Page 192

53. 42$\frac{2}{3}$ %.
54. 43 %.
55. 52 %.
56. 51 %.
57. 51$\frac{11}{14}$ %.
58. 62$\frac{1}{2}$ %.
59. 66$\frac{1}{4}$ %.
60. 58 %.
61. 52$\frac{1}{2}$ %.
62. 68 %.

Pages 192–195

3. $26.20.
4. $41.06.
5. $31.85; $32.50.
6. $73.75; $75.26.
7. $31.66.

Pages 197–201

1. 18.88 %; 26.03 %; 23.28 %.
2. 29.02 %; 19.65 %; 24.67 %; 25.25 %; 47.58 %; 26.97 %; 36.44 %; 38.16 %.
3. .38 %; .26 %; .43 %; .37 %.
4. 9.98 %; 7.51 %; 7.63 %; 8.56 %.
5. 47.69 %; 20.88 %; 31.43 %. Yes, sum = 100 %.
6. 18.87 %; 17.68 %; 21.47 %.
7. $1320; $1408.
8. 20 %; 33$\frac{1}{3}$ %.
9. $5.81; $6.23.
10. 3$\frac{1}{3}$ %; 1$\frac{2}{3}$ %; 17$\frac{2}{15}$ %.
11. 17.58 %; .53 %.
12. $157,220; 9.42 %.
13. 8.56 %; 10.7 %.
14. 11.44 %.
15. 20.77 %.
16. 11.84 %.
17. $18,159.60.
18. $676,344; $553,744; 35.52 %.
19. 57.76 %; 5.46 %; 2.20 %.
20. 35.88 %; 44.87 %; 12.09 %; 7.16 %.
21. 36.35 %.
22. 10.22 %.
23.
a. 1. $15,537.72.
2. $26,473.96.
3. $22,513.92.
4. $25,249.02.
5. $26,667.20.
6. $11,315.98.
b. 1. 26.96 %.
2. 26.96 %.
3. 29.29 %.
4. 29.22 %.
5. 29.28 %.
6. 22.51 %.
c. 1. $6772.37.
2. $14,411.56.
3. $12,673.32.
4. $15,158.60.
5. $15,686.95.
6. $3350 78.
d. 1. 11.75 %.
2. 14.67 %.
3. 16.49 %.
4. 17.54 %.
5. 17.23 %.
6. 6.66 %.
e. $127,757.80.
f. 27.75 %.
g. $68,053.58.
h. 14.78 %.

Pages 201–205

2. $414.90; $432.19.
3. .65; .5; .4; .6; .75.
4. $30.25; $30.94.
5. $42.96; $44.75.
6. $47.71; $48.41.
7. $80.28; $84.36.
8. $45.31; $46.83.
9. $31.63; $32.35.
10. $45.63; $48.67.
11. $109.46; $120.29.
12. $53.73; $61.23.
13. $87.42; $91.06.
14. $435; $464.
15. $769.36; $795.12.
16. $1071; $1275.
17. $1194.67; $1344.

18. \$1085 ; \$1116.
19. \$1312.50 ; \$1400.
23. $3\frac{4}{7}$ %.
25. $12\frac{1}{2}$ % ; 10 %.
26. $6\frac{1}{4}$ % ; 5 %.
27. $2\frac{1}{2}$ % ; 2 %.
28. 20 % ; 15 %.
29. $13\frac{1}{3}$ % ; 10 %.
30. $6\frac{2}{3}$ % ; 5 %.
31. $28\frac{4}{5}$ % ; $20\frac{4}{7}$ %.
32. $17\frac{3}{5}$ % ; $12\frac{4}{7}$ %.
33. 5 % ; $3\frac{4}{7}$ %.
34. $23\frac{1}{5}$ % ; $16\frac{4}{7}$ %.
35. 23 % ; $15\frac{1}{3}$ %.
36. 20 % ; $13\frac{1}{3}$ %.
37. $12\frac{1}{2}$ % ; $8\frac{1}{3}$ %.
38. 44 % ; $27\frac{1}{2}$ %.
39. 36 % ; $22\frac{1}{2}$ %.
40. 28 % ; $17\frac{1}{2}$ %.
41. 20 % ; $12\frac{1}{2}$ %.
42. $48\frac{3}{4}$ % ; $27\frac{6}{7}$ %.
43. 5 % ; $2\frac{6}{7}$ %.
44. 8 % ; $4\frac{4}{9}$ %.
45. \$74.36.
46. \$78.32.
47. \$123.93.
48. \$161.86.
49. \$130.53.
50. $9\frac{18}{23}$ % ; $13\frac{1}{23}$ %.
51. 20 %.
52. $14\frac{2}{7}$ %.
53. $36\frac{4}{11}$ % ; $26\frac{2}{3}$ %.
54. 19.05 % ; 9.6 %.
55. $8\frac{8}{29}$ % ; $13\frac{23}{29}$ %.
56. $41\frac{2}{3}$ % ; $24\frac{1}{6}$ % ; 35.44 % ; 19.17 %.

Page 206

1. \$3400.
2. \$3500.
3. \$12,500.
4. \$4000.
5. \$12,000.
6. \$11,200.
7. \$5700.
8. \$3300.
9. \$11,400.
10. \$3300.
11. \$9200.
12. \$6500.
13. \$16,200.
14. \$18,700.
15. \$22,000.
16. \$18,200.
17. \$9540.
18. \$3060.
19. \$3292.
20. \$8175.

Pages 206–209

1. \$18 ; 3.6 ¢.
2. \$7.20 ; 1 ¢ ; 65 ¢ ; $71\frac{3}{7}$ ¢.
3. \$250.
4. \$3437.50 ; \$125.
5. \$1687.50 ; \$75.
6. \$168,175 ; \$175.
7. \$5880.
8. \$17.50 ; $3\frac{1}{2}$ ¢.
9. \$233.86.
10. \$34.80.
11. \$390.15.
12. \$3306.03.
13. \$561.15.
14. \$40.55 ; 8.55 %.
15. \$327.20.
16. \$123.40.
17. \$520.90.
18. \$2403.97 ; \$379.97.

Pages 211–215

15. \$117.
16. \$113.20.
17. \$73.50.
18. \$192.50.
19. \$149.42.
20. \$52.62.
21. \$71.36.
22. \$132.73.
23. \$196.
24. \$73.13.
25. \$96.
26. \$33.50.
27. \$81.67.
28. \$165.63.
29. \$381.58.
30. \$464.10.
31. \$288.75.
32. \$216.
34. \$8.12.
35. \$21.78.
36. \$11.76.
37. \$9.69.
38. \$24.
39. \$11.56.
40. \$10.13.
41. \$92.50.
42. \$17.80.
43. \$19.64.
44. \$11.76.
45. \$50.40.
47. \$302.72.
48. \$48.75.
49. \$107.64.
50. \$121.50.
51. \$306.94.
52. \$42.47.
53. \$71.17.
54. \$46.04.
55. \$15.60.
56. \$33.33.
57. \$54.54.
58. \$127.44.
59. \$29.23.
60. \$45.33.
62. \$1528.43.
63. \$1006.24.
64. \$1793.04.
65. \$209.20.
66. \$2918.48.
67. \$272.
68. \$1605.88.
69. \$1590.
70. \$300.
71. \$170.
72. Gain \$95.
73. Gain \$15.

Pages 216–217

2. \$29.29.
3. \$47.79.
4. \$101.70.
5. \$54.85.
6. \$8.89.
7. \$4.94.
8. \$12.09.
9. \$36.75.
10. \$19.57.
11. \$93.09.
12. \$27.60.
13. \$10.38.
14. \$22.60.
15. \$7.52.
16. \$56.04.
17. \$51.52.
18. \$43.00.

19. $67.68.
20. $19.80.
21. $14.24.
22. $7.80.
23. $41.60.
24. $16.20.
25. $44.10.
26. $49.00.
29. $19.38.
30. $42.00.
31. $52.00.
32. $18.75.
33. $18.00.
34. $47.25.
35. $71.50.
36. $27.71.
37. $50.25.
38. $162.75.
39. $50.00.
40. $128.80.
41. $62.89.

Pages 217–218

3. $10.56.
4. $13.13.
5. $12.60.
6. $15.36.
7. $9.00.
8. $8.00.
9. $13.47.
10. $17.22.
11. $16.50.
12. $13.49.
13. $7.58.
14. $6.25.
15. $6.30.
16. $11.25.
18. $7.97.
19. $8.31.
20. $9.30.
21. $8.75.
22. $11.45.
23. $1.25.
24. $.83.
25. $5.19.
26. $4.06.
27. $8.44.
28. $9.33.
29. $13.37.
30. $10.13.
31. $7.44.

Pages 219–220

1. $864.27.
2. $1552.37.
3. $1516.62.
4. $3460.60.
5. $693.57.
6. $4345.71.
7. $4309.86.
8. $5311.32.

Page 226

1. $36.75.
2. $28.37.
3. $43.55.
4. $42.60.
5. $1568.
6. $1250.55.

Pages 228–231

1. 146 da.
2. 98 da.
3. 74 da.
4. 197 da.
5. 124 da.
6. 235 da,
7. 98 da.
8. $9.45.
9. $25.46.
10. $39.43.
11. $24.42.
12. $24.42.
13. $11.70.
14. $42.47.
15. $45.71.
16. $23.52.
17. $26.45.
18. $35.25.
19. $40.48.
34. $16.33; $858.67; $875.
36. Nov. 10, 1917.
38. $1477.50.
39. $1500 on Nov. 10, 1917.
41. Jan. 24, 1918; $15.63.
42. $30.75 each.
44. $1831.48.
45. $1830.19.
46. $2791.49.
47. $3776.50.
48. $1697.61.
49. $2430.48.
50. $3894.27.
51. $4403.67.
52. $1284.24.
53. $1689.30.
54. $3365.27.
55. $4684.84.
56. $12,350.
57. $2593.10.
58. $1020.93.

Page 232

2. $2.55.
3. $5.34.
4. $4.76.
5. $7.13.
6. $8.84.
7. $40.23
8. $77.47.
9. $109.20.
10 $161.97.
11. $90.16.
12. $2.90.
13. $5.44.
14. $6.64.
15. $16.56.
16. $17.71.
17. $19.69.
18. $19.23.
19. $6.53.
20. $7.58; $442.42.
21. $26.40; $933.60.
22. $55.18; $4244.82.
23. $33.92; $1816.08.
24. $41.17; $3758.83.
25. $52.42; $4197.58.

Pages 237–239

1. $1970.
2. $1243.75.
3. $21.65.
4. $1835.82.
7. $2765.
9. $151.14.
10. $69.36.
11. $114.17.
12. $197.45.
13. $100.31.
14. $80.97.
16. $1632.62.

Pages 242–245

2. $703.02.
4. $3198.96.
5. $13,266.50.
6. $155,822.52.
8. $9989.04.

9. $6848.09.
10. $9913.28.
11. $23,395.56.
12. $2182.47.
13. $1560.04.
14. $59,060.50.
15. $32,649.63.
16. $2393.85.
17. $10,579.79 more.
18. Simp. Int. $3750; comp. int. $6664.50.

Page 247

2. $12,810.89 for 10 yr., then $4801.92.
3. $41,179.38.
4. $96,871.07.
5. $423.29.

Pages 252–255

1. $100.
2. 7 %.
3. $7; $1050.
4. Stock.
5. $133.33.
10. $4331.25.
14. $1050.
16. $166.67.
17. $83.33.
18. $50; $133.33; $200; $250; $333.33; $500.
19. 6 %.
20. 5 %; $185,412.
21. $14,000; 6 %.
22. 4 %; $13,387.02.
23. $24.57 more.
24. W–O about ½ %.
25. $1762.50.
26. $1010.25.
27. $1570.50.
28. Stock $10.63.

Page 258

2. 4.81 %; 5.12 %; 5.29 %.
3. 1st makes 6.19 %; 2d makes 4.85 %.

Pages 260–261

1. $5200; $5000.
2. $7520; $8000.
4. Less.
5. More.
7. First.
9. $9720.
10. 40 bonds.
11. $1350.
12. $17,786.25.
22. About 4.8 %.

Pages 262–263

1. 1.8 %; $297.
2. 1.96 %; $699.72.
3. 1.85 %; $190.55.
4. 1¼ %; $1.25; $12.50.
5. $30.70.
6. $346.15.
7. $.75.
8. $2.00.
9. $1.75.
10. $1.25.
11. $1.47.
12. $.67.
13. $1.29.
14. $1.26.
15. $2.02.
16. $1.60.

Pages 264–266

1. $1380.
2. $2.55.
3. $766,990.50.
4. $6,996,050.40.
5. $7,317,885.
6. $18,667,266.
7. $75,874,-141.42.
8. $10.80.
9. $1.80 less; $16.20 more; $20.80 less.

Pages 266–267

2. $1970.
3. $100.
4. $470.
5. $2220.
6. $3320.
7. $5670.
8. $11,420.
9. $20,920.
10. $37,420.
11. $72,920.

Pages 269–270

1. $2000; $1562.50.
2. $12,000; $1200; $600.
3. $42.
4. ⅙; $2333.33.
5. $900; $1500.
6. $1440; $960.
7. $900; $1500; $900; $600.
9. $1200; $1800; $3000.
10. $500.
11. $62.22.
12. $63.36.

Pages 275–277

1. $9.51.
2. $848.59.
3. $320.23 more in insurance.
4. $3349.40, Ins.
5. $1216.37, Bank.
7. About the same as in Prob. 6.
8. Same as Prob. 7.
10. Same as Prob. 8.
11. Same.
12. Ins. $812.31.
13. Ins. $583.97.
14. Ins. $306.11.
15. Bank, $31.89.
16. Bank, $443.16.
17. Bank, $943.49.
18. Bank, $1552.25.
19. $1.64; $13.04.

Pages 278–284

1. 6 % ; $ 14,400.
2. $ 63.40.
3. $ 82.90 ; $ 79.25.
4. $ 101.04.
5. $ 1850.
6. $2\frac{1}{2}$ % loss.
7. $ 52.80 ; $ 55.
8. 7 %.
9. $ 21.
10. $22\frac{1}{2}$ %.
11. 20 %.
12. 70 %.
13. 52 %.
14. 40 % and 40 %.
16. $ 58.96.
17. $ 41.60 more.
18. $ 1736.76.
19. $ 37.17.
20. \$ $6.92\frac{3}{8}$ per share less.
21. $ 176.63 increase.
22. $ 405 less.
23. $ 1736.11.
24. $ 3000.
25. $2000; $4000.
26. 4.42 %.
27. $ 75 ; $ 80; $ 120 ; $ 135.
28. 16.92 % ; 7 % ; $ 1,148,420.
29. 14.66 % ; 8 %.
30. About 135.
31. $21\frac{7}{8}$ % ; $ 2031.25.
32. $ 1000 ; $ 1030.80 ; $ 1287.06.
33. $ 233.58.
34. $ 1634.88.
35. $ 645.81.
40. $ 12.08.
43. 5.44 %.
44. 12.63 %.
45. $33\frac{1}{3}$ %.
46. 10.71 %.
47. $ 34.80.
48. $ 872.72 ; $ 1454.54 ; $ 2327.27 ; $ 1200 ; $ 2000 ; $ 3200.
49. 1.75 %.
50. 26.32 %.
51. $ 275.93.
52. $ 682.75.

Page 286

22. $\frac{9}{25}$.
23. $\frac{4}{25}$.
24. $\frac{1}{27}$.
25. $\frac{12}{25}$.
26. $\frac{1}{4}$.
27. $\frac{3}{5}$.
28. $\frac{40}{51}$.
29. $\frac{9}{10}$.
30. $\frac{18}{17}$.

Pages 286–287

2. $ 187.50.
3. $ 141.18.
4. $\frac{3}{32}$.
5. $\frac{5}{8}$; $\frac{5}{3}$.
6. 50 %.
7. .7854 ; 1.9635 lb.
8. .9.
9. 8.377.
10. $\frac{2}{5}$; 25°.

Pages 288–290

1. 706.25 lb. ; 437.5 lb.
2. 843.75 lb. ; 537.5 lb.
3. 556.25 lb. ; 156.25 lb. ; 556.25 lb.
4. 11.17 oz. ; 6.08 oz. ; 12.43 oz.
5. 23.61 oz. ; 59.03 oz.
6. 251.56 lb.
7. 318.78 T.
8. No, it should weigh 536.6 g.
9. 3.
10. 1.58 oz.
11. 272 g.
12. 10.32 in.
13. 259.31 lb.
14. 43.87 oz.
15. 70,169.98 lb.
16. About 12,568 lb.

Page 292

1. 3.
2. $\frac{2}{3}$.
3. $3\frac{3}{8}$.
4. $13\frac{1}{3}$.
5. $31\frac{1}{2}$.
6. $13\frac{1}{3}$.
7. $5\frac{5}{8}$.
8. $11\frac{2}{3}$.
9. $3\frac{1}{5}$.
10. $4\frac{1}{5}$.
11. $7\frac{7}{8}$.
12. $7\frac{1}{3}$.
13. 6.
14. 10.
15. 9.
16. 15.
17. 16.
18. 12.
19. $\frac{1}{8}$.
20. 15,000.

Pages 292–293

1. $ 333.33 ; $ 533.33 ; $ 733.33.
2. $ 30 ; $ 40 ; $ 80.
3. $ 1000 ; $ 1000 ; $ 1125 ; $ 1375.
4. $ 4000 ; $ 12,000 ; $ 12,000 ; $ 32,000.
5. $ 900 ; $ 1200 ; $ 1500.

Pages 296–300

1. 160 lb.
2. 100 lb.
3. 144 lb.
4. 125 lb.
5. 96 lb.
6. 82.5 lb.
7. 12 lb.
8. 1507.97 lb.
9. 12,000 lb.
10. 42 lb.
11. $71\frac{3}{11}$ lb.

12. 280 lb.
13. $42\frac{1}{3}$ lb.
14. 2000 lb.
15. $\frac{25}{98}$.
16. $6666\frac{2}{3}$ lb.
17. 75 lb.; 90 lb.
18. $177\frac{1}{12}$ lb.
19. $1085\frac{4}{7}$ lb.
20. 3000 lb.
21. 80 lb.
22. 12 lb.
23. 2638.9 lb.
24. 14.9 lb.
25. 25,132.8 lb.
26. 35,185.92 lb.
27. 2.8 in.
28. 90,478 lb.; 226,195 lb.

Pages 301–304

1. 15 ft. and 18 ft.
2. 217 sq. rd. nearly.
3. 75.54 lb.
4. 524.28 lb.
5. 5000 ft.; 120 ft.
6. 68 ft.
7. 50 ft.; 108 ft.
8. 180 ft.
9. 66 ft.; 58.46 ft.
11. 16.5 in. by 22.5 in.
12. 3.72 in.
14. 855 mi.
15. 9 in.
18. $2\frac{17}{44}$ in.
19. $91\frac{1}{8}$ cu. cm.

Pages 306–307

1. 64 ft.; 144 ft.; 256 ft.; 400 ft.
2. 48 ft.; 80 ft.; 112 ft.
3. 1600 ft.; 304 ft.; 57,600 ft.
4. 64 ft.; 96 ft.; 320 ft.; 1920 ft.
5. 6 sec.
6. 3 sec.
7. 100 ft.
8. 144 ft.
9. 144 ft.; 324 ft.
10. 1024 ft.; 192 ft.
11. 192 ft.; 576 ft.

Pages 307–308

1. About $\frac{1}{2}$ sec.
2. 4.34 in.
3. About 22 in.
4. 5 38 sec.
5. .11 in.
6. Nearly .11 in.
7. .74 in.

Pages 308–309

1. $10\frac{5}{7}$ cu. yd.
2. 9.6 cu. in.
3. 1.361 g.
4. .5 qt.
5. 18 lb.
6. 150 lb.

Page 310

1. 4.
2. 10.24.
3. 60 in.
4. 16.
5. 1.64.
6. 1200.
7. About 52.